Decoding

Statistical Analysis and Modeling of Neural Data

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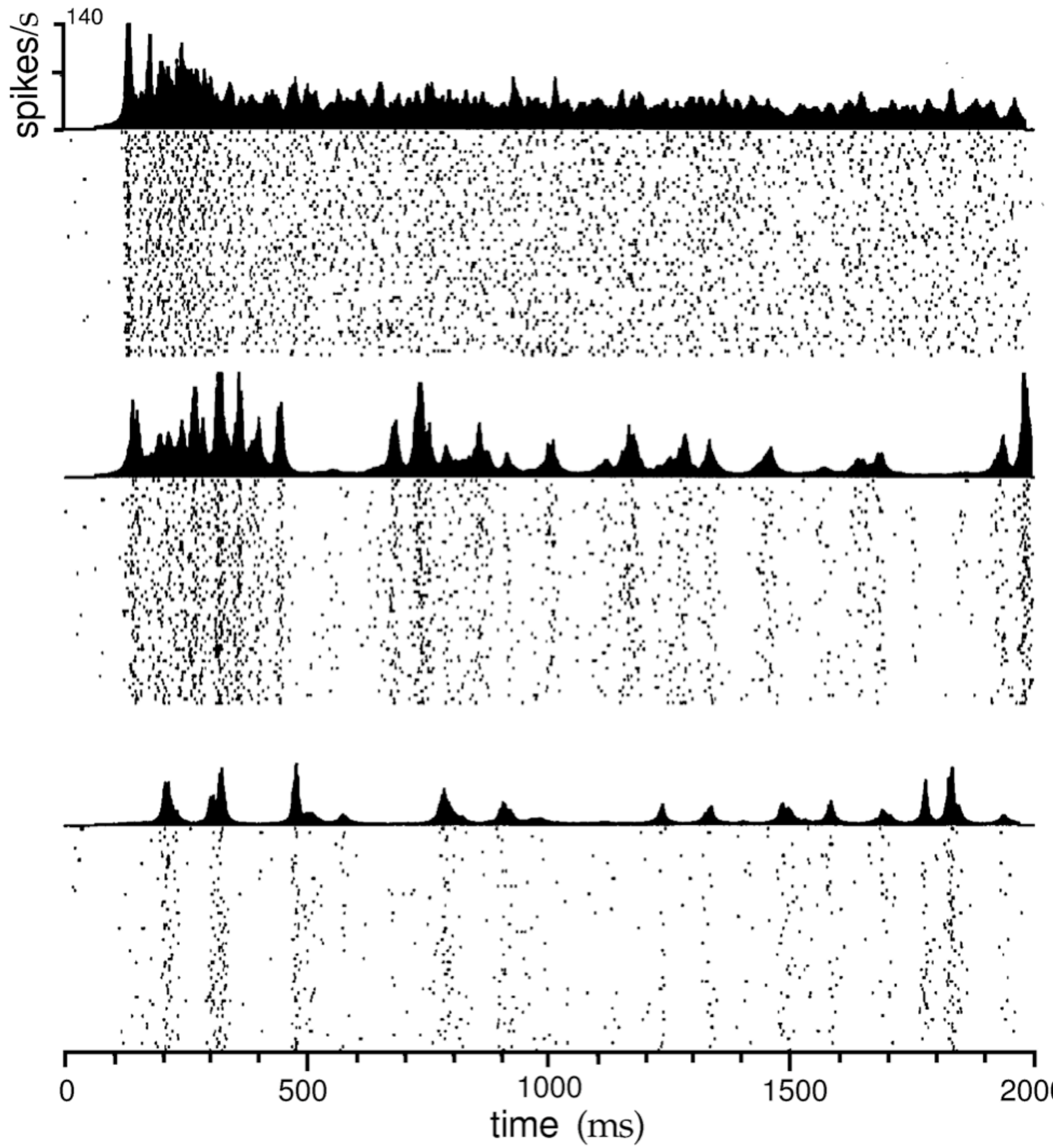
the scientist's perspective

P(spikes | stim)



the organism's perspective

P(stim | spikes)

The organism receives sensory responses, and must make judgements about the stimulus, remember it, or act on it.

the organism's perspective

P(stim | spikes)

