Online Supplement

Bump-discrimination model

To characterize the pattern of results observed in our data, we fit a model to observers' trial-by-trial decisions in the bump-discrimination task. On every trial, the observer compared one bump with depth d_X under illumination condition X to another bump with depth d_Y under illumination condition Y. We assumed that the observer's estimate of bump depth ρ was a linear transformation of the actual bump depth d dependent on the illumination condition (Ho et al. 2006, 2007) and write the bump depth transformation functions as

$$\rho_X = L_X\left(d_X\right)$$
$$\rho_Y = L_Y\left(d_Y\right) \tag{1}$$

On each trial, these estimates are perturbed by normally distributed noise with zero mean,

$$\begin{aligned} D_X &= \rho_X + \varepsilon_X \\ D_Y &= \rho_Y + \varepsilon_Y. \end{aligned} \tag{2}$$

We allow for the possibility that the variance of the error depends on the magnitude of perceived bump depth, in a manner analogous to Weber's Law. Since our choice of a bump depth scale was arbitrary, we formulate a generalization of Weber's Law. We assume that the standard deviation of the error is proportional to a power function of the perceived bump depth level:

$$\varepsilon_X \sim N\left(0, \sigma^2 \rho_X^{2\gamma}\right). \tag{3}$$

Here, σ^2 is the variance when ρ_X equals one, and γ scales variance with bump depth. If $\gamma = 1$, then Weber's Law holds for our arbitrary bump-depth scale. If $\gamma = 0$, then variance does not depend on bump depth level.

We next assume that the observer forms a decision variable Δ on each trial to decide whether the bigger bump appeared in the first or second interval,

$$\Delta \quad = D_Y - D_X \quad = \rho_Y - \rho_X + \varepsilon, \tag{4}$$

where ε is normal with mean 0 and variance $\sigma^2 \left(L_X \left(d_X \right)^{2\gamma} + L_Y \left(d_Y \right)^{2\gamma} \right)$. The observer responds "second interval" if $\Delta > 0$, and otherwise responds "first interval".

We assume that the bump depth transformation functions are linear,

$$\begin{aligned} L_X(d) &= c_X d \\ L_Y(d) &= c_Y d\,. \end{aligned} \tag{5}$$

We define the *contour of indifference* to be the (d_X, d_Y) pairs such that $L_Y(d_Y) = L_X(d_X)$. These pairs are predicted to appear equal in bump depth to the observer under the corresponding illumination conditions. We refer to this contour as the *transfer function* τ_{XY} connecting the two illumination conditions *X* and *Y*,

$$d_Y = \tau_{XY}(d_X) = L_Y^{-1} \circ L_X(d_X) = \frac{c_X}{c_Y} d_X = c_{XY} d_X, \tag{6}$$

where c_{XY} is as defined above. Note that if $c_{XY} = 1$, the observer's judgments of bump depth are unaffected by a change of illumination. That is, the observer is bump-depth constant, at least for this pair of illumination conditions. We cannot directly observe $L_X(d)$ for any illumination condition *X* or estimate the constant c_X in the form of $L_X(d)$ we have assumed. However, we can estimate the transfer function parameter c_{XY} from our data. If bump-depth constancy holds, then $c_X = c_Y$ for any two illumination conditions *X* and *Y* and the value of c_{XY} should equal one.

A pseudocue model

We propose a pseudocue model to explore a possible mechanism driving the effects of visuo-haptic training found here. We begin by assuming that the observer bases judgments on noisy estimates of illuminant-invariant cues such as disparity D_d and the pseudocue directly manipulated here, the proportion of cast shadow D_s. Each is an unbiased estimate of its corresponding physical measure, e.g., $E\left[D_s\right] = d_s(d, \varphi_p)$. We assume that cues and pseudocues are scaled and linearly combined by a weighted average (Landy et al. 1995). In viewing a bump of depth *d* under illumination condition *X,* the observer forms the bump depth estimate

$$D = w_d D_d + w_s D_s \,, \tag{7}$$

where the values w_d and w_s combine the scale factors and weights and do not necessarily sum to 1 as typical weight values do. In the 2-IFC task used here, observers compared the bump depth estimate for one bump to another bump with depth d' under a different illumination condition *Y*,

$$D' = w_d D'_d + w_s D'_s, \tag{8}$$

to decide which bump depth was larger. The PSE represents the case in which $D = D'$. Subtracting Eq. 7 from Eq. 8 yields

$$0 = w_d \Delta D_d + w_s \Delta D_s, \tag{9}$$

Where $\Delta D_d = D'_d - D_d$ and $\Delta D_s = D'_s - D_s$. We assume that w_d was nonzero; therefore we can rearrange Eq. 9 as

$$\Delta D_d = a_s \Delta D_s, \tag{10}$$

where $a_s = -w_s / w_d$. We define $\Delta d_s = E\left[\Delta D_s\right] = d_s - d'_s$, and similarly for Δd_d. Eq. 10 expresses the tradeoff of the pseudocue and illuminant-invariant cues so as to maintain subjective equality. If we take the expected values of both sides of Eq. 10, we have

$$\Delta d_d = a_s \Delta d_s. \tag{11}$$

If an observer were bump depth constant across illumination conditions, we would expect the PSE bias across illumination conditions Δd_d to be 0, as in this case $d_d = d'_d = d$ (the physical bump depth). If not, Δd_d is the systematic deviation from the line of constancy for each test condition. Consequently, we can treat Eq. 11 as a regression equation,

$$\Delta \overline{d}_d = a_0 + a_s \Delta \overline{d}_s + \varepsilon, \tag{12}$$

where $\Delta \overline{d}_d, \Delta \overline{d}_s$ are the mean estimates of $\Delta d_d, \Delta d_s$ obtained from data and we have included a constant term a_0 so that we can directly test whether $a_0 = 0$ as expected; this value of the constant term corresponds to the assumption that two perfectly flat surfaces should be judged to have equal depth independent of illumination.

We are interested in comparing the weight of the pseudocue in the pre- and post-test sessions. We did not find any patterned deviation of values of $\hat{a}_0$ from 0, therefore we recomputed regressions, forcing $\hat{a}_0$ to be 0, separately for the data from the pre- and post-test sessions. This allowed us to compare the values of $\hat{a}_s$ between the pre- and post-test sessions.