Vision Research Vol. 21, pp. 755 to 764, 1981
Printed in Great Britain

0042-6989/81/060755-10$02.00/0
Pergamon Press Ltd

A SIMPLE EXPLANATION OF THE INDUCED SIZE EFFECT

ARIES ARDITI, LLOYD KAUFMAN and J. ANTHONY MOVSHON
Department of Psychology, New York University, 6 Washington Place, New York, NY 10003, U.S.A.

(*Received* 28 *July* 1980; *in revised form* 8 *December* 1980)

Abstract—When a meridional lens is oriented axis vertical before one eye, the horizontal magnification produces a tilting of visual space in the third dimension which is predictable from the geometry of binocular parallax. When, however, the lens is oriented axis horizontal so that the magnification is vertical, a distortion of space occurs which is similar to the tilting caused by the presence of a lens oriented axis vertical over the other eye. It has commonly been believed that this phenomenon, known as the induced size effect, has no geometric explanation, and is an anomalous stereoscopic response to vertical disparities. An explanation is presented which accounts for the induced effect in terms of actual horizontal disparities between vertically magnified oblique lines, and disparities of the two-dimensional spatial spectra of the two eyes.

INTRODUCTION

When a weak positive cylindrical lens or a meridional magnifying lens is placed before one eye with its axis oriented vertically, the image of that eye is magnified in the horizontal meridian, and not in the vertical meridian. The resulting expansion of one eye's retinal image introduces binocular disparities, which in turn cause an apparent skewness of the viewed scene. Ogle (1938) referred to this type of distortion as the "geometric effect" since its occurrence is consistent with the geometrical conditions that normally lead to stereopsis. The underlying geometry is diagrammed in Fig. 1.

If the meridional lens is rotated 90 deg so that its axis is oriented horizontally rather than vertically, the resulting magnification is in the vertical meridion, and not in the horizontal meridian. Although the resulting image expansion of one eye's view is vertical, observers report a distortion of the scene similar to that of the geometric effect but in the *opposite* direction. For example, if the lens is worn over the right eye with its axis horizontal (producing vertical magnification), an objectively fronto-parallel plane appears tilted so that its left side is perceived as farther from the observer than its right side. Ogle (1938) called this phenomenon the "induced size effect" and believed it to be quite different from the geometric effect: In the induced effect, vertical expansion of one eye's image seems to induce a horizontal expansion of the other eye's mental image prior to the stereoscopic process.

The term "induced size effect" is due to one class of explanation for the phenomenon, an example of which can be traced to Lippincott (1889, 1917). Using weak cylindrical lenses, he attributed the effect to differences in apparent distance of objects imaged in the two eyes, brought about by the dioptric power added to one eye. A positive cylindrical lens, he believed, would tend to make an object appear closer because its power is convergent. This reduction in apparent distance will, by Emmert's Law, cause the object to appear smaller. Thus in this theory, the mental image of the eye with the vertically magnifying lens before it, undergoes an *overall* shrinkage due to size scaling prior to stereopsis. The horizontal component of this shrinkage brings about the disparities which cause the apparent tilt.

Ogle (1938) showed that the effect occurs even with lenses having no dioptric power (meridional afocal

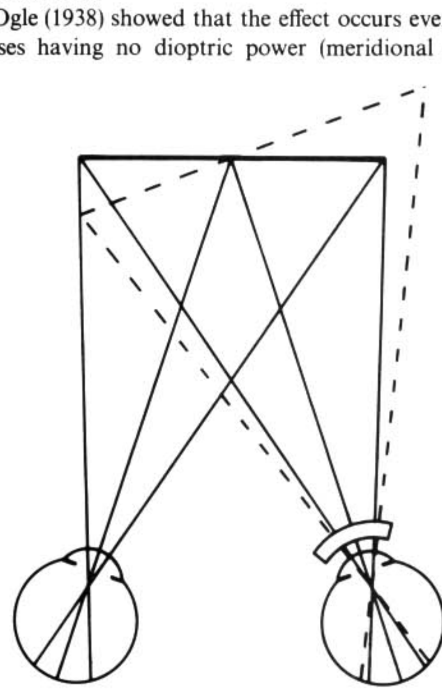

Fig. 1. Diagram showing the apparent tilt of a fronto-parallel surface produced by a meridional magnifying lens oriented axis vertical over the right eye.

lenses), and thus that Lippincott's explanation was wrong. While Ogle's name for the effect retains the idea of a size change in the mental representation of one half-image prior to stereopsis, his own explanation does not require such changes. Ogle's theory attributes the effect to the action of a hypothetical mechanism whose purpose is to compensate for naturally occurring magnification differences in the retinal images of objects viewed with asymmetric convergence at very close distances. Under such conditions, nearby objects are closer to one eye than the other, and because of this images of different overall size appear on the two retinae. In the interests of veridicality, the binocular visual system responds to the vertical disparities in a direction which offsets the apparent tilt which would otherwise result from the horizontal size differences between the two eyes. The introduction of a vertically magnifying lens over one eye, then, stimulates this compensatory mechanism alone.

As will be pointed out below, there are many other theories of the induced effect, but they are all conceptually similar to the theories proposed by Lippincott and Ogle. Moreover, they all share a common flaw: the inability to explain why the induced effect occurs only for certain kinds of stimuli and not for others. To illustrate this point, consider Fig. 2, which contains several variations on the familiar Wheatstone stereogram.