[the homunculus, from "Men in Black"]

the organism's perspective

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Probabilistic description

- P(spikes | stim) : forward model

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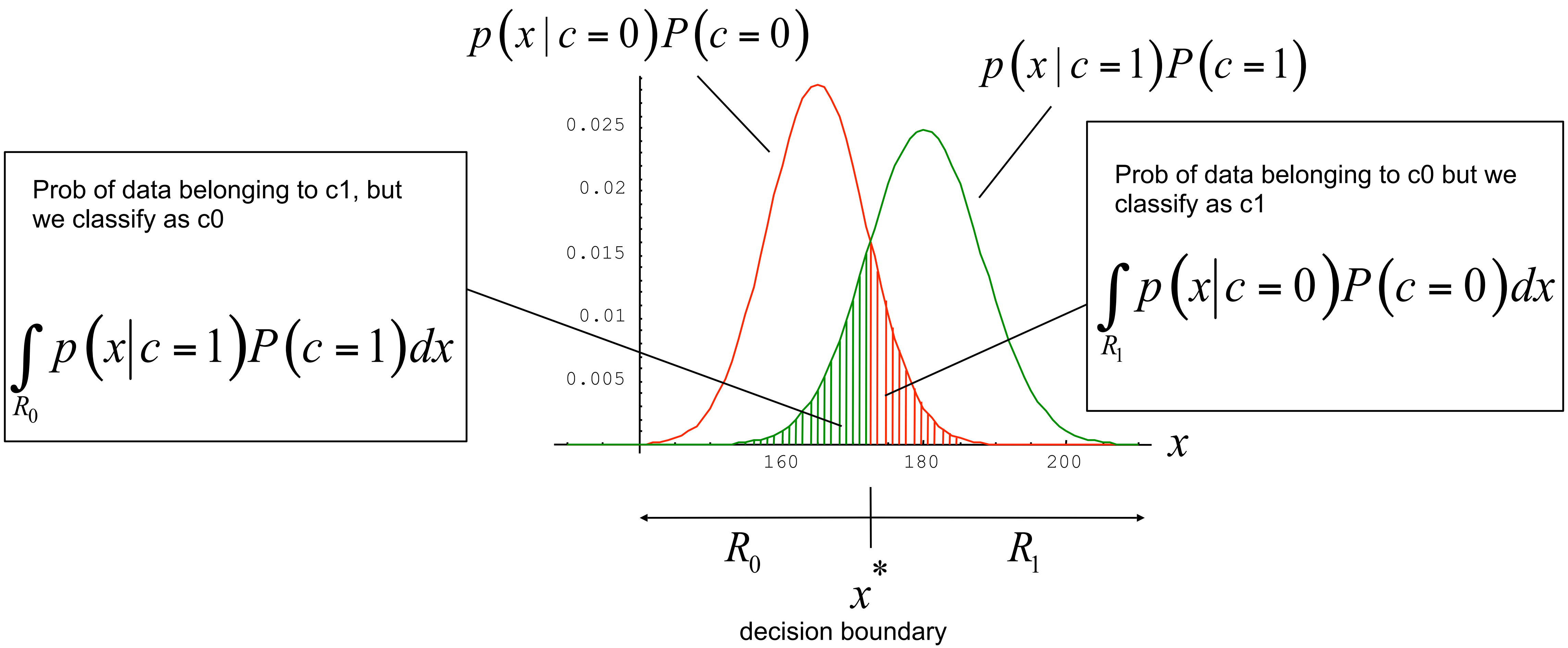
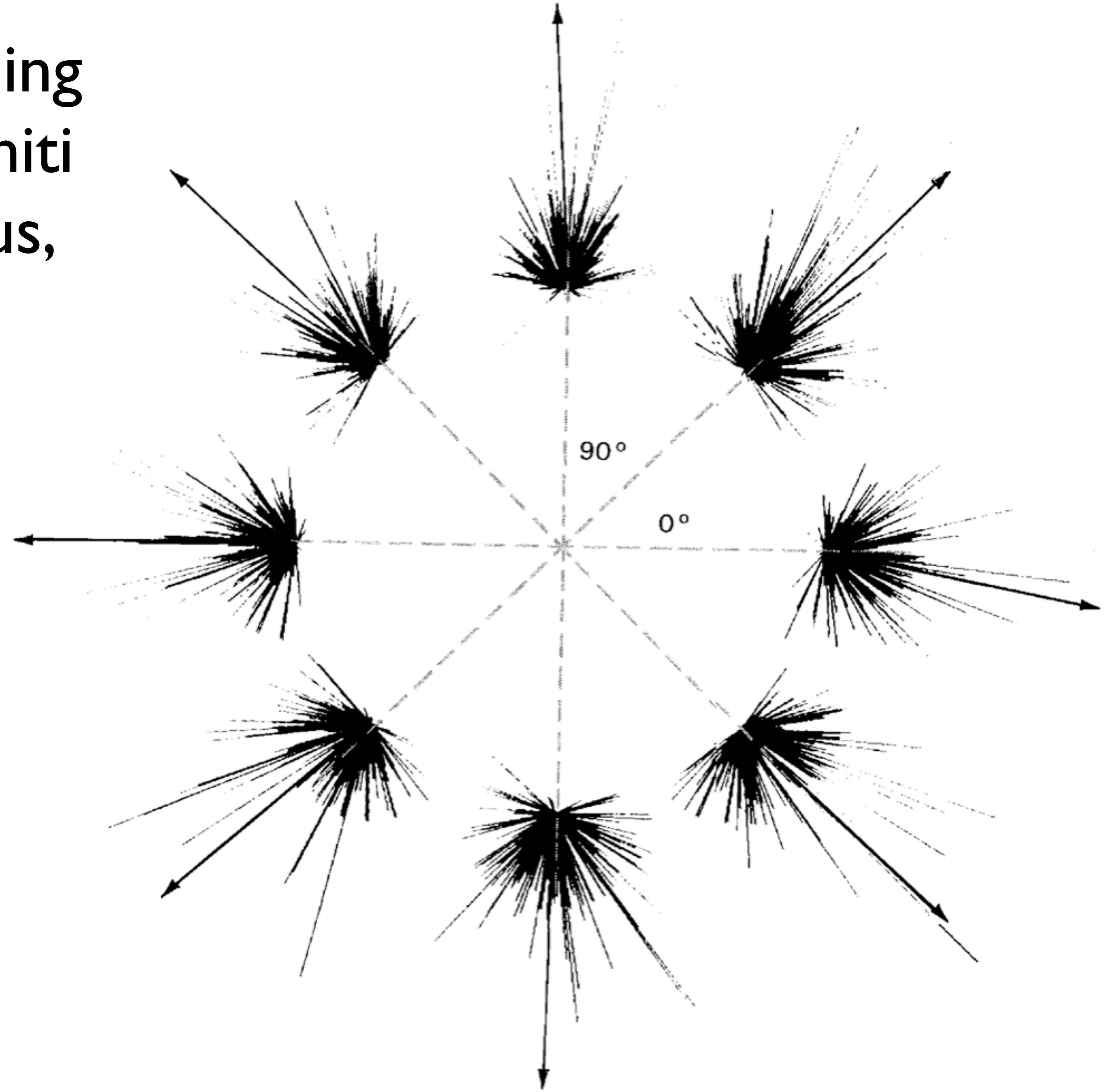
  - two neurons: linear discriminators, correlations

