Mathematical Tools
for Neural and Cognitive Science

Fall semester, 2022

Section 1a: Trichromacy
(an extended linear algebra example)
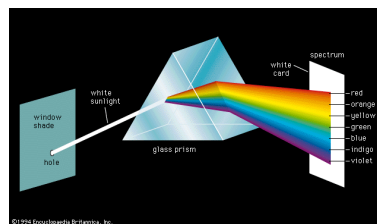

Trichromacy

A spectacular multi-disciplinary story …

- Physics: spectral nature and additivity of light [Newton, and others, 1600's]

- Perception: Trichromatic matching experiments [Grassman, Young, and others, 1850's]

- Mathematics: Human color matching is explained by a 3-dimensional linear model [late 1800's]

- Engineering: Devices for color reproduction require only three color channels [early 1930's]

- Neurobiology: Trichromacy in humans (and some other primates) arises from 3 cone types [late 1980's!]

[See Wandell, *Foundations of Vision*, ch2]

Spectral nature of light

[Newton, 1665]

Representing spectra with vectors
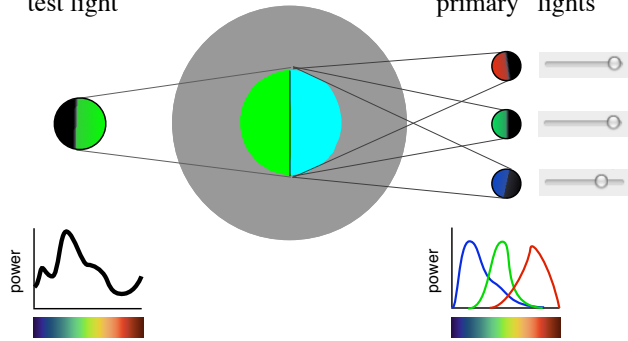
$$\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{31} \end{bmatrix}$$

Power (arbitrary units) vs Wavelength (nm)

[c/o David Brainard]

Intensity adjustment (scalar multiplication)

$\vec{b}$
$0.5\,\vec{b}$

Power (arbitrary units) vs Wavelength (nm)

[c/o David Brainard]

Additive mixtures

$\vec{b_1}$
$\vec{b_2}$
$\vec{b}$

$$\vec{b} = \vec{b_1} + \vec{b_2}$$

Power (arbitrary units) vs Wavelength (nm)

[c/o David Brainard]

Color perception: what's it for?

[c/o David Brainard]

Basic Color Matching
Perceptual color matching experiment

Test Light

Arbitrary
test light

Mixture of three
"primary" lights

Primary
Mixture

power

Wavelength (nm)

power

Monday, June 21, 2010

[Young, Helmholtz, Grassman, etc, 1800's]

Basic Color Matching
Perceptual color matching experiment

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test light

Mixture of three
"primary" lights

Primary
Mixture

power

Wavelength (nm)

power

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[Young, Helmholtz, Grassman, etc, 1800's]

Slide 1

CRT Phosphor emission spectra

— Red
— Green
— Blue

Power (watts/sr/m ^{2}/5 nm)

0.02

0.01

0

380   460   540   620   700   780

Wavelength (nm)

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Slide 2

Grassmann's Laws (1853)

1) Any light can be matched with a mixture of 3 primaries

2) Rescaling the light results in a rescaled mixture

3) Adding two lights results in a sum of their mixtures

➡ Color matching can be described by an Nx3 linear system*

* Fine print: i) Normal human observer; ii) photopic intensity levels, not too bright, not too dim; iii) independent primaries.

Slide 3

Experimental measurement:

$\begin{bmatrix} \\ \vec{b} \\ \\ \end{bmatrix}$ $\begin{bmatrix} \vec{p_r} & \vec{p_g} & \vec{p_b} \end{bmatrix}$

p_r p_g p_b

Human subject

b   0.75   0.50

b Linear model: $\begin{bmatrix} 0.25 \\ 0.75 \\ 0.50 \end{bmatrix} = \begin{bmatrix} & \mathbf{H} & \end{bmatrix} \begin{bmatrix} \\ \vec{b} \\ \\ \end{bmatrix}$

[c/o David Brainard]

Implications

- If *P* is an *Nx3* matrix containing the primary spectra, then the color matching experiment is captured by a *3xN* "matching matrix" *H* that maps a light $\vec{l}$ to 3 "knob settings":
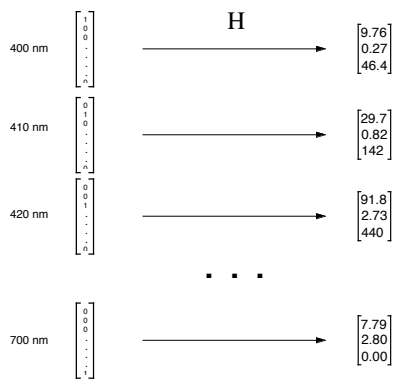
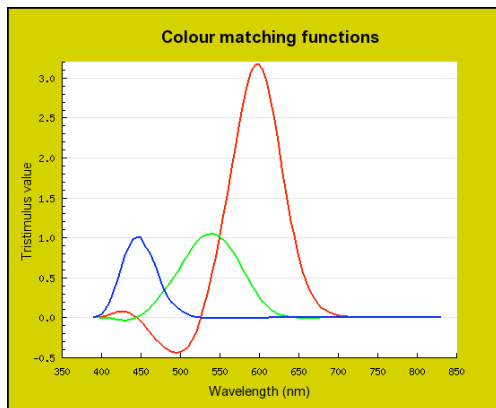$$\vec{l} \sim PH\vec{l}$$

where $\sim$ means "looks the same as"

- Two lights look the same if (and *only* if) they produce the same match settings:

$$\vec{l_1} \sim \vec{l_2} \quad \Leftrightarrow \quad H\vec{l_1} = H\vec{l_2}$$

Characterization

[c/o David Brainard]

Colour matching functions

Stiles-Burch Data, Monochromatic RGB Primaries of 645, 526, and 444 nm.

A matrix that mimics a human!   [Stiles & Burch, 1959]

Useful calculations

- Scientific: given results of one matching experiment, predict the results of another one (with different primaries).

- Practical: calibrate a display device, so as to generate mixtures of three colors that match the appearance of any desired real-world spectrum.
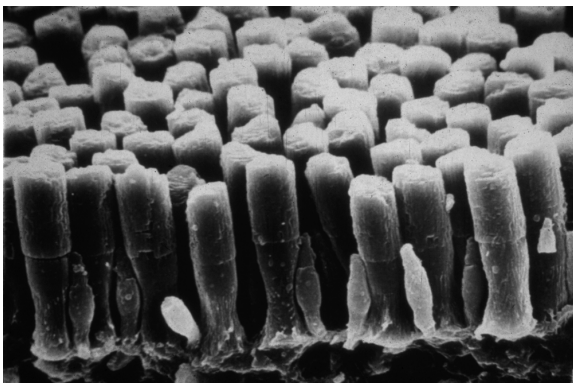
[derive on board]

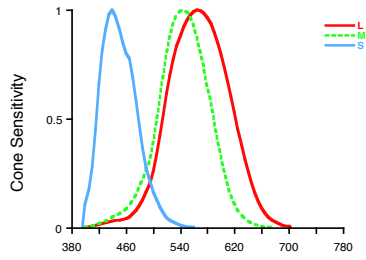