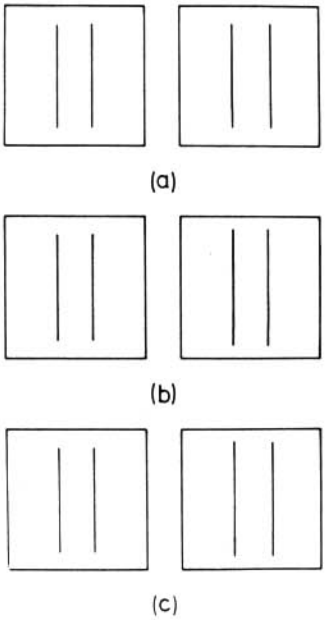

Fig. 2. Ordinary Wheatstone-type stereograms in which (a) the right half-image is horizontally expanded 10%; (b) the right half-image is vertically expanded 10%; (c) the right half-image is expanded 10% in both the horizontal and vertical dimensions.

Figure 2a is a normal Wheatstone stereogram in which the space separating the lines of the right half-field is 10% wider than that separating the lines of the left half-field. This disparity could have been produced by looking at two lines with both eyes while the right eye is wearing a meridional lens that magnifies its image by 10%. Therefore the depth produced by stereoscopically viewing this stereogram is an instance of the so-called geometric effect and is predictable from the known principles of stereopsis.

Figure 2b is a Wheatstone stereogram with no horizontal disparity. However, the right half-image is vertically expanded as in the induced effect. Even so, there is no depth between the lines when this stereogram is viewed in a stereoscope or when its half-images are combined in free stereoscopy. When the right half-image is expanded both vertically and horizontally as in Fig. 2c, the depth effect is the same as that obtained in Fig. 2a. Ogle's theory would predict the absence of depth since the opposing induced and geometric effects are both operative and would cancel each other. Yet there is not even a tendency toward reduction of depth in this stereogram. The theories of Householder (1943) and Julesz (1971), to be described later, can be criticized on similar grounds. Furthermore, there are no published theories which, to our knowledge, make specific predictions about which kinds of stereoscopic presentations will or will not produce the effect.

Despite the fact that the induced effect occurs only for some kinds of stimuli and not for others, the phenomenon is still of importance to theories of stereopsis because it reinforces the idea that non-depth-producing vertical, and horizontal stereoscopic fusional processes are fundamentally the same (e.g. Julesz, 1971; Nelson, 1975). Although some controversy surrounds the size and shape (Mitchell, 1966) and indeed the existence (Kaufman and Arditi, 1976a,b; Arditi and Kaufman, 1978; Duwaer and van den Brink, 1981) of the fusional areas, some investigators (e.g. Kertesz and Jones, 1970; Sullivan and Kertesz, 1975) claim that the areas have substantial vertical as well as horizontal dimensions.

Sheedy and Fry (1979) are the most recent proponents of the view that the fusional areas have appreciable vertical extent. Moreover, they make the claim that the induced effect itself is important evidence for the existence of sensory fusion and, *ipso facto*, the fusional areas.

The view of Sheedy and Fry is consistent with the wide-spread notion that sensory fusion underlies all instances of stereopsis, including the induced effect. The argument is roughly this: Sensory fusion (e.g. in a projection field) underlies stereopsis. Stereopsis occurs when horizontally disparate elements fuse. If stereopsis occurs when there is only vertical disparity, then the fusion of the vertically disparate elements must also underly the resulting stereoscopic effect. In essence, since sensory fusion is the basis of stereopsis, then sensory fusion of vertically disparate stimuli must also account for the induced effect. Indeed the impact of the requirement that the visual system respond selectively to vertical as well as horizontal disparities is illustrated by Nelson's (1975) theory which postulates neural mechanisms capable of accommodating and in principle computing depth from vertical disparities.

Certain constraints, of course, prevent vertical fusion from producing stereopsis. Since binocular parallax arises from the horizontal separation of the eyes in the head, virtually all of the retinal disparities are horizontal. Under all but the rarest of circumstances (e.g. asymmetric near convergence), there is no meaningful way to assign depth values to vertical disparities. Since vertical disparities *per se* cannot signify depth, theories of the induced effect usually invoke a secondary mechanism which produces horizontal disparities from processes such as the overall shrinkage of the mental representation of the vertically expanded retinal image of the lensed eye or the horizontal expansion of the other eye's image. Such mechanisms would, of course, require a neural configuration considerably more complex than one which processes disparities arising only from horizontal parallax.

In view of the complexities introduced by requiring the visual system either to process vertical disparities or accommodate to them in some other way so that stereopsis is induced, it is important to be clear as to the stimulus conditions that actually produce the induced effect. As pointed out earlier, not all vertical disparities lead to an induced effect. Consequently, we set out to analyze the stimulus conditions leading to the effect and arrived at the conclusion that it is not in fact caused by vertical disparity at all. If this conclusion is valid, it follows that theories of stereopsis need not incorporate mechanisms such as a change in the representation of the size of the image of one eye.

The theory proposed here is essentially geometrical. While Ogle (1950) and more recently Westheimer (1978) were aware of the fundamental geometrical relations incorporated in this theory, neither of these investigators chose to advance them as a full explanation. We do so here.

THEORY

An understanding of our theory requires classification of some basic geometrical consequences of vertical magnification of patterns containing oblique lines or obliquely oriented spatial frequency components. Therefore, we begin by considering the effect of vertically magnifying an oblique line as a result of placing an appropriately oriented meridional lens over one eye. The solid line in Fig. 3 represents a circle of unit radius, while the dashed line depicts its vertically magnified counterpart, an ellipse of major axis $2M$, where M is the vertical magnification factor. Also shown are lines drawn from the origin to points bisecting the arcs of the first quadrant of the circle and the ellipse.

Since the magnification is vertical, the maximum vertical disparity of the two lines is at their end points. The precise value of this disparity depends on M and also upon the lengths of the lines. Moreover, there is a gradient of vertical disparity of corresponding points along the lengths of the two lines ranging from zero at the origin to the maximum at the ends of the lines.