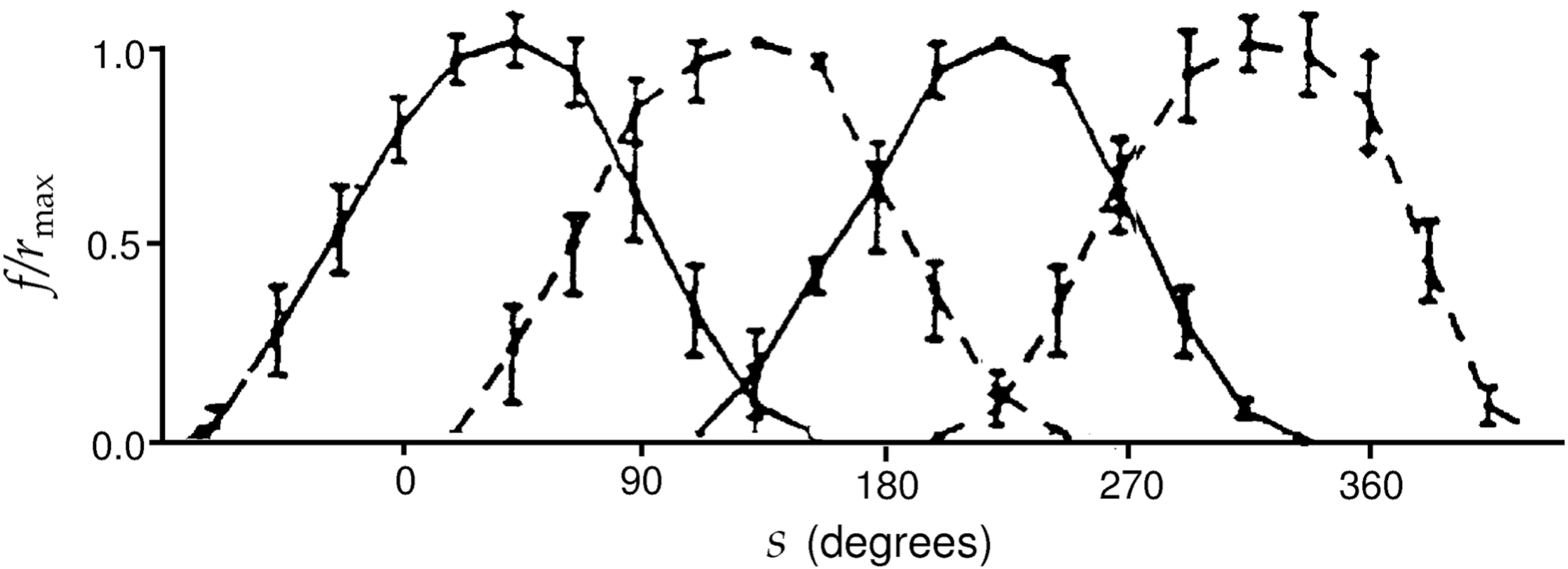


$p(x \mid c=0)P(c=0)$
$p(x \mid c=1)P(c=1)$
Prob of data belonging to c1, but we classify as c0
$\int_{R_0} p(x|c=1)P(c=1)dx$
Prob of data belonging to c0 but we classify as c1
$\int_{R_1} p(x|c=0)P(c=0)dx$
0.025
0.02
0.015
0.01
0.005
160
180
200
x
R_0
R_1
x^*
decision boundary

“vector” decoding [Kalaska, Caminiti Georgopoulous, 1983]

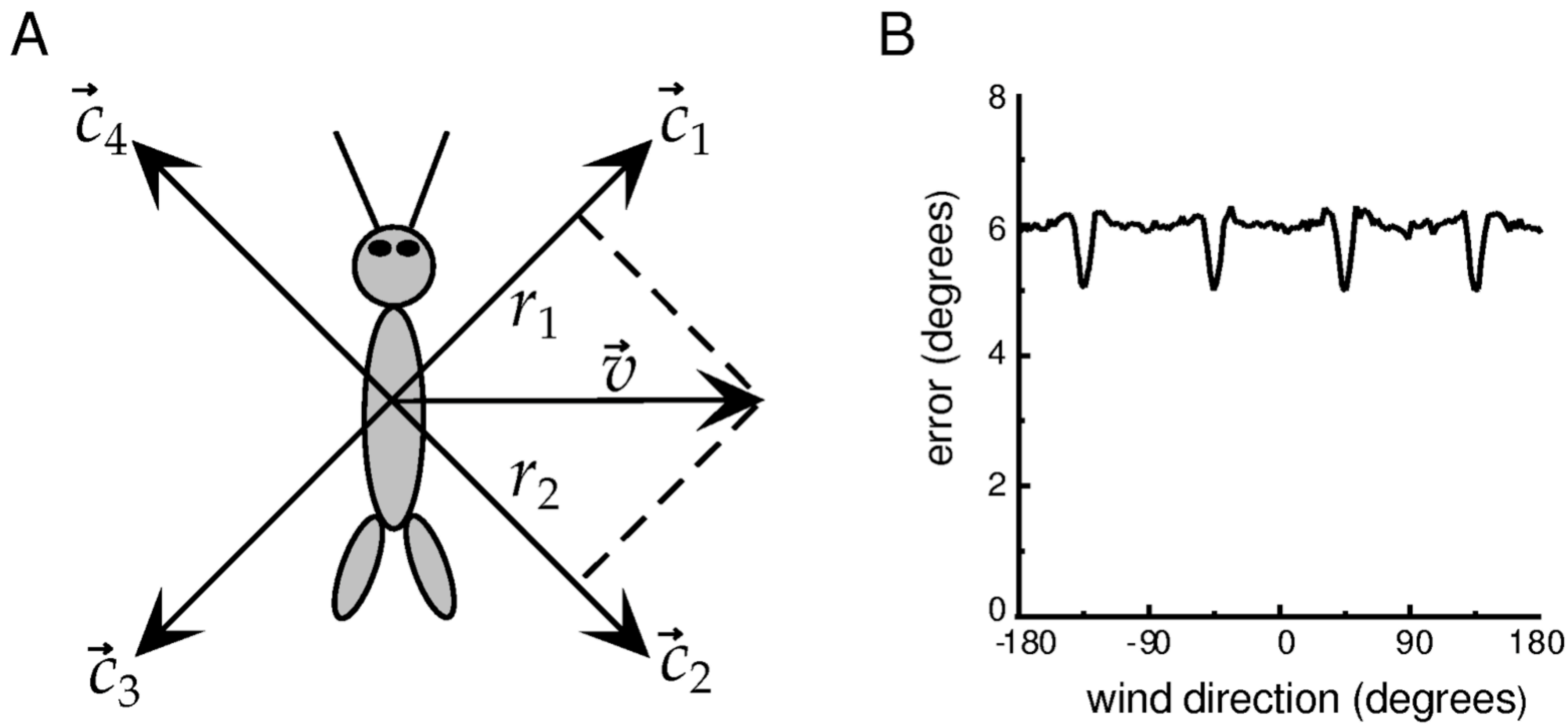


A sum of vectors, weighted by firing rate, predicts arm movement...



