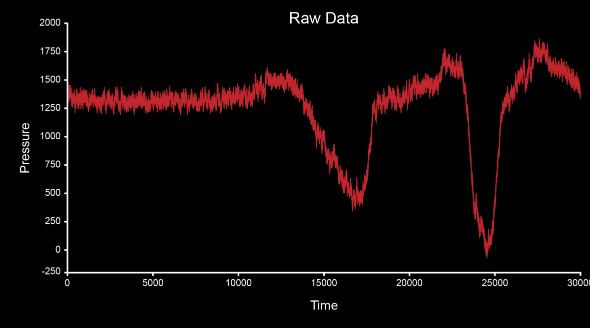
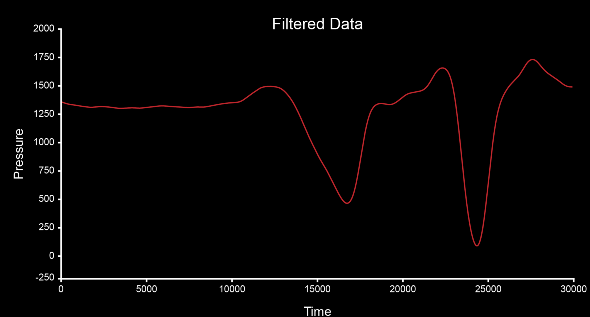
Lab V: Fourier Transform

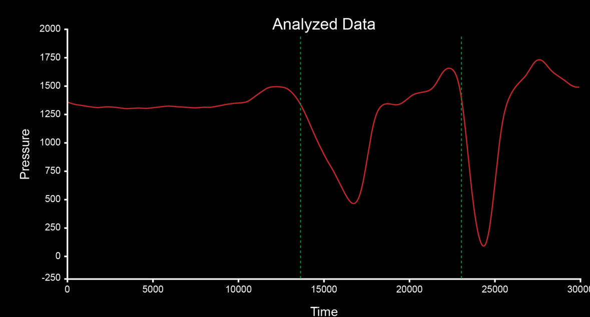
Motivating Example: removing noise



Motivating Example: removing noise



Motivating Example: removing noise



How do we process signals?

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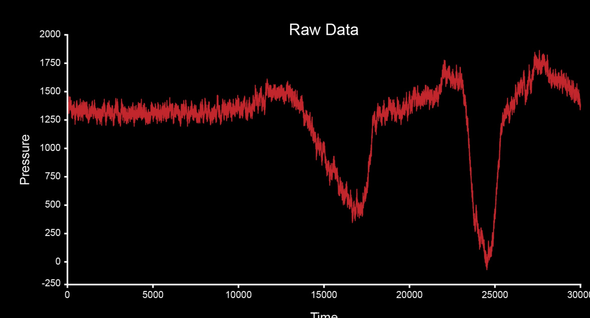
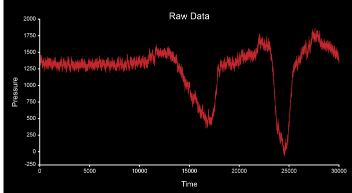
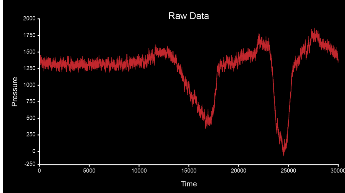


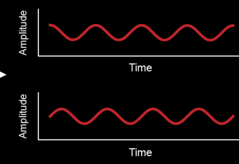
How do we process signals?



Simpler signals that are easier to process

How do we process signals?

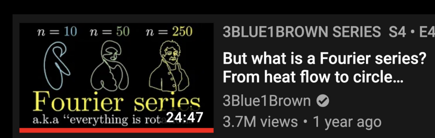




Signals are combinations of signals…

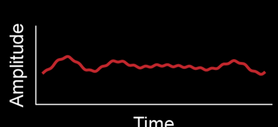
- We can create any signal using a sum of sines and cosines
- Trust me
- Or trust 3blue1brown:

