

Mathematical Tools for Neural and Cognitive Science

Fall semester, 2022

Section 1a: Trichromacy (an extended linear algebra example)

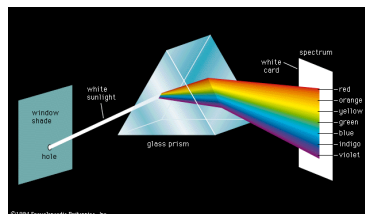
Trichromacy

A spectacular multi-disciplinary story ...

- Physics: spectral nature and additivity of light [Newton, and others, 1600's]
- Perception: Trichromatic matching experiments [Grassman, Young, and others, 1850's]
- Mathematics: Human color matching is explained by a 3-dimensional linear model [late 1800's]
- Engineering: Devices for color reproduction require only three color channels [early 1930's]
- Neurobiology: Trichromacy in humans (and some other primates) arises from 3 cone types [late 1980's!]

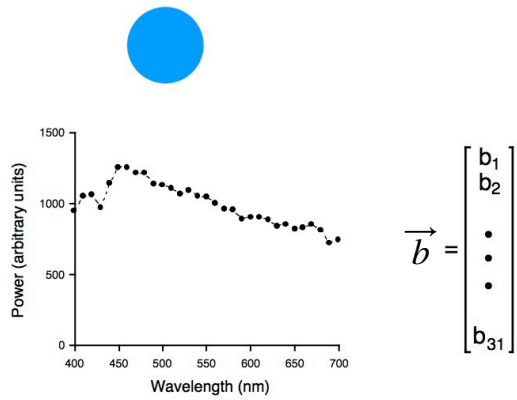
[See Wandell, *Foundations of Vision*, ch2]

Spectral nature of light



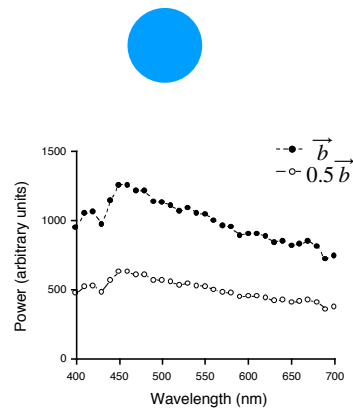
[Newton, 1665]

Representing spectra with vectors



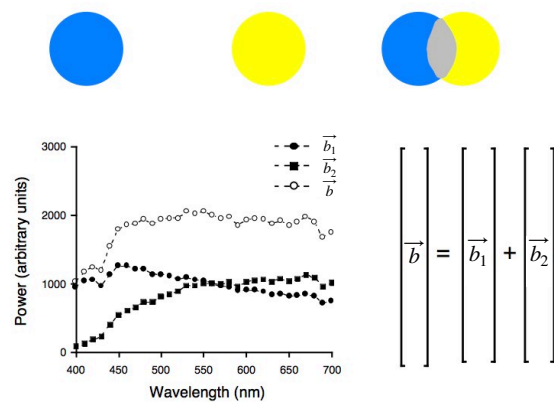
[c/o David Brainard]

Intensity adjustment (scalar multiplication)



[c/o David Brainard]

Additive mixtures



[c/o David Brainard]

Color perception: what's it for?

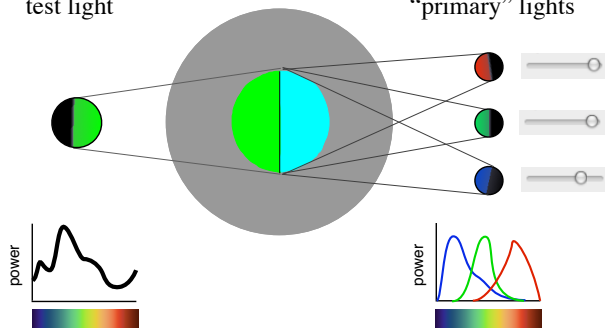


[c/o David Brainard]