As already indicated, it is widely accepted that these vertical disparities produce the induced effect. What is less obvious is the fact that there are horizontal disparities between all points on the unmagnified line and the magnified line except at the end. Because the lines are of different length, there is no one-to-one correspondence between points on the lines of the two eyes' view. For this reason, it is difficult to decide if it is the vertically disparate or the horizontally disparate points which interact to produce the effect. The only purely vertical disparity is at the endpoints of the lines. At the end point of the unmagnified line there is a maximum horizontal disparity of length $'$ between that point and the point horizontally adjacent on the magnified line nearest to it. By similar triangles, $(v + v')/h = v/(h - h')$. Since $M = (v + v')/v$,

$$h' = h(1 - 1/M). \tag{1}$$

As noted in the introduction, for small magnifications the geometric and induced effects give approximately equal but opposite values of perceived tilt. One can seen from Fig. 3 that the horizontal disparity produced by vertical expansion of the line in one eye is roughly equal and opposite to the horizontal disparity which would be produced by horizontal expansion of the line in the other eye.*

Vertical magnification of oblique lines, then, produces horizontal disparity. According to our simple theory, it is this horizontal disparity which produces a depth effect equal in magnitude but opposite in direction to the depth which results from horizontal magnification of that same eye's image. Therefore, the induced effect is every bit as geometrical as the so-called geometric effect. Both effects result from horizontal disparities that are physically present.

It is now easy to see why the induced effect does not occur with the vertical lines of a Wheatstone stereogram. Vertical expansion of one half-field of the stereogram leads to no change in horizontal disparities. Such changes would occur if the lines making up the stereogram were oblique.

Thus far we have dealt only with the classic concept of point-for-point horizontal disparity. There are other useful ways to define disparity, however, such as when the retinal images are considered as composed of spatially extended harmonic components. Meridional magnification may produce differences in orien-

* To show this more rigorously, consider the ratio $h\dagger/h'$, where $h\dagger$ is the maximum horizontal disparity which would be produced if the line were *horizontally* magnified by M. Since $h\dagger = h(M - 1)$ and by equation (1), $h' = h(1 - 1/M)$, then $h\dagger/h' = \mathrm{M}$, after substitution and reduction of terms. Thus when M is close to 1, as it is under conditions of the induced effect the lengths $h\dagger$ and h' must be nearly equal.

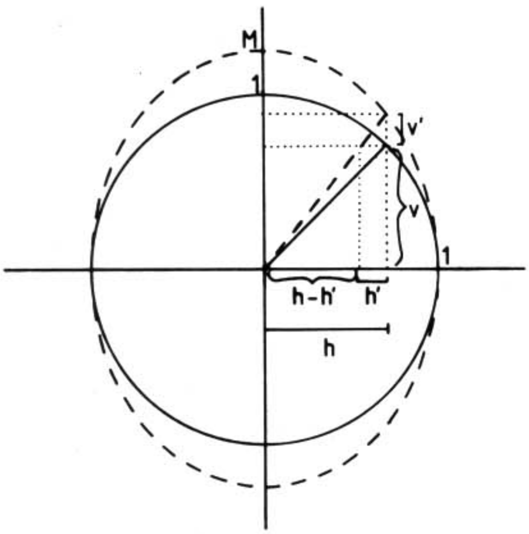

Fig. 3. Diagram illustrating classical point-for-point disparities which arise from vertical magnification of an oblique line by factor M. Lengths v and h are vertical and horizontal extents of the unmagnified line; v' and h' are maximum vertical and horizontal disparities between the unmagnified (solid) and magnified (dashed) lines.

tation of these components, and in their spatial frequency. Figure 4 illustrates this fact schematically. Obviously, all of the oblique components in any pattern will be affected similarly to the single component represented in the figure. Specifically, vertical magnification produces shifts in orientation towards the vertical (axis of magnification), and decreases in spatial frequency, of all oblique spatial components. Indeed, considerable support exists for the notion that disparities of orientation and/or spatial frequency are relevant cues to depth independent of point-for-point disparities, for certain stimuli (Blakemore, 1970; Fiorentini and Maffei, 1971; Blakemore *et al.*, 1972; Levinson and Blake, 1979; Braddick, 1979; Tyler and Sutter, 1979).

The extension of the theory to allow disparity processing of oblique harmonic components introduced by vertical expansion of one eye's image allows it to account for the induced effect in the absence of explicit oblique lines, and in scenes in which the obliques are masked or embedded within other or more salient components of the pattern, such as the leaf room used by Ogle (1950).

We now provide some demonstrations that verify the geometrical theory described above. These demonstrations produce strong induced effects when there are oblique lines present. However, stimuli in which there are no oblique lines but which do contain oblique harmonic components lead to somewhat weaker effects since the oblique components have relatively low energy as compared with other elements in the displays. For these stimuli we report the results of an experiment designed to test for the presence of the effect. We turn first to the demonstrations.

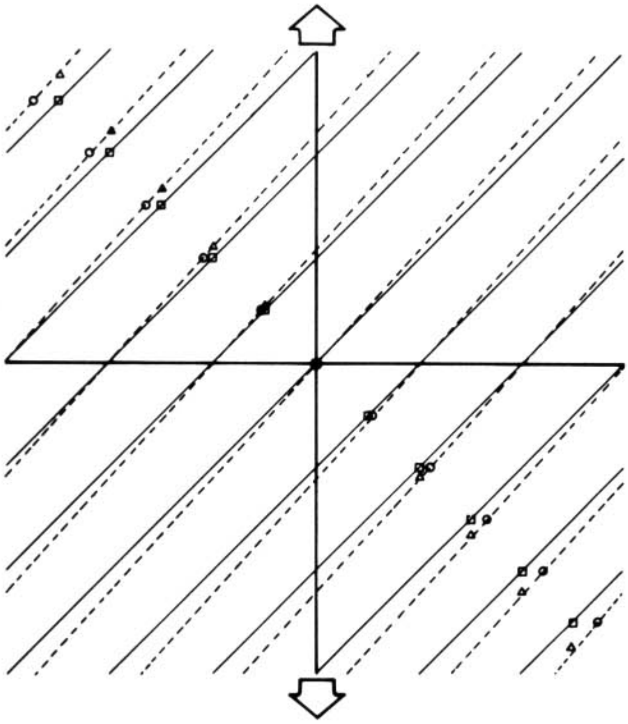

Fig. 4. Schematic representation of oblique spatial harmonic component. Solid lines represent peaks of unmagnified component, while dashed lines represent peaks of vertically magnified component. The arrows indicate the meridian of magnification. The difference between the squares and circles represents the horizontal disparity between peaks at that horizontal meridian, whereas the difference between squares and triangles represents the vertical disparity between peaks at that vertical meridian.