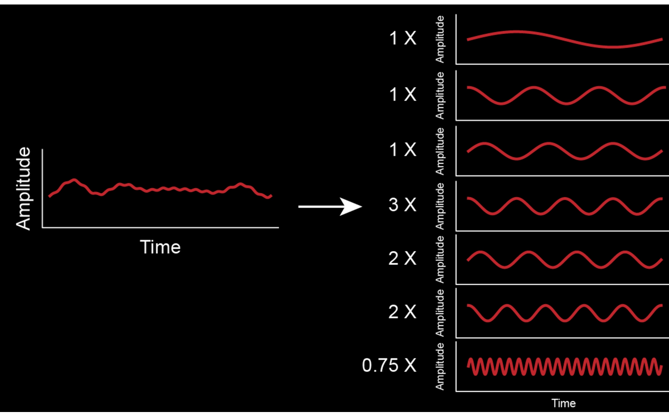
Signals are combinations of signals…

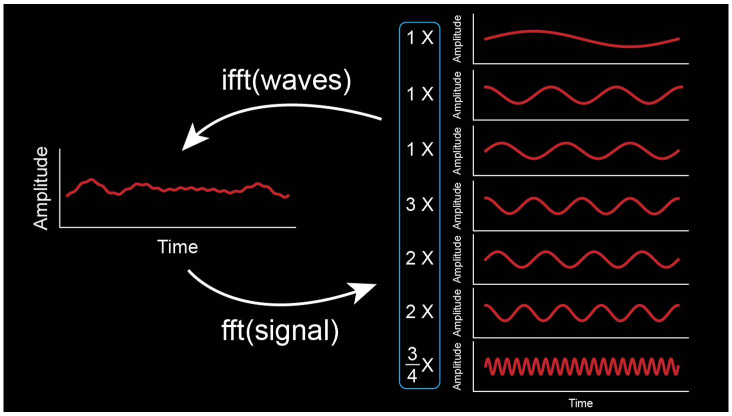


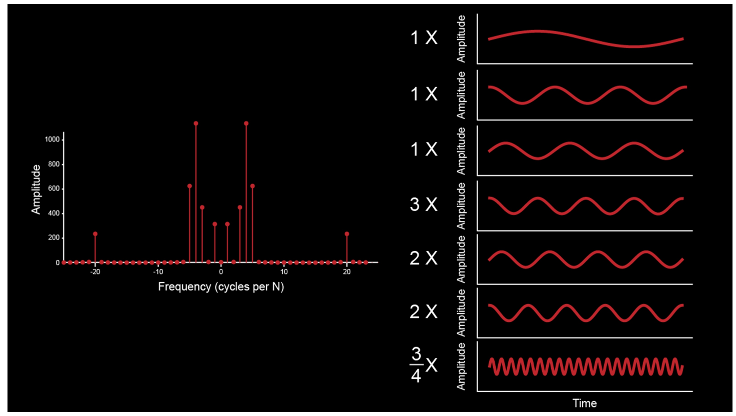
https://www.youtube.com/watch?v=r6sGWTCMz2k