Perceptual color matching experiment

Arbitrary
test light

Mixture of three
“primary” lights

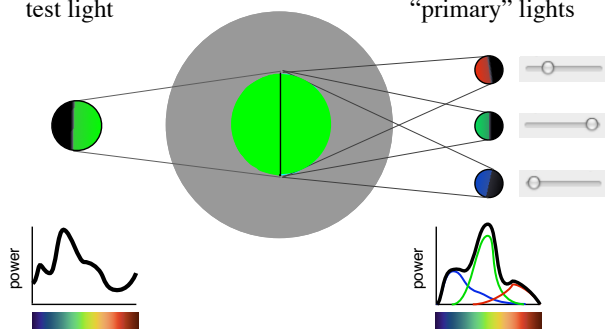


[Young, Helmholtz, Grassman, etc., 1800's]

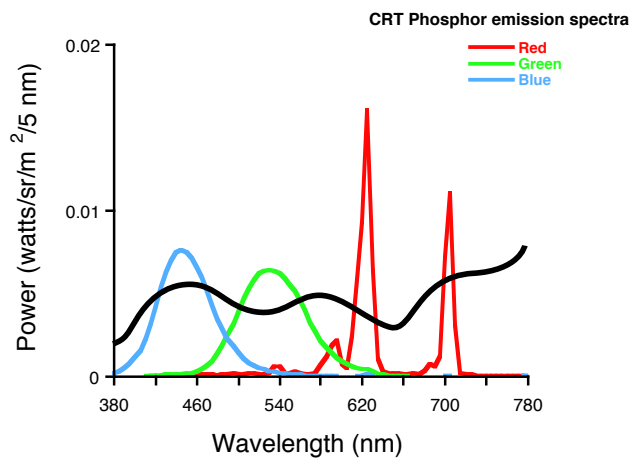
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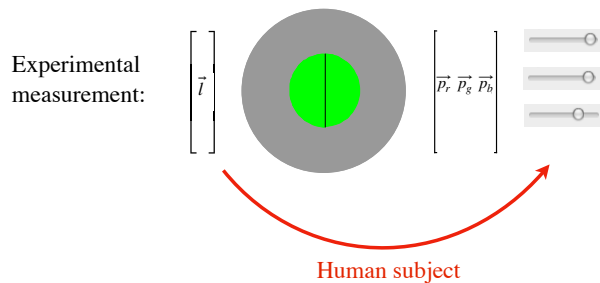
Grassmann's Laws (1853)



- 1) Any light can be matched with a mixture of 3 primaries
- 2) Rescaling the light results in a rescaled mixture
- 3) Adding two lights results in a sum of their mixtures

→ Color matching can be described by an $N \times 3$ linear system*

* Fine print: i) Normal human observer; ii) photopic intensity levels, not too bright, not too dim; iii) independent primaries.



Linear model:

$$\begin{bmatrix} 0.25 \\ 0.75 \\ 0.50 \end{bmatrix} = \begin{bmatrix} \mathbf{H} \end{bmatrix} \begin{bmatrix} \vec{l} \end{bmatrix}$$

[c/o David Brainard]

Implications

- If P is an $N \times 3$ matrix containing the primary spectra, then the color matching experiment is captured by a $3 \times N$ “matching matrix” H that maps a light \vec{l} to 3 “knob settings”:

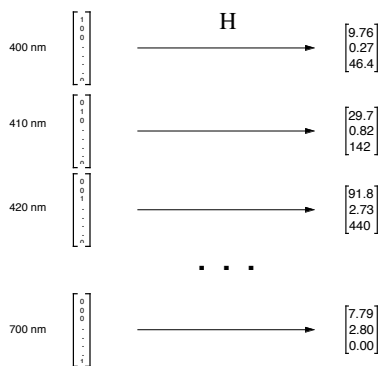
$$\vec{l} \sim PH\vec{l}$$

where \sim means “looks the same as”

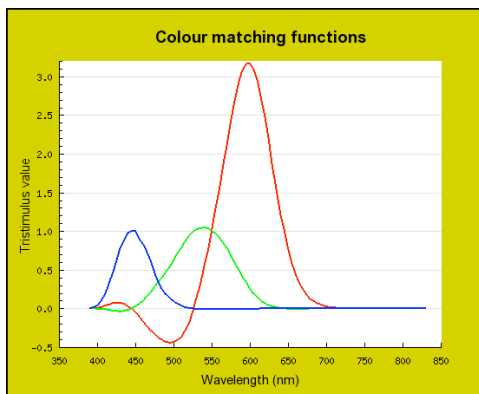
- Two lights look the same if (and *only* if) they produce the same match settings:

$$\vec{l}_1 \sim \vec{l}_2 \Leftrightarrow H\vec{l}_1 = H\vec{l}_2$$

Characterization



[c/o David Brainard]



Stiles-Burch Data, Monochromatic RGB Primaries of 645, 526, and 444 nm.