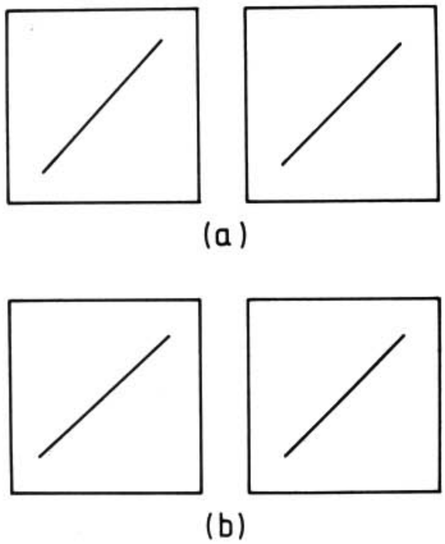

Fig. 5. Oblique line stereograms demonstrating (a) the "induced effect". (b) the "geometric effect". Magnification is 10%.

DEMONSTRATIONS

An "induced effect" may be observed by viewing the oblique lines of Fig. 5a. For comparison, a similar "geometric effect" stereogram is provided in Fig. 5b. When these stereograms are viewed in a stereoscope or with divergent eyes, the upper portion of the line in 5a and the lower portion of the line in 5b tilt away from the observer. Figure 5a was produced by vertically expanding the left half-field whereas in Fig. 5b the left half-field is expanded horizontally.

As already shown in the case of the Wheatstone stereogram, if the lines in Fig. 5a were perfectly vertical there would be no depth effect. Similarly, there would be no depth if the lines were perfectly horizontal because vertical expansion would not change the patterns. However, if the lines were horizontal and one of them was horizontally expanded, tilting would occur because of disparity at their end points.

As stated above, disparity of oblique spatial harmonic components makes it possible to predict the occurrence of the induced effect in the absence of explicit oblique contours. Patterns containing elements bearing an oblique relationship to one another often contain such components, such as the series of stereograms in Fig. 6. The right half-images are vertically expanded by a factor of 1.1 relative to the left half-images. Even though the dots can be resolved, the pattern serves as a sufficient stimulus to produce the induced effect. The Fourier transforms of such patterns show that they contain more energy at oblique orientations than they do at non-oblique orientations.*

An even more dramatic example of an induced effect in the absence of explicit contours is that which may be produced by viewing suitably constructed random-dot stereograms. The random dots in the two half-images of Fig. 7 are completely uncorrelated except for an inner circular region in the left half-field and an elliptical region with major axis vertical in the right half-field. The ellipticity is due to a 12% vertical expansion of the inner region. When this stereogram is viewed stereoscopically with divergent eyes the observer perceives a "circle" with the right side closer than the left. It is important to note that this stereogram too contains energy at oblique orientations, and that the effect still occurs despite the fact that the point-for-point disparities of the individual dots are purely vertical.

It is perhaps worth noting the similarity between this stimulus to one employed by Ogle (1938, 1950), consisting of a pane of glass splattered with ink. Viewed with a meridional lens over one eye, the pane

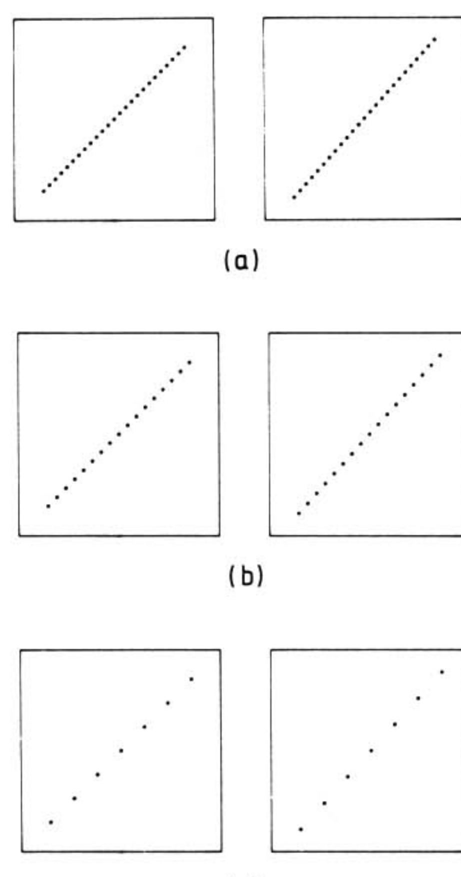

Fig. 6. A series of vertical magnification oblique "line" stereograms composed of dots, with varying degrees of line definition. Magnification is 10%.

* Not all patterns which contain oblique spatial relationships have Fourier energy at oblique orientations, however. One such counterexample is the physical sum of a vertical and a horizontal sinusoidal grating. Here the maxima and minima or spatial "beats" of the pattern occur in regular spacing along the diagonals. The authors have failed to see any tilt in such "sine-wave checkerboards" when they are vertically magnified in one eye; on the other hand, tilt is quite vividly experienced when the same patterns are rotated so that all of the Fourier energy is oblique.