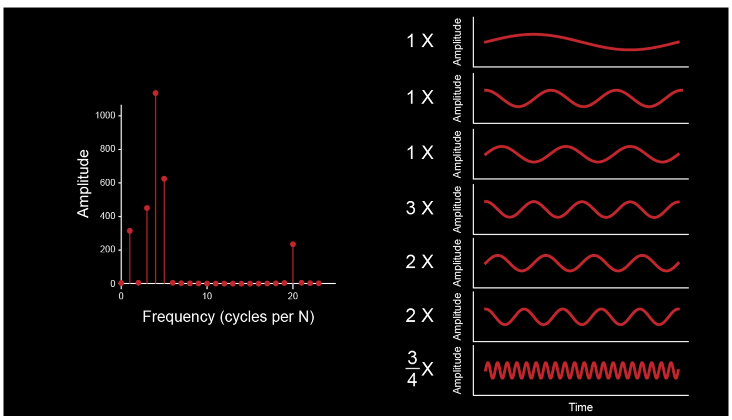
- 5:11-5:50
- 0:34-1:36

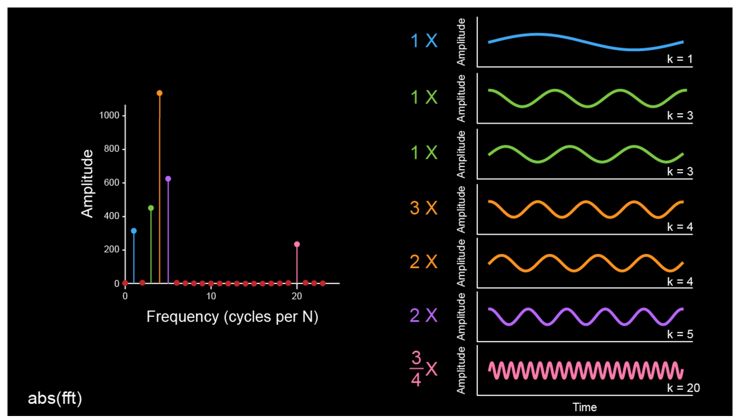


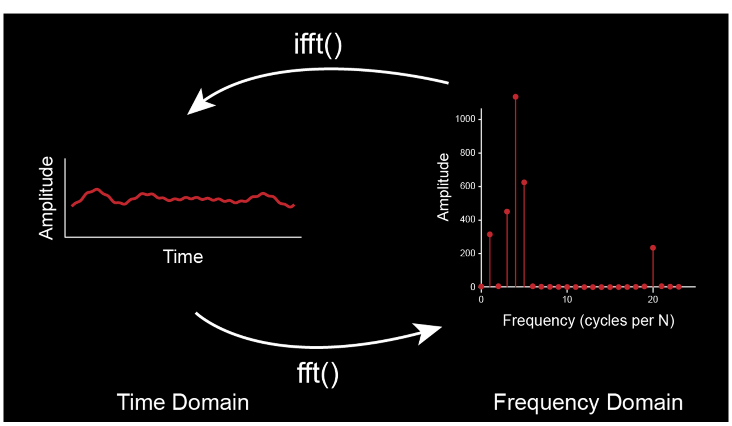




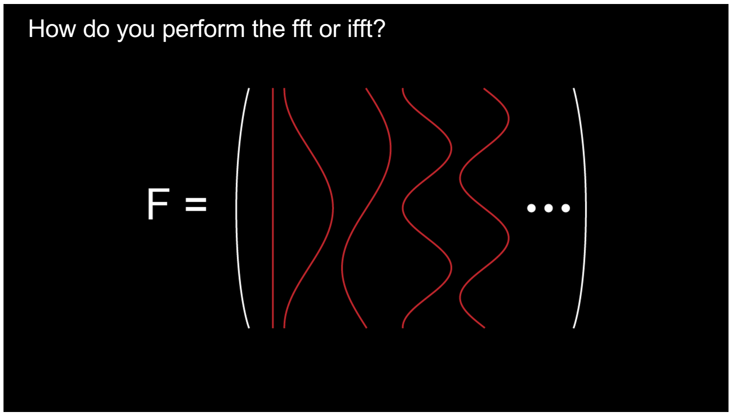


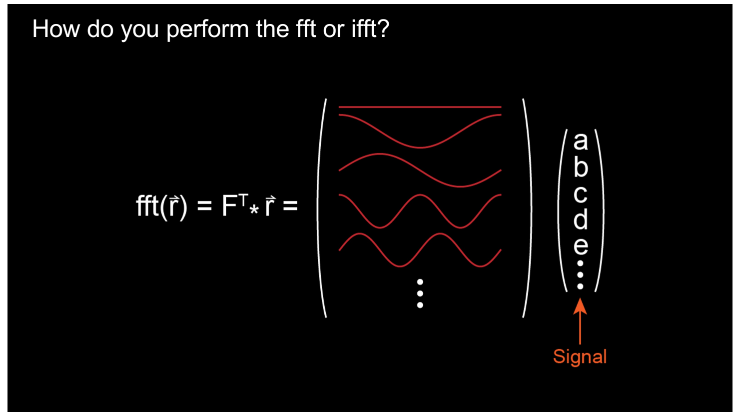


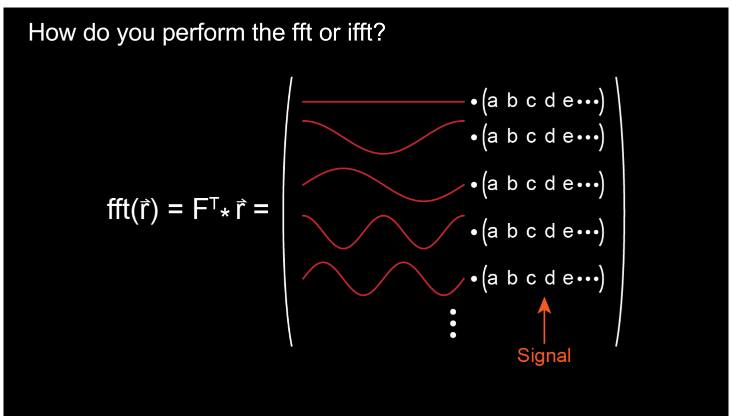


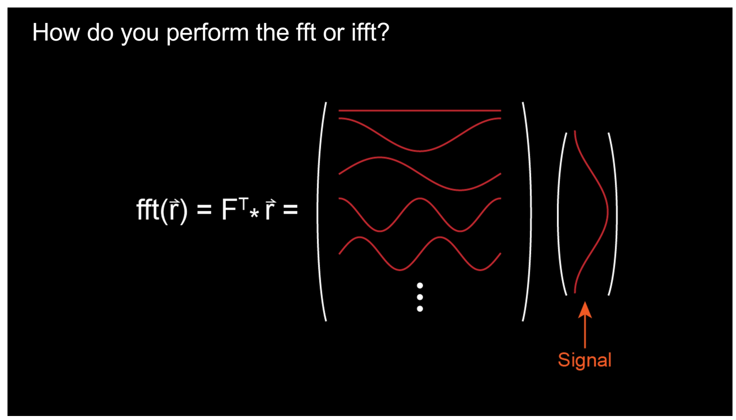


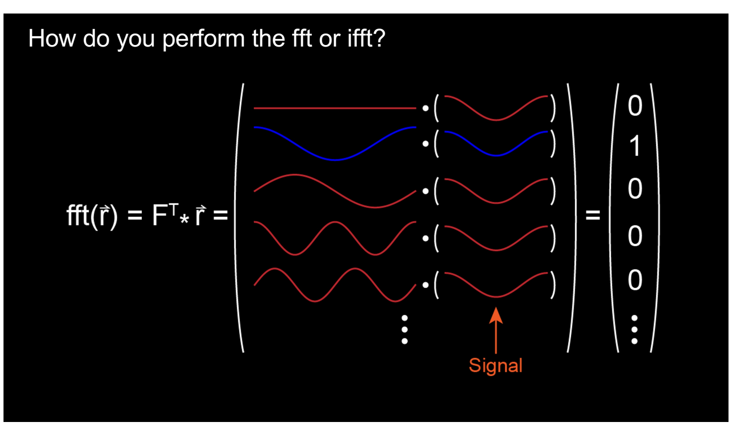
How do you perform the fft or ifft?