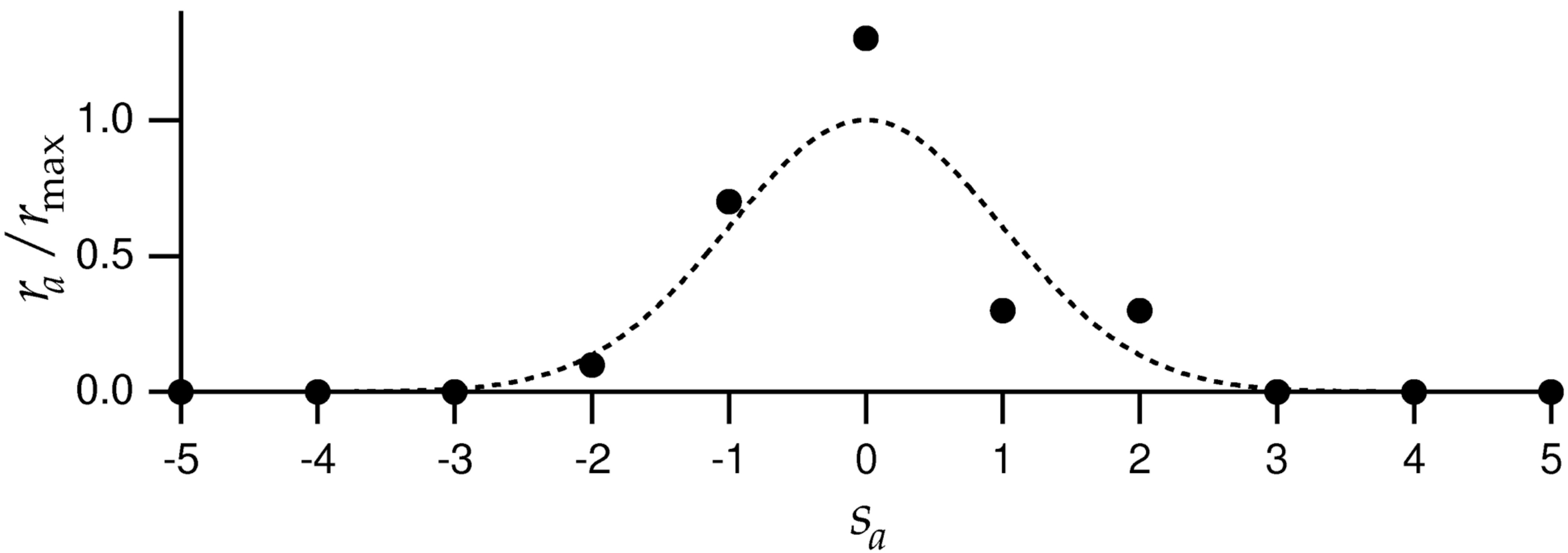
Response to wind direction of
4 cercal cricket interneurons [Theunissen & Miller '91]

Linear population decoding, cricket cercal interneurons [Salinas & Abbott '94]



- ML for independent Poisson neurons
- ML with Gaussian tuning curves
- ML with von Mises tuning curves
- “vector rule” with cosine tuning curves
- OLE
- Cramer-Rao Bound

ML decoding



For neurons with homogeneous tuning curves f_k(x) and independent Poisson spiking, ML gives:

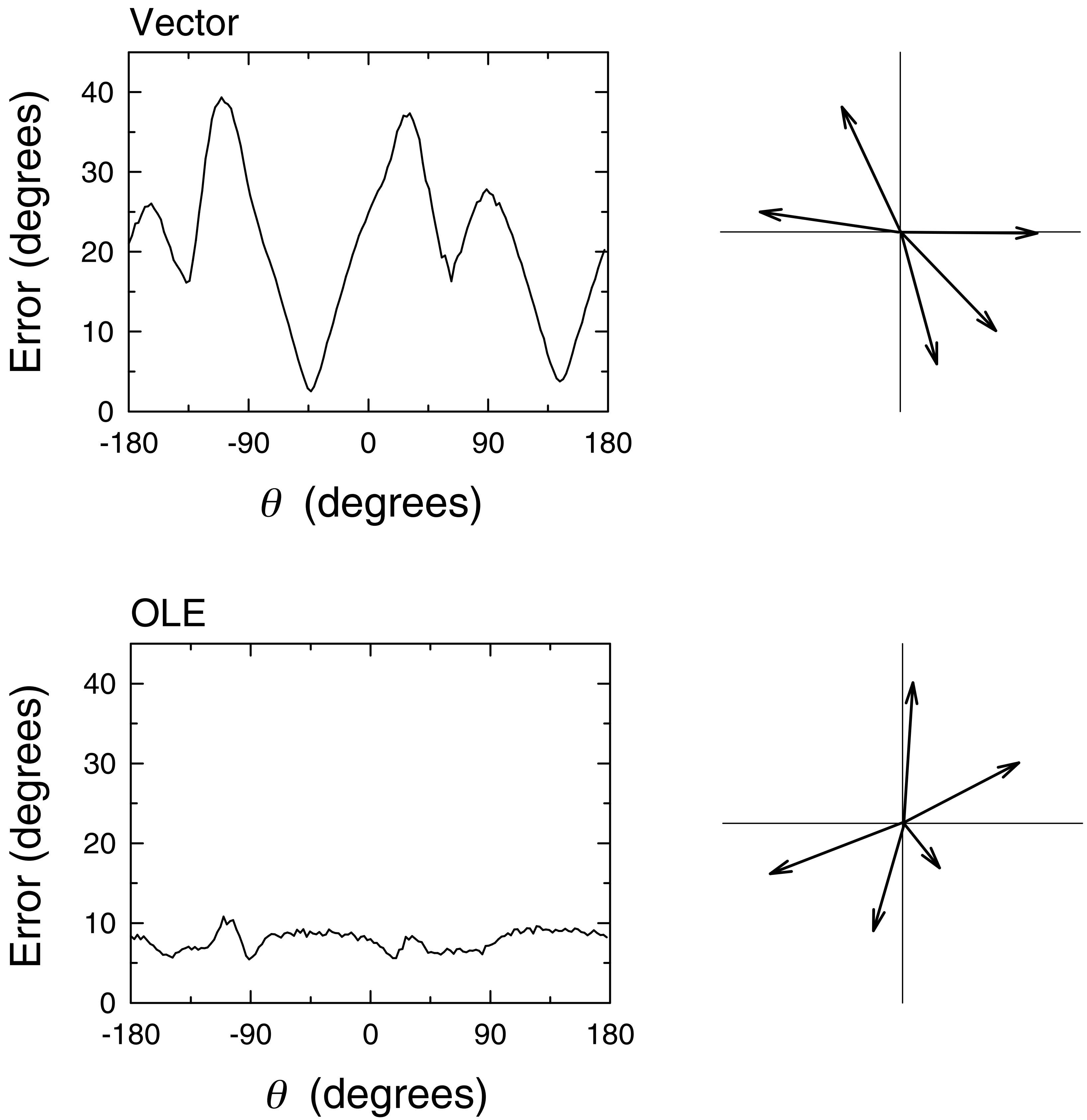
$$\frac{\partial}{\partial x} \log p(N_k|x) = \sum_k N_k \frac{\partial}{\partial x} \log f_k(x) = 0$$

In the special case of Gaussian tuning curves, ML estimate is simply a sum of the peak locations of each tuning curve, weighted by the number of spikes

$$\hat{x} = \frac{\sum_k N_k x_k}{\sum N_k}$$

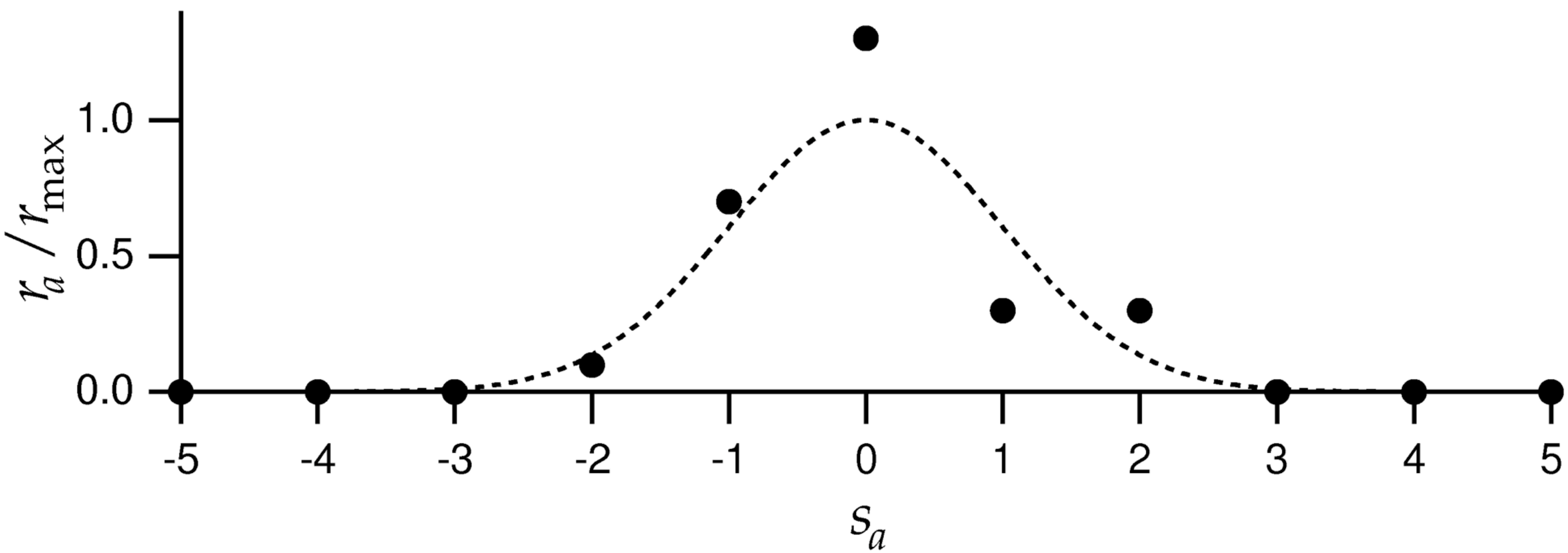
In the special case of von Mises tuning curves (exponential of cosine), ML estimate is angle of a vector computed as the weighted sum of unit vectors in the peak direction of each tuning curve, weighted by the number of spikes