A matrix that mimics a human!

[Stiles & Burch, 1959]

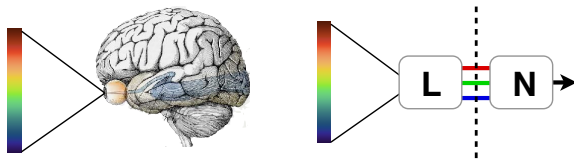
Useful calculations

- Scientific: given results of one matching experiment, predict the results of another one (with different primaries).
- Practical: calibrate a display device, so as to generate mixtures of three colors that match the appearance of any desired real-world spectrum.

[derive on board]

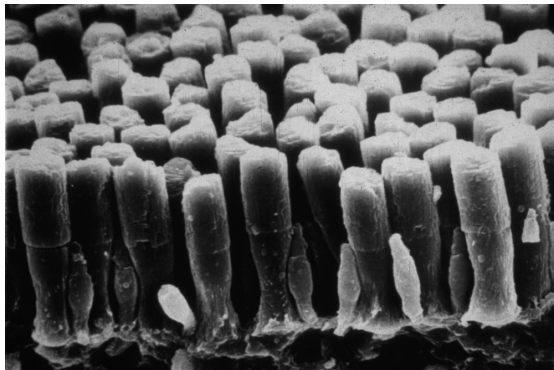
Summary:

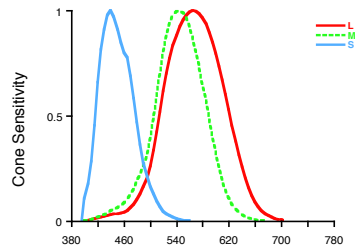
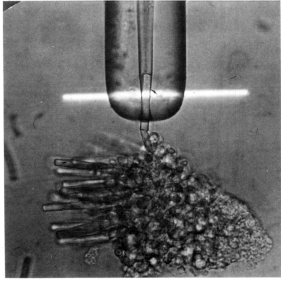
- Perceptual measurements: any light with can be matched with a mixture of 3 primaries
- Theory/model: the visual system projects the wavelength spectra of light onto a 3-dimensional subspace



- Prediction/generalization: perceptual “metamers” - lights that appear identical, but have physically distinct wavelength spectra
- Engineering: CIE standards for color representation (1931).
- Underlying physiological mechanism (cones), verified in 1987!

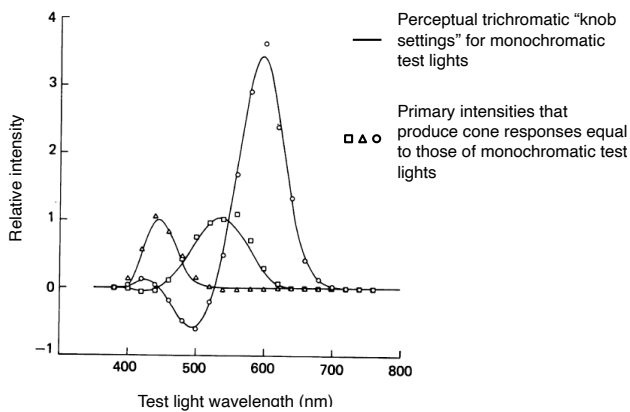
The underlying mechanism...





[Baylor, Nunn & Schnapf, 1987]

Confirmation of prediction (100+ years later!):



[Baylor, Nunn & Schnapf, '87]

Dimensionality caveats

- Normal human vision, at high (“photopic”) light levels is tri-chromatic (cones)
- At low (“scotopic”) light levels, we are all monochromats (rods only)
- At intermediate (“mesopic”) light levels, we are quadrachromats (rods&cones)!
- Common genetic forms of color blindness are due to lack of one or two cone types