Fig. 7. Random-dot stereograms in which the inner correlated region is expanded 12% in the right half-field.

appeared to be tilted when in fact it was frontoparallel.

Perhaps the simplest kind of stimulus in which there are no explicit contours but in which there is oblique spatial energy, is one composed simply of two dots. Figure 8a is a stereogram consisting of such a pattern, in which the right half-image is expanded horizontally relative to the left. Binocularly combining the two half-fields, of course, causes the lower dot to be perceived as nearer than the upper dot.

Figure 9 shows the two half-fields superimposed and schematizes three types of disparity that exist in the stimulus. τ is the spatial period of a low spatial frequency component in the left half-field, whose orientation is 45 deg. Note that the period of the corresponding component in the right half-image is $\tau + \Delta\tau$, and its orientation is 45 deg + θ. In fact, disparities of spatial frequency exist for all components in this pattern except those oriented horizontally; disparities of orientation exist in this pattern for all components save those near horizontal and vertical. Finally, ρ is the classic horizontal disparity of the dots, assuming fixation in the plane of the frame. The classic theory of stereopsis explains the percept of

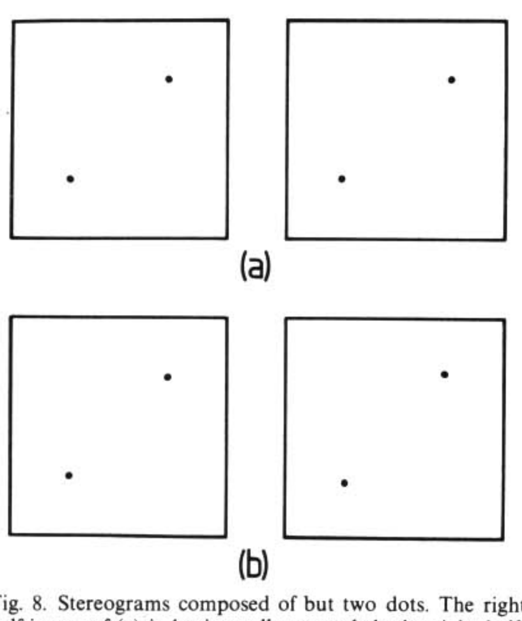

Fig. 8. Stereograms composed of but two dots. The right half-image of (a) is horizontally expanded; the right half-image of (b) is vertically expanded. Magnification is 10%. Prolonged viewing may be necessary to see depth in (b).

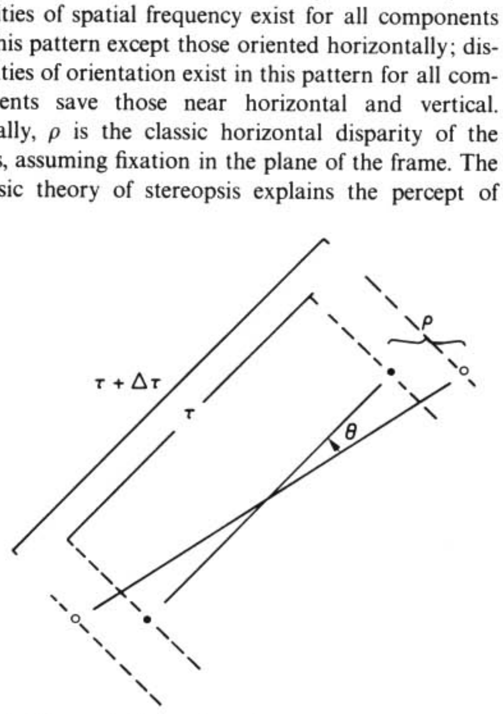

Fig. 9. Schematic illustration of the stereogram in Fig. 8a with the half-fields superimposed. The left half-image is represented by filled symbols; the right by open symbols. The solid lines indicate the orientation of the most prominent spatial component, and θ is the disparity of orientation between them. The dashed lines indicate the most prominent spatial component, and the lines τ and $\tau + \Delta\tau$ indicate its spatial period in the two half-fields. The ratio $\tau/(\tau + \Delta\tau)$ gives the frequency ratio for these components. For both stereograms in Fig. 8, $\theta = 4.0$ deg, and $\tau/(\tau + \Delta\tau) = 0.93$, ρ and its associated brace indicate the point-for-point horizontal disparity of each dot, assuming fixation in the plane of the frame.

depth by a computation of this horizontal point-for-point disparity.

Consider now the stereogram in Fig. 8b which is identical to that of 8a except that the right half-image is expanded vertically relative to the left half-image. Rotating Fig. 9 through 90 deg represents the superimposed half-fields of Fig. 8b. The disparities of both orientation and spatial frequency are the same here as in Fig. 8a except that they are reversed in sign. The classical horizontal disparity of visible elements—the dots—is absent.

Stereoscopically viewing Fig. 8b produces a depth effect similar in magnitude to that produced by Fig. 8a despite the absence of point-for-point horizontal disparity. To be sure, the depth effect occurs less easily but it is present. Although this is consistent with the finding that point-for-point disparity is not a necessary condition for stereopsis (Kaufman, 1964; Kaufman and Pitblado, 1965), this is the first demonstration that such an effect can occur when only two dots are employed as stimuli.

That the vertical point disparities are not themselves responsible for the impression of depth may be demonstrated by covering one of the two dots in each half-field: depth is no longer seen. Also, theoretically, cyclotorsional eye movements could cause the vertical sections of the frame to become disparate and thus induce an impression of relative depth of the dots, i.e. through "depth contrast" (Wallach and Lindauer, 1961). However the depth effect occurs in the absence of a frame, which the reader may verify by covering the outline of the square.