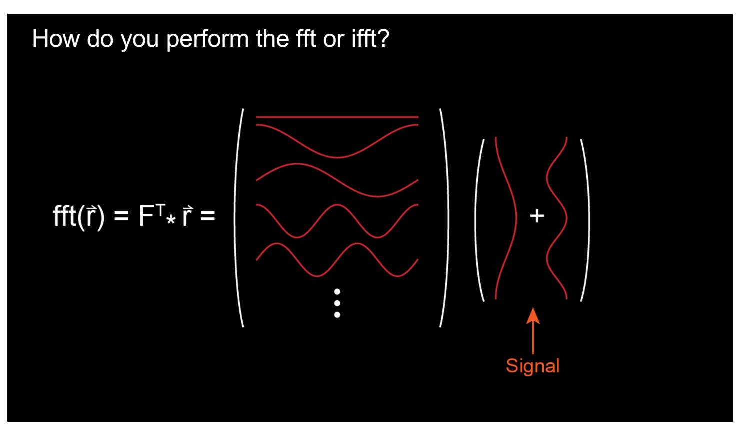

How do you perform the fft or ifft?
F = •••

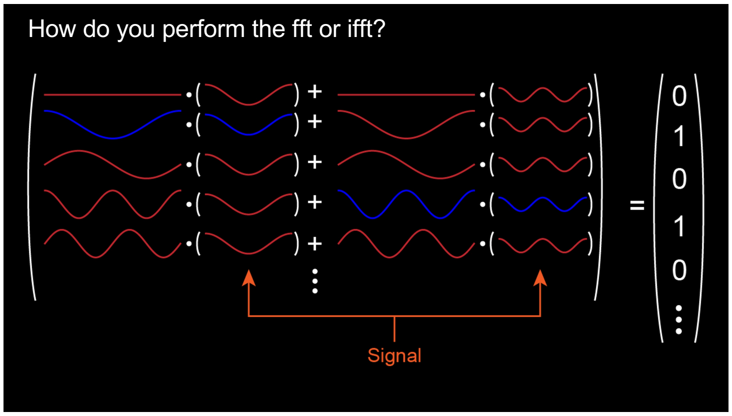

How do you perform the fft or ifft?
fft(r⃗) = Fᵀ* r⃗ =
a b c d e
Signal

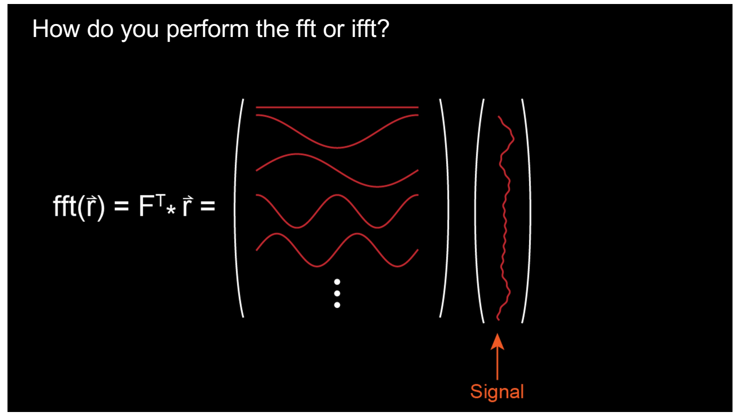

How do you perform the fft or ifft?
fft(r⃗) = Fᵀ* r⃗ =
•(a b c d e•••)
•(a b c d e•••)
•(a b c d e•••)
•(a b c d e•••)
•(a b c d e•••)
Signal

