2. Physical and Physiological Optics

I. Radiometry

Energy (erg = dyne cm; joule = 10^7 erg)
Power P (erg/sec, watt = joule/sec, horsepower, calorie)
Radiant flux, also *P* (watts, passing through an area of a surface coming from one side of the surface)
Irradiance E_e = radiant flux/area (watt/cm^2) = $\frac{dP}{dS}$ where *S* is area
Primary/Secondary sources
Solid angle ω (steradians) vs angle θ (radians)
Point sources

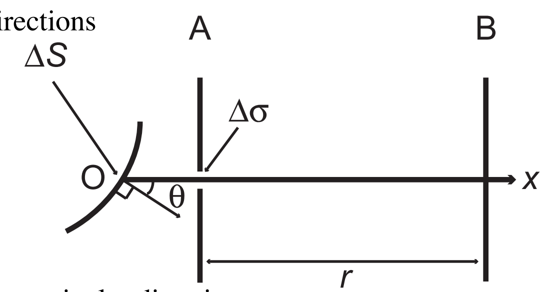
Radiant intensity $I_e = \frac{dP}{d\omega}$ (watts/steradian emitted in a particular direction)

Therefore, irradiance $E_e = I_e \cos\theta / r^2$ on a surface situated along that direction

Extended sources (the sky, a piece of paper, a monitor screen)

Radiant emittance $M_e = \frac{dP}{dS}$ = watt/cm^2 emitted in all directions

Next, consider the case when the source is a surface. Plane A is close to the source, and has a small hole of size Δσ small enough to be treated as a point source. The observer is at plane B a large distance *r* away. Thus, as seen from B, the hole subtends solid angle $\Delta\omega = \Delta\sigma / r^2$. The observers sees a patch of area Δ*S* of the surface through the hole.



Radiance L_e = watt/cm^2/steradian emitted by a patch in a particular direction

$= dI_e / d\sigma$ = radiant intensity per area of hole

$= dE_e / d\omega$ = received irradiance per solid angle of source (i.e., per steradian)
(because $\Delta E_e = \Delta I_e / r^2$ and $\Delta\omega = \Delta\sigma / r^2$)

Lambertian (source/surface), cosine law, here's the derivation:

$\Delta\sigma = \Delta S \cos\theta$

$\Delta P = \int_\omega L_e \Delta\sigma d\omega$

$M_e \Delta S = \int_\omega L_e \Delta\sigma d\omega$

$M_e = \int_\omega L_e \cos\theta d\omega$

Change integration to annuli around the normal, and for Lambertian surface, L_e is independent of θ:

$d\omega = 2\pi \sin\theta d\theta$

Hence: $M_e = \pi L_e \int_0^{\frac{\pi}{2}} \sin 2\theta d\theta = \pi L_e$

Light as a wave

speed of light $c \approx 3 \times 10^{10}$ cm/sec
wavelength $\lambda = c / \nu$ (in nm = 10^{-9} meters)

(visible = blue 400 to red 700)

Light as particles
- Photons, each of energy $h\nu$ ($h = 6.6245 \times 10^{-27}$ erg sec; ν = cycles/sec)
- Expected number of quanta = energy/energy-per-photon

Spectral content
- Spectral power distribution (e.g. M_e per λ)
- Spectral reflectance function
- Spectral transmittance function
- Spectral absorption function

Blackbody radiators, color temperature, standard illuminants

II. Photometry

Radiance vs luminance
- Matching
- Flicker photometry
- MDB, Motion, etc.

CIE standard observer (V_λ, V_λ' based on light level and target size)

Luminance $L = K_m \int V_\lambda L_{e\lambda} d\lambda$ vs radiance

Choice of K_m leads to the candle/candela (*luminous intensity*) = lumens/steradian

Luminous flux (lumen = candela steradian), which is the photometric unit analogous to the watt, lumens/watt is a measure of efficiency

Illuminance or *Illumination* (phot = lumen / cm^2 = 10000 lux; lux = lumen/m^2 = meter-candle; lumen/ft^2 = foot-candle = 10.764 lux)

Luminous emittance = Illuminance or Illumination for emitted flux

Luminance (candela/m^2 = nit, etc.)

Measures for spots and lines

III. Optics and the Eye (more on this next week)

Reflection

Refraction (index of refraction $n = c/v$, Snell's law: $\sin\theta/\sin\theta' = n'/n$)

Dispersion

Diffraction

The eye
- Anatomy
  - Cornea, iris, lens, aqueous, vitreous, retina, pigment epithilium
  - Optic nerve/blind spot, fovea, rod-free area, macula
- Visual angle
- Eccentricity
- Pupil
  - Dynamic range
  - Speed
  - Retinal illuminance (LS, in trolands), where L is in cd/m^2 and S is in mm^2
  - Stiles-Crawford effect, effective pupil area

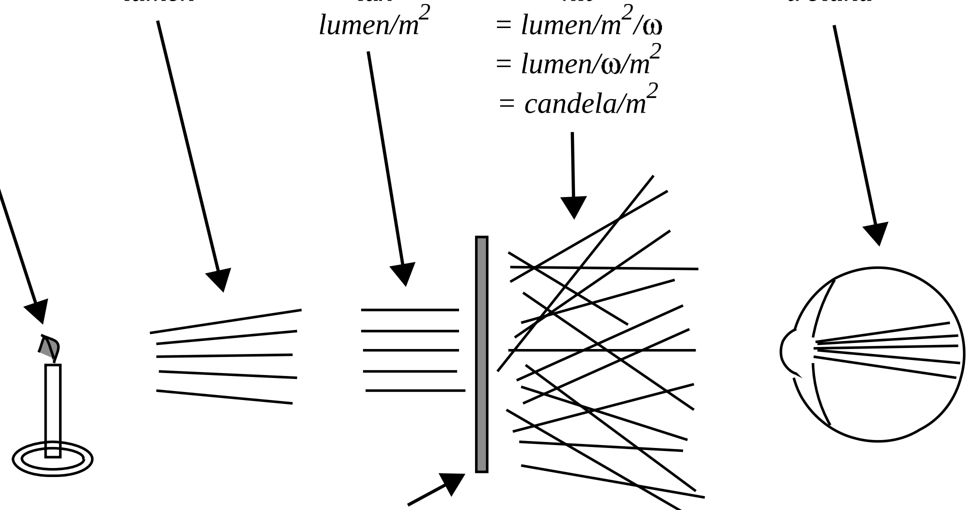
Contrast
Weber ($C_w = (L - L_{bg})/L_{bg}$)
Michelson
$(L_{\max} - L_{\min})/(2L_{mean})$
$(L_{\max} - L_{\min})/(L_{\max} + L_{\min})$
RMS (root mean square)
$\frac{1}{A}\int C_w(x,y)dx\ dy$
Weber's law, Rose-de Vries (square root) law

IV. Stimulus Generation

Maxwellian View
Monitors
Raster scan, rates, interlace
Calibration issues
Phosphor nonlinearity (gamma)
Phosphor time constant
Geometric distortion
Video amplifier bandwidth
Spatial nonhomogeneity
Gun independence
Phosphor constancy (spatially and across intensity)
Phosphor additivity
LCD displays
MEMS projection displays
Laser interferometry

Photometric Units

Luminous Intensity of the source	**Luminous Flux** (total output of the source per unit of time)	**Falling flux Illuminance**	**Emitted flux Luminance**	**Falling flux Retinal Illuminance**
candela	*lumen*	*lux*	*nit*	*troland*
lumen/ω		*lumen/m*^{2}	= *lumen/m*^{2}*/ω* = *lumen/ω/m*^{2} = *candela/m*^{2}	



Some calculations:

(a) candelas $\times$ $1/d^2$ $\times$ cos(angle of incidence) = lux

lumen/ω $\times$ 1/m^2 $\times$ ω = lumen/m^2

(b) lux $\times$ 1/π = nits

lumen/m^2 $\times$ 1/ω = lumen/ω/m^2

(c) nits $\times$ A = trolands, where A is the area of the pupil in mm^2

Notes: d = distance in meters

A = pupil area in mm^2

π = 3.14159

Eq. (b) is for a perfect diffusing and reecting (or transmitting) Lambertian surface. For white paper, multiply the result by about .95.

Synonyms:

lux = meter-candle, nit = candela/m^2
For a uniform point source: candelas $\times$ 4π steradians = lumens
1 radian = 59.3 degrees

Illuminance

The number to multiply the number of Lux to get:

meter-candle	1
milliphot	.1
phot	.0001
foot-candle	p = .0929

Note that $p = \left(\frac{12\text{inches/ft}}{39.4\text{inches/meter}} \right)^2 = .0929\,m^2/ft^2$

Luminance

The number to multiply Nits to get:

Equivalent lux, apostilb or blondel	$3.14 = \pi$
Candela per square meter	1
Millilambert	$.314 = \pi/10$
foot-lambert, equivalent foot-candle	$.292 = \pi\, p$
candela per square foot	$.0929 = p$
lambert, equivalent phot	$.00314 = \pi/10^4$
stilb, candela/cm^2	.0001

Obsolete system

Illuminance = candela/ft^2
The resulting luminance (from a perfect surface) yields foot-candles = foot-lamberts
Thus, foot-lamberts times pupil area (in mm^2) times 0.292 = Trolands

Conversion Factors

(1) Luminance (Photometric Brightness) Conversion Factors

1 nit = 1 candela/m^2
1 stilb = 1 candela/cm^2
1 apostilb (international) = 0.1 millilambert = 1 blondel
1 apostilb (German Hefner) = 0.09 millilambert
1 lambert = 1000 millilamberts

Multiply Number of → To Obtain Number of ↓	Footlambert By	Nit	Millilambert	Candela/in^2	Candela/ft^2	Stilb
Footlambert	1	0.2919	0.929	452	3.142	2,919
Nit	3.426	1	3.183	1,550	10.76	10,000
Millilambert	1.076	0.3142	1	487	3.382	3,142
Candela/in^2	0.00221	0.000645	0.00205	1	0.00694	6.45
Candela/ft^2	0.3183	0.0929	0.2957	144	1	929
Stilb	0.00034	0.0001	0.00032	0.155	0.00108	1

(2) Illumination Conversion Factors

1 lumen = 1/673 lightwatt
1 lumen-hour = 60 lumen-minutes
1 footcandle = 1 lumen/ft^2
1 watt-second = 10^7 ergs
1 phot = 1 lumen/cm^2
1 lux = 1 lumen/m^2 = 1 meter-candle

Multiply Number of → / To Obtain Number of ↓	Footcandles (By)	Lux	Phot	Milliphot
Footcandles	1	0.0929	929	0.929
Lux	10.76	1	10,000	10
Phot	0.00108	0.0001	1	0.001
Milliphot	1.076	0.1	1,000	1

Exercises

(1) A subject is seated 2 meters from a CRT display screen. The screen uniformly and diffusely reflects 0.5 of the incident light. The screen is illuminated by a standard candle at a distance of 1 meter. A bright disk on the screen is made to appear continuously present by a computer program, and its luminance (measured in the dark) is 0.02 cd/m^2. The diameter of the disk is 10 mm, and it is viewed through a 2 mm diameter artificial pupil.

(a) What is the retinal illumination of the background (the CRT surface)?
(b) What is the retinal illumination of the disk?
(c) What is the contrast of the disk?
(d) What is the visual angle subtended by the disk?

(2) Given: A standard candela (point source), a large piece of opaque gray paper with albedo (reflectance factor) 0.64 which reflects diffusely (i.e. is a Lambertian surface), a ruler and scissors, and an observer. Show how you could produce a disk stimulus subtending 1 deg of visual angle, viewed by the observer at a distance of 1 meter, and which produces a retinal illumination of 1 troland through a 1 mm artificial pupil.

(3) Given: the same materials as in problem (2), plus a can of dark paint with a brush and thinner that enables you to paint the paper any arbitrarily darker shade of gray, a flat wall surface, glue. Show how you would produce a square wave grating on the wall to be viewed in the left half of the visual field. The grating frequency is to be 0.1 cycles per degree, four cycles are required, the contrast modulation depth is to be 0.1, the mean luminance is to be 0.1 nit, and the observer is to be 1 meter from the wall.

(4) A careless graduate student made a 25 mm diameter finger-smudge in the middle of the department's large, perfect reflecting-diffusing plate. The plate reflects 1.00 of incident light but now only 0.95 in the smudge area. The professor is doing a luminous calibration. He is holding the department's standard candle. The candle and the professor are both 4 meters from the plate. The professor peers at the plate with his pupils dilated (5 mm diameter pupil). Are the pupils dilated because s/he sees the smudge?

(a) Describe the professor's retinal image (Trolands, visual angle, contrast).
(b) A student carries the candle closer. Does this improve the professor's visibility of the smudge? Explain.
(c) The candle is fixed and the professor comes closer. How does this affect visibility? Explain.

(5) A reading specialist wishes to study the rate at which the New York Times can be read aloud in dim light. His subjects are fitted with 4 mm pupils and seated 0.5 meter away from the newspaper. The white part of the paper diffusely reflects 0.8 of the incident flux; s/he wishes this to have a retinal illumination of 1 troland. S/he has a 25W bulb (≈ 300 lumens) available for this purpose. Assume the bulb emits light equally in all directions.

(a) How far from the newspaper should s/he place the bulb.
(b) New York Times print typically has about 7 letters per cm. What is the number of letters in words that just fit into the one-degree area of the subject's sharpest foveal vision?

(6) A 1 degree diameter disk has a luminance of 1 cd/m^2 and consists of 550 nm monochromatic light. It is viewed through a 2 mm diameter pupil. What is the average number of quanta/second arriving at the retina (assuming no absorption in air, cornea, lens, or ocular media)?.