EXPERIMENT

Depth in Fig. 8b is not easily seen. Experienced stereo viewers have no trouble perceiving unambiguous depth but less experienced observers are less likely to see it. This may be due to the presence of conflicting cues to depth. For while the disparity of the oblique spatial components may signal the presence of depth, the vertical alignment of the dots (and absence of disparity of vertical spatial components) may signal the absence of depth. Because of the difficulty of seeing depth in such stereograms, we conducted a more rigorous test of depth discrimination for stimuli geometrically similar to those in Fig. 8b, with the frame omitted. Two of the authors and one experienced observer who was unaware of the purpose of the experiment or of the composition of the stimuli served as subjects.

The stimulus was presented as bright dots in an other-wise dark field on the face of a CRT by a computer. The dots of one half-field were oriented at 45 deg and, at the viewing distance of 152 cm, were separated by 2.1 deg. The dots of the other half-field were expanded vertically so that the vertical disparity of the corresponding dots in the corresponding dots in the two half-fields (ρ in Fig. 9) was 6.5 min arc. The expanded half-field was viewed by the right eye on half the trials and by the left eye on the other of the randomly intermixed trials. At the start of each trial the observer fixated a dot located midway between the positions at which the test dots would appear. Observer J.A.M. required the aid of a prism in viewing the two half-fields while A.A. and R.B. obtained bifoveal fixation by crossing their eyes. Following a button press, the stimuli replaced the fixation dot for 250 msec, and the observers had to decide whether the left- or right-hand dot appeared closer. No feedback was given. Correct responses, i.e. those that were consistent with the induced effect, indicated that the subjects could correctly perceive the cross of the disparity.

The probability of responding correctly was 0.90 for A.A., 0.80 for J.A.M. and 0.82 for R.B., over 50 trials for each observer. At longer exposure durations, all three observers approached perfect performance.

DISCUSSION

The demonstrations and experiment described above support an explanation for the induced effect which we believe to be much simpler than any previous theory.

Ogle's theory, as described in the introduction, relates the effect to a mechanism which compensates for the differential magnification of one eye's image during asymmetric convergence to a nearby object. Householder (1943) offered a similar explanation, noting that the image of any object, whether fixated or not, is larger in one eye than the other if it is closer to that eye. However, his theory ascribes a different functional significance to the effect; that is, it represents the utilization of the size difference as a cue for localizing the median plane. This view is somewhat more general than Ogle's since it recognizes the fact that naturally occurring interocular image-size differences may occur without asymmetric convergence, as when an object is nearer to one eye than another but some other object in the sagittal direction is fixated.

Both of these theories explain the induced effect by giving it a functional significance, but neither addresses the issue of how it might work. In fact, these theories require the visual system to solve a complicated problem in projective geometry. The degree to which there are natural interocular image-size differences depends upon the distance between the object and the interocular axis, the angle formed by the interocular axis and the line bisecting the visual directions to the object, and the vertical distance between the object and eye-level. More importantly, the incorporation of such explanations into viable models of stereopsis requires the existence of a mechanism which can alter the horizontal extent of one half-field *prior* to the occurrence of stereopsis. At the same time, the amount of alteration required depends on the occurrence of "fusion" or at least the computation of vertical disparity. Hence such theories require by logical necessity either feedback from ster-

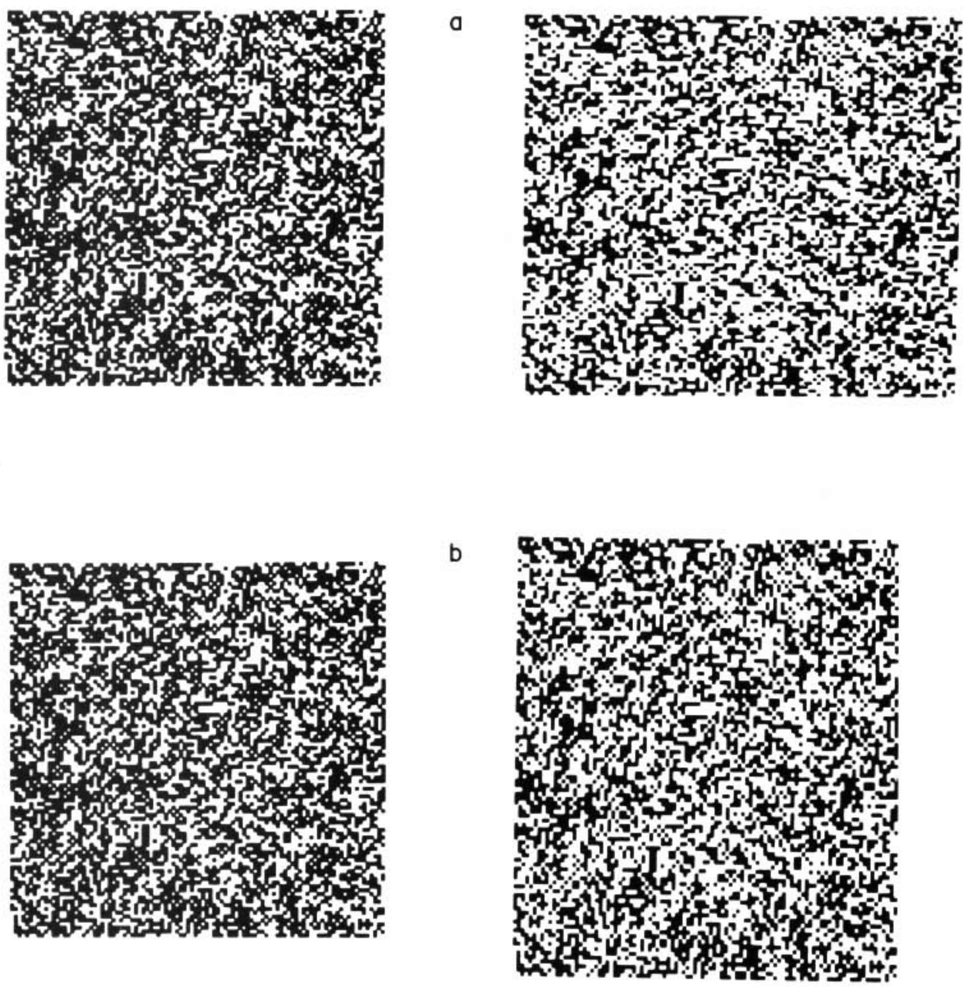

Fig. 10. See text.

eopsis mechanisms to prior monocular processing, or some system in which vertical disparities are computed prior to and in series with horizontal disparities.