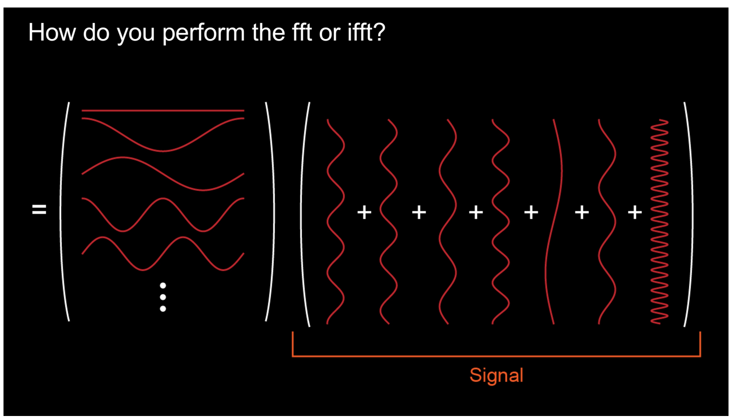

How do you perform the fft or ifft?
fft(r⃗) = Fᵀ* r⃗ =
Signal

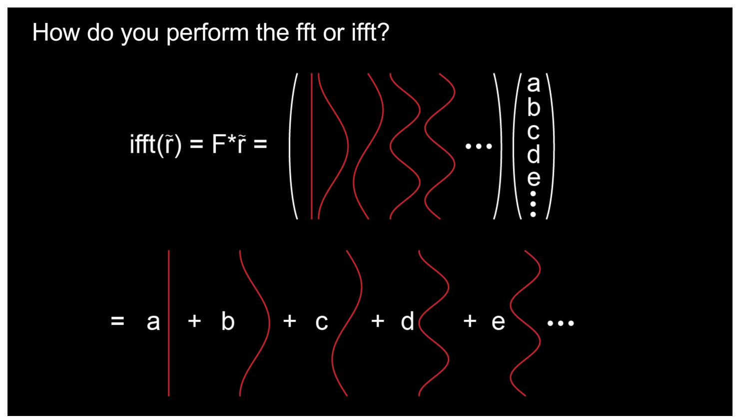

How do you perform the fft or ifft?
fft(r⃗) = Fᵀ* r⃗ =
=
0
1
0
0
0
Signal

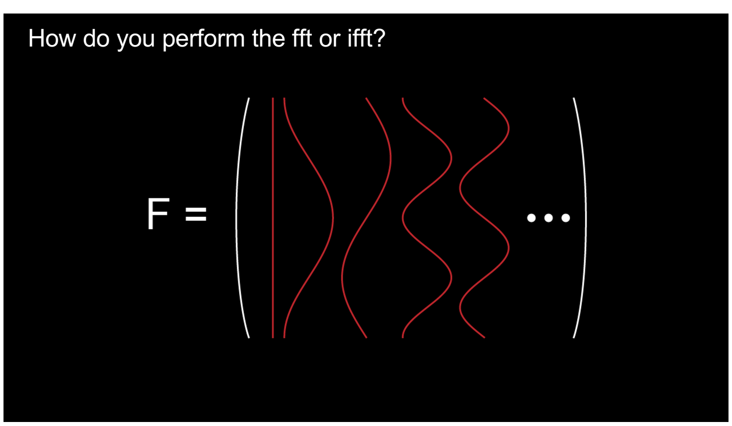

How do you perform the fft or ifft?
fft(r⃗) = Fᵀ* r⃗ =
+
Signal

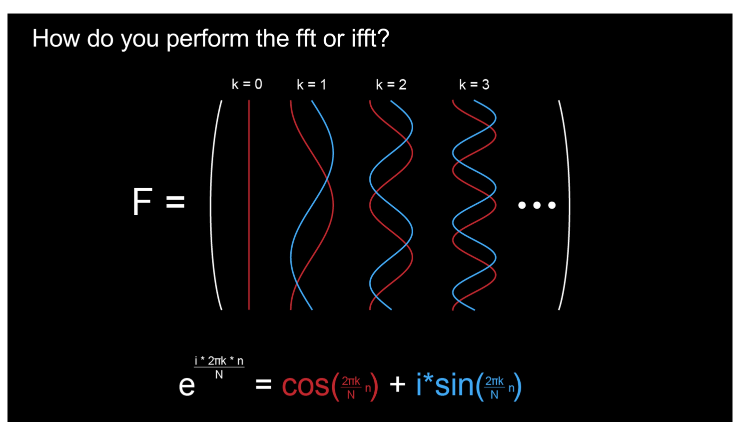

How do you perform the fft or ifft?
0
1
0
1
0
Signal


How do you perform the fft or ifft?
fft(r⃗) = Fᵀ* r⃗ =
Signal


How do you perform the fft or ifft?
Signal


How do you perform the fft or ifft?
ifft(r̃) = F*r̃ =
a
b
c
d
e
= a + b + c + d + e


How do you perform the fft or ifft?
F =


How do you perform the fft or ifft?
k = 0
k = 1
k = 2
k = 3
F =
e^(i * 2πk * n / N) = cos(2πk/N n) + i*sin(2πk/N n)

How do you *actually* perform the fft?