$$\hat{\theta} = \angle \sum_k N_k u_k$$



Salinas & Abbott, ‘94

ML decoding



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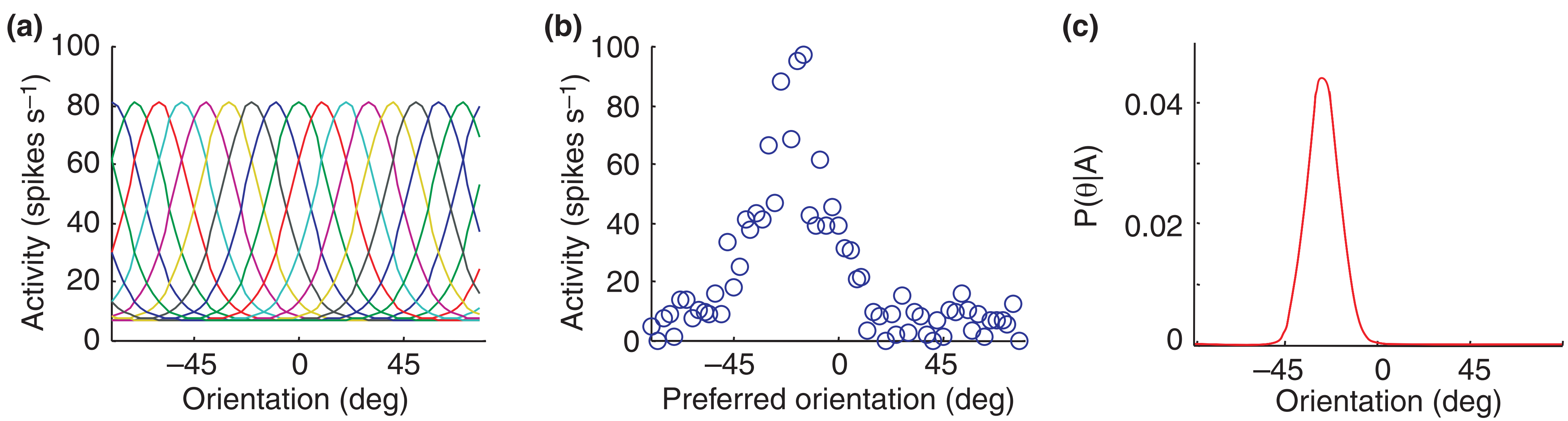


Figure 4. Inferences with gain encoding. **(a)** Idealized Gaussian tuning curves to orientation for 16 cells in primary visual cortex. **(b)** Response of 64 cells with Gaussian tuning curves similar to the ones shown in (a), in response to an orientation of $-20°$. The cells have been ranked according to their preferred orientation and the responses have been corrupted by independent Poisson noise, a good approximation of the noise observed *in vivo*. **(c)** The posterior distribution over orientation obtained from applying a Bayesian decoder to the noisy hills shown in (b). With independent Poisson noise, the peak of the distribution is given by the peak of the noisy hill, and the width of the distribution (i.e. the uncertainty) is inversely proportional the amplitude of the noisy hill. Adapted from Ref. [47].

- Knill & Pouget, 2005

Cramer-Rao Bound

$$\sigma_{\mathrm{est}}^2 \geq \frac{[1 + b'_{\mathrm{est}}(s)]^2}{I_F(s)}$$

$$b_{\mathrm{est}}(s) = \mathbf{E}(\hat{s}(s)) - s$$

$$I_F(s) = \mathbf{E}\left[-\frac{\partial^2 \log p(r|s)}{\partial s^2}\right]$$

$$I = -E\left[\frac{\partial^2 \ln \mathrm{P}\left(\mathbf{A} \mid \theta\right)}{\partial \theta^2}\right]$$

$$\mathrm{P}\left(\mathbf{A} \mid \theta\right) = \prod_{i=1}^{n} \mathrm{P}\left(a_i = k_i \mid \theta\right) = \prod_{i=1}^{n} \frac{f_i\left(\theta\right)^{k_i} e^{-f_i\left(\theta\right)}}{k_i\,!}$$