Julesz (1971) constructed random dot stereograms with vertical, horizontal or overall magnification. His primary interest in these patterns was in the fact that they seemed to demonstrate plasticity of oculocentric visual directions in the cyclopean field. He obtained good "fusion" (stereopsis) even though one half-field was 15% larger than the other. Figure 10a shows one such pattern in which the right half-image is horizontally expanded by 15%. This is, of course, analogous to Ogle's geometric effect and results in a tilting forward of the left-hand side of the pattern. Figure 10b shows the same stereogram but with the right half-field expanded vertically by 15%. As in the induced effect, binocular viewing of this pattern results in a tilting forward of the right side of the pattern.

Julesz (1971) suggests a zooming mechanism which in effect matches the vertical dimensions of the half-fields of Fig. 10b. This alters the overall size of that half-field and gives rise to the tilt. Such a mechanism might be accomplished by Julesz' spring-loaded dipole model. Vertical expansion of one half-field causes vertical expansion of that eye's dipole array. This vertical "stretching" results in horizontal contraction of the same eye's dipole array, thus yielding the depth effect. The parallel to Ogle's theory is obvious; it proposes a modification of the monocular inputs to the stereopsis mechanism on the basis of a binocular comparison of the images.

Nelson's (1977) theory attributes the induced effect to cooperative processes in stereopsis. As in the theories of Sperling (1970) and Julesz (1971), Nelson assumes mutual facilitation of neurons tuned to the same disparity. He adds the assumption that the facilitation is independent of the orientation of the disparity; that is, facilitation occurs between detectors sensitive to the same amount of disparity whether it is measured horizontally, obliquely or vertically. Thus, vertical disparity detectors of say, +3 units, can facili-

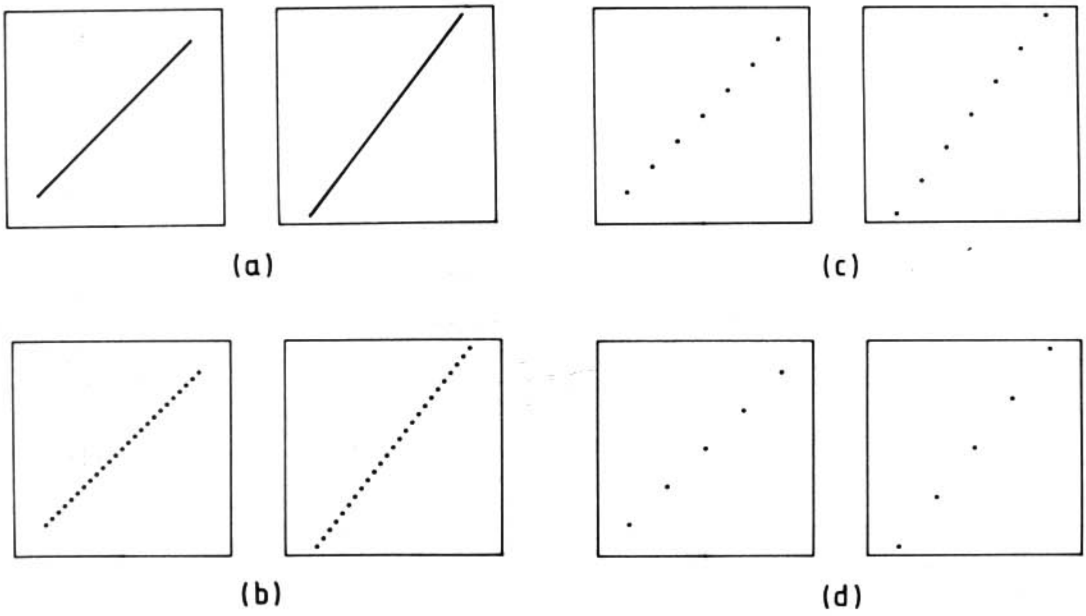

Fig. 11. Same as Fig. 6, but with 30% vertical magnification.

tate horizontal disparity detectors of +3 units to produce the induced effect.

One problem with this theory is that the sign of any given vertical disparity is inherently ambiguous since vertical disparities do not alter convergence angle of the eyes, and are thus unsigned. If the sign of vertical disparities were uniquely assigned, this theory would predict opposite directions of depth for the upper and lower portions of a stereogram with one half-field vertically expanded since the vertical disparities would be of opposite sign above and below the focus of expansion. Furthermore, it even fails to predict the observed left-right tilt since vertical disparities and thus any facilitated horizontal disparities are constant along any horizontal meridian.

Finally, Westheimer (1978) recently questioned the very existence of the induced effect. Using vertically expanded line stereograms rather than optically magnified textured surfaces, he found no evidence that the direction of depth could be determined by the eye in which the patterns were vertically expanded. He concluded that the effect is a higher-order cognitive phenomenon rather than one which is basically sensory in nature. In accordance with the present theory, however, he found slight but statistically insignificant depth discrimination for patterns which contained "implicit" diagonal contours (e.g. a square).

Before concluding, we must come to grips with a possible discrepancy between our observations those reported by Ogle. He found that beyond a certain magnification, typically about 8%, apparent tilt ceased to grow and often declined with higher magnifications. Using oblique lines, we have observed no such maximum in the function relating apparent tilt to vertical magnification. Ogle's primary stimulus was an ink-splattered pane of glass; hence we have no way of assessing the relative energy of harmonic spatial components in his patterns. But it is plausible that these components were weak and, with large magnification, were offset by other components that had no horizontal disparity under the conditions of the induced effect.

To explore this possibility we constructed stereograms similar to those of Fig. 6, but with higher magnifications. Figure 11a is an oblique line stereogram in which the right half-image is expanded 30%. The depth effect here is quite strong. Figures 11b through 11d are similar except that the lines are replaced by strings of dots, yielding oblique spatial energy without explicit lines. In the limit, this type of stereogram becomes the two-dot pattern of Fig. 8b. With magnification as large as 30% there is less and less of a depth effect as dot density increases. Relative to the energy at non-oblique meridians, the energy in the meridian of the diagonal is strongest in 11a and weakest in 11d. So we propose that the limit to the effect found by Ogle is not only due to the amount of differential magnification of the images in the two eyes but also to the strengths of the oblique Fourier components. When these components are sufficiently strong, as when they are produced by lines themselves or by relatively dense strings of dots, the effect occurs with very large magnification and is equal and opposite to the geometric effect.

The theory of the induced effect presented here requires no modification of the monocular images prior to neural binocular comparison, as implied by earlier theories. It accounts for the effect solely in terms of known principles of stereopsis which parsimoniously relate the phenomenon to horizontal, orientational or spatial frequency disparities. Such disparities are present whenever oblique lines or oblique harmonic components are present in the stimulus. The failures of the effect when large magnifications are employed may well be due to the attenuation of these oblique components. Thus, the term "induced effect" may well be a misnomer, since no induced disparities or size changes are needed. Moreover, the interpretation that vertical disparity can lead to stereopsis seems to be unfounded.

Acknowledgements—This research was supported by grants to J.A.M. from NSF (BNS 76-18904) and NIH (EY 2017) and by a United States Air Force contract to Robert Sekuler (AFOSR-79-0064). Some of the work described here was carried out at the Cresap Neuroscience Laboratory of Northwestern University. J.A.M. is an Alfred P. Sloan Research Fellow. We thank George Sperling and Bela Julesz for their insightful comments on an earlier version of this manuscript.

REFERENCES

Arditi A. and Kaufman L. (1978) Singleness of vision and the initial appearance of binocular disparity. *Vision Res.* **18,** 117–120.

Blakemore C. (1970) A new kind of stereoscopic vision. *Vision Res.* **10,** 1181–1199.

Blakemore C., Fiorentini A. and Maffei L. (1972) A second neural mechanism of binocular depth discrimination. *J. Physiol., Lond.* **226,** 725–740.

Braddick O. J. (1979) Binocular single vision and perceptual processing. *Proc. R. Soc. Lond. B* **204,** 503–512.

Duwaer A. L. and van den Brink G. (1981) What is the diplopia threshold? *Percept. Psychophys.* in press.

Fiorentini A. and Maffei L. (1971) Binocular depth perception without geometrical cues. *Vision Res.* **11,** 1299–1362.

Householder A. E. (1943) A theory of the induced effect. *Bul. Math. Biophys.* **5,** 155–161.

Julesz B. (1971) *Foundations of cyclopean perception.* Univ. of Chicago Press, Chicago.

Kaufman L. (1964) On the nature of binocular disparity. *Am. J. Psychol.* **77,** 398–401.

Kaufman L. and Arditi A. (1976a) The fusion illusion. *Vision Res.* **16,** 535–543.

Kaufman L. and Arditi A. (1976b) Confusion prevails! A reply to Kertesz and Sullivan. *Vision Res.* **16,** 551–552.

Kaufman L. and Pitblado C. (1965) Further observations on the nature of effective binocular disparities. *Am. J. Psychol.* **78,** 379–391.

Kertesz A. E. and Jones R. W. (1970) Human cyclofusional response. *Vision Res.* **10,** 891–896.

Levinson E. and Blake R. (1979) Stereopsis by harmonic analysis. *Vision Res.* **19,** 73–78.

Lippincott J. A. (1889) On the binocular metamorphopsia produced by correcting glasses. *Archs Ophthal.* **18,** 18–30.

Lippincott J. A. (1917) On the binocular metamorphopsia produced by optical means. *Archs Ophthal.* **46,** 397–426.

Mitchell D. E. (1966) A review of the concept of Panum's fusion areas. *Am. J. Ophthal.* **43,** 387–401.

Nelson J. (1975) Globality and stereoscopic fusion in binocular vision. *J. Theoret. Biol.* **49,** 1–88.

Nelson J. (1977) The plasticity of correspondence: aftereffects, illusions and horopter shifts in depth perception. *J. Theoret. Biol.* **49,** 1–88.

Ogle K. N. (1938) Induced size effect. I. A new phenomenon in binocular space perception associate with the relative sizes of the images of the two eyes. *Archs Ophthal.* **20,** 604–623.

Ogle K. (1950) *Researches in binocular vision.* Saunders, Philadelphia.

Sheedy J. E. and Fry G. A. (1979) The perceived direction of the binocular image. *Vision Res.* **19,** 201–211.

Sperling G. (1970) Binocular vision: a physical and a neural theory. *Am. J. Psychol.* **83,** 416–534.

Sullivan M. J. and Kertesz A. E. (1975) The nature of fusional effort in diplopic regions. *Vision Res.* **15,** 79–83.

Tyler C. and Sutter E. (1979) Depth from spatial frequency difference: an old kind of stereopsis? *Vision Res.* **19,** 859–865.

Wallach H. and Lindauer J. (1961) On the definition of binocular disparity. *Psychol. Beitr.* **6,** 521–530.

Westheimer G. (1979) Vertical disparity detection: Is there an induced size effect? *Invest. Ophthal. Visual. Sci.* **18,** 614–621.