Signal (N x 1)

1) $\tilde{r} = \text{fft}(\vec{r})$

Frequency components (N x 1, complex)

1) $\tilde{r} = \text{fft}(\vec{r})$

(i) $\tilde{r} = \text{real}(\text{fft}(\vec{r}))$

(ii) $\tilde{r} = \text{imag}(\text{fft}(\vec{r}))$

(iii) $\tilde{r} = \text{abs}(\text{fft}(\vec{r}))$

(iv) $\tilde{r} = \text{angle}(\text{fft}(\vec{r}))$

$$e^{\frac{i \cdot 2\pi k \cdot n}{N}} = \cos\left(\frac{2\pi k}{N} n\right) + i{*}\sin\left(\frac{2\pi k}{N} n\right)$$

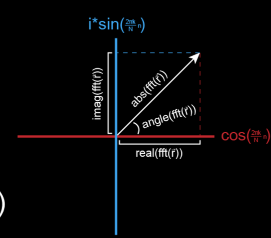
1) $\tilde{r} = \text{fft}(\vec{r})$

(i) $\tilde{r} = \text{real}(\text{fft}(\vec{r}))$

(ii) $\tilde{r} = \text{imag}(\text{fft}(\vec{r}))$

(iii) $\tilde{r} = \text{abs}(\text{fft}(\vec{r}))$

(iv) $\tilde{r} = \text{angle}(\text{fft}(\vec{r}))$



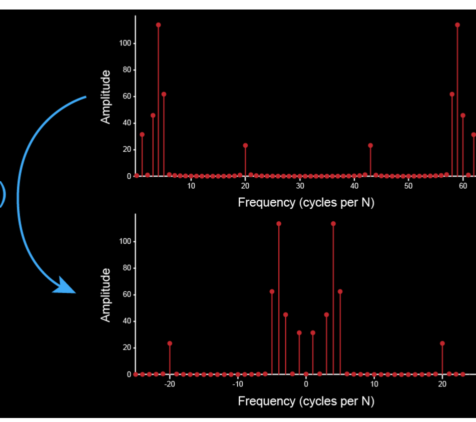
1) $\tilde{r} = \text{fft}(\vec{r})$

Frequency components (N x 1, complex)

2) $\tilde{r}_{shifted} = \text{fftshift}(\tilde{r})$

Frequency components (N x 1, complex)
Rearranged to make interpretation easier

fftshift(r)



1) $\tilde{r} = \text{fft}(\vec{r})$

2) $\tilde{r}_{shifted} = \text{fftshift}(\tilde{r})$

3) $\text{stem}(\, x,\ \tilde{r}_{shifted}\,)$

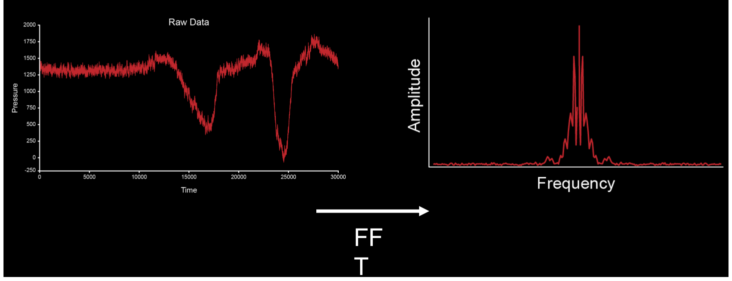
Frequencies
-N/2 -> N/2

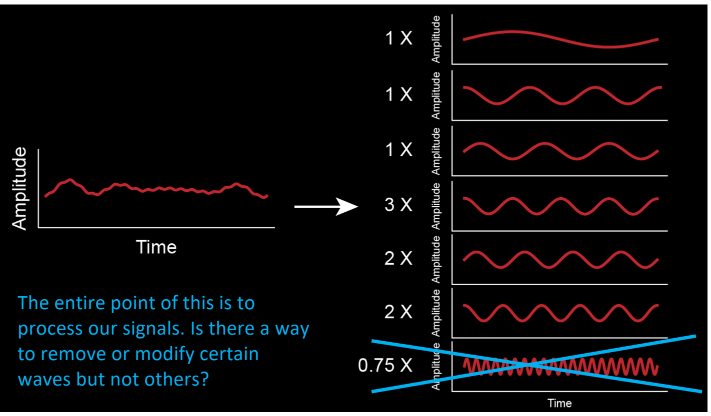
Frequency components (N x 1, complex)
Rearranged to make interpretation easier

Exercise #1

Fourier Transform and Shift-Invariant Systems

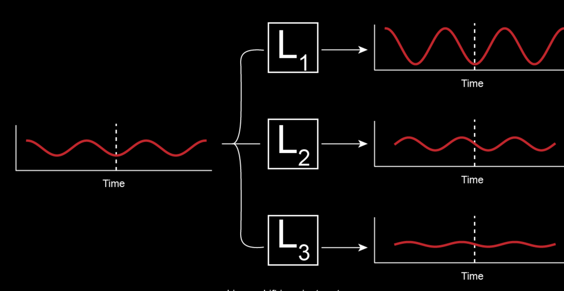
What now?

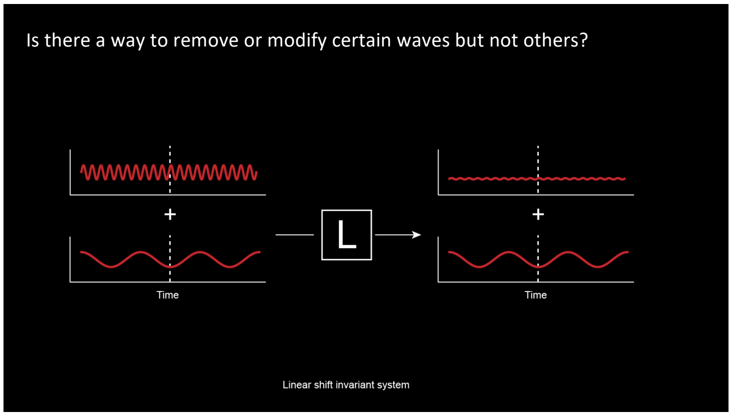


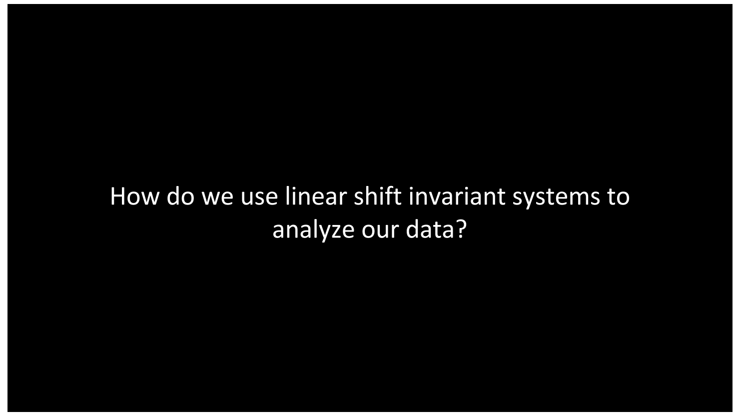


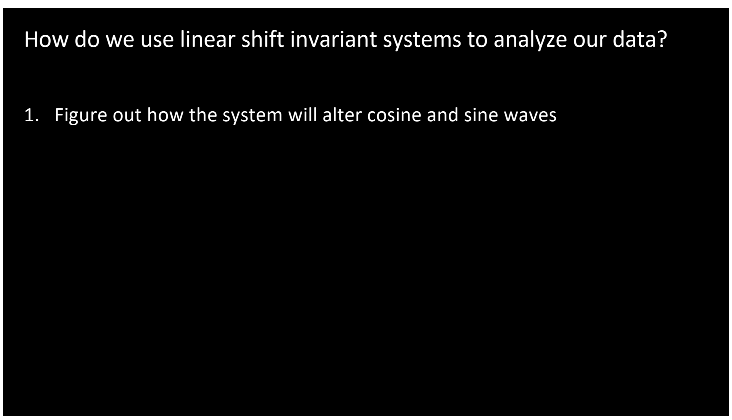
The entire point of this is to process our signals. Is there a way to remove or modify certain waves but not others?

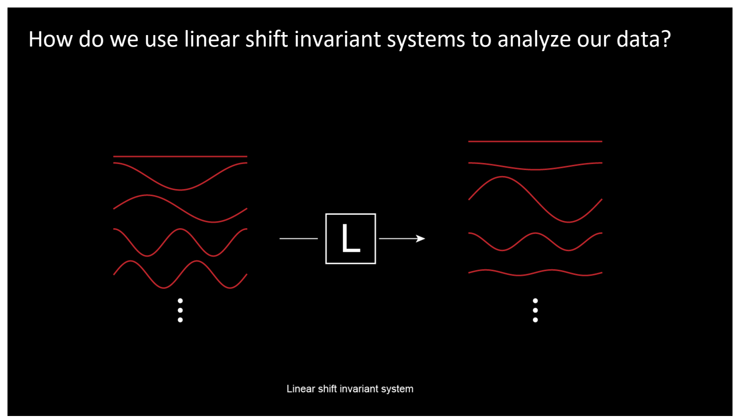
Is there a way to remove or modify certain waves but not others?

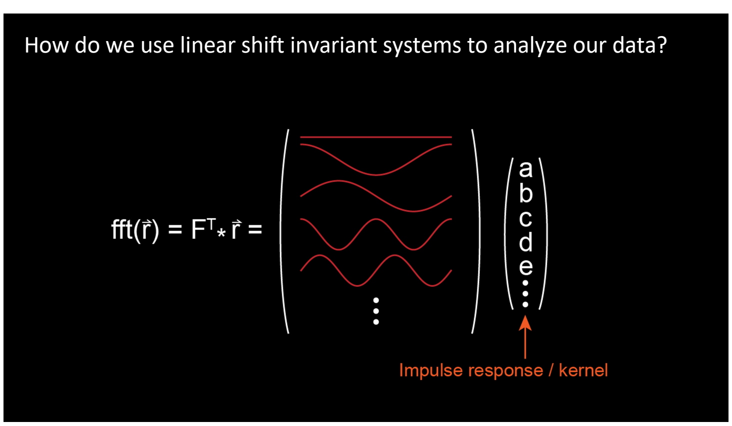


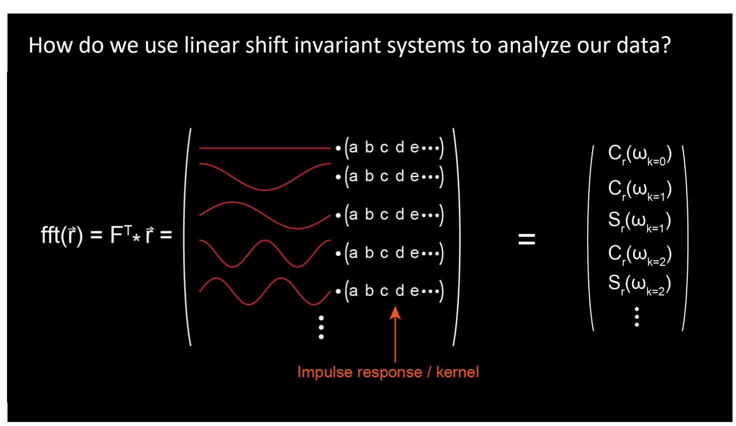

Is there a way to remove or modify certain waves but not others?
+
L
+
Time
Time
Linear shift invariant system


How do we use linear shift invariant systems to analyze our data?


How do we use linear shift invariant systems to analyze our data?
1. Figure out how the system will alter cosine and sine waves


How do we use linear shift invariant systems to analyze our data?
L
Linear shift invariant system


How do we use linear shift invariant systems to analyze our data?
fft(r⃗) = Fᵀ* r⃗ =
a
b
c
d
e
Impulse response / kernel

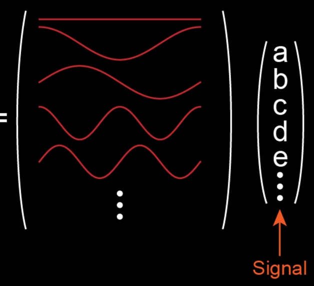

How do we use linear shift invariant systems to analyze our data?
fft(r⃗) = Fᵀ* r⃗ =
•(a b c d e•••)
•(a b c d e•••)
•(a b c d e•••)
•(a b c d e•••)
•(a b c d e•••)
=
C_r(ω_k=0)
C_r(ω_k=1)
S_r(ω_k=1)
C_r(ω_k=2)
S_r(ω_k=2)
Impulse response / kernel

How do we use linear shift invariant systems to analyze our data?

1. Figure out how the system will alter cosine and sine waves
   - Take fft() of impulse response
2. Figure out how your signal of interest is broken down into cosine and sine waves

How do we use linear shift invariant systems to analyze our data?

$\text{fft}(\vec{x}) = F^T * \vec{x} =$



How do we use linear shift invariant systems to analyze our data?

1. Figure out how the system will alter cosine and sine waves
   - Take fft() of impulse response
2. Figure out how your signal of interest is broken down into cosine and sine waves
   - Take fft() of signal
3. Use #1 and #2 to figure out how the system alters your signal

How do we use linear shift invariant systems to analyze our data?

$$\tilde{R} \qquad \text{fft}(\vec{x})$$

$$\begin{pmatrix} C_r(\omega_{k=0}) & 0 & 0 & 0 & 0 \\ 0 & C_r(\omega_{k=1}) & 0 & 0 & 0 \\ 0 & 0 & S_r(\omega_{k=1}) & 0 & 0 \\ 0 & 0 & 0 & C_r(\omega_{k=2}) & 0 \\ 0 & 0 & 0 & 0 & S_r(\omega_{k=2}) \; \cdots \\ & & & & \vdots \end{pmatrix} \begin{pmatrix} C_x(\omega_{k=1}) \\ S_x(\omega_{k=1}) \\ C_x(\omega_{k=2}) \\ S_x(\omega_{k=2}) \\ \vdots \end{pmatrix}$$

What happens to each cosine and sine wave in the form of a diagonal matrix

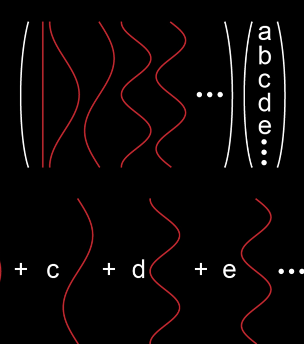
Amount of each cosine and sine wave in signal

How do we use linear shift invariant systems to analyze our data?

1. Figure out how the system will alter cosine and sine waves
   - Take fft() of impulse response
2. Figure out how your signal of interest is broken down into cosine and sine waves
   - Take fft() of signal
3. Use #1 and #2 to figure out how the system alters your signal
   - R * fft(x)
4. Convert back into a signal

How do you perform the fft or ifft?

$\text{ifft}(\tilde{r}) = F^* \tilde{r} =$



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1. Figure out how the system will alter cosine and sine waves
   - Take fft() of impulse response
2. Figure out how your signal of interest is broken down into cosine and sine waves
   - Take fft() of signal
3. Use #1 and #2 to figure out how the system alters your signal
   - R * fft(x)
4. Convert back into a signal
   - ifft(R * fft(x))

How do we use linear shift invariant systems to analyze our data?