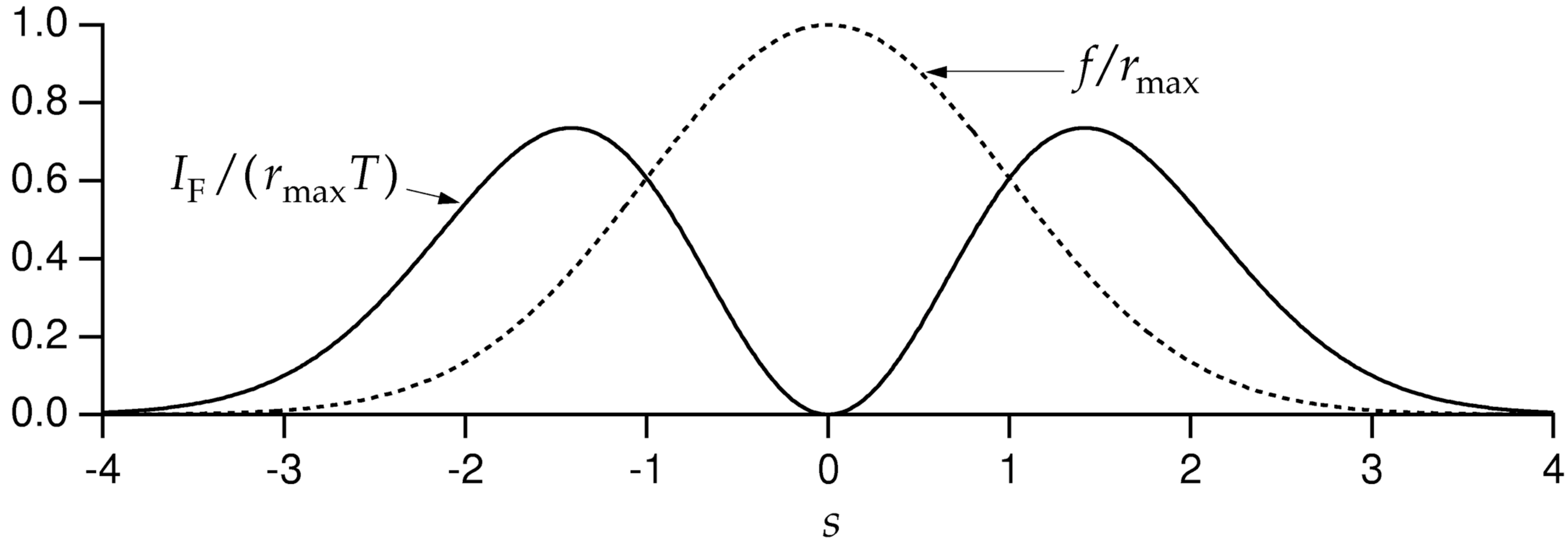
$$\ln \mathrm{P}\left(\mathbf{A} \mid \theta\right) = \sum_{i=1}^{n} k_i \ln f_i\left(\theta\right) - f_i\left(\theta\right) - \ln\left(k_i\,!\right)$$

$$\frac{\partial \ln \mathrm{P}\left(\mathbf{A} \mid \theta\right)}{\partial \theta} = \sum_{i=1}^{n} \frac{k_i f_i^{'}\left(\theta\right)}{f_i\left(\theta\right)} - f_i^{'}\left(\theta\right)$$

$$\frac{\partial^2 \ln \mathrm{P}\left(\mathbf{A} \mid \theta\right)}{\partial \theta^2} = \sum_{i=1}^{n} -\frac{k f_i^{'}\left(\theta\right)^2}{f_i\left(\theta\right)^2} + \frac{k_i f_i^{''}\left(\theta\right)}{f_i\left(\theta\right)} - f_i^{''}\left(\theta\right)$$

$$-E\left[\frac{\partial^2 \ln \mathrm{P}\left(\mathbf{A} \mid \theta\right)}{\partial \theta^2}\right] = \sum_{i=1}^{n} \frac{f_i\left(\theta\right) f_i^{'}\left(\theta\right)^2}{f_i\left(\theta\right)^2} - \frac{f_i\left(\theta\right) f_i^{''}\left(\theta\right)}{f_i\left(\theta\right)} + f_i^{''}\left(\theta\right)$$

$$I = \sum_{i=1}^{n} \frac{f_i^{'}\left(\theta\right)^2}{f_i\left(\theta\right)}$$



$$I_F(s) = \sum_n \frac{[f_n'(s)]^2}{f_n(s)}$$

Fisher Information of model neuron, assuming Poisson firing, Gaussian tuning

Linear temporal decoding [Rieke etal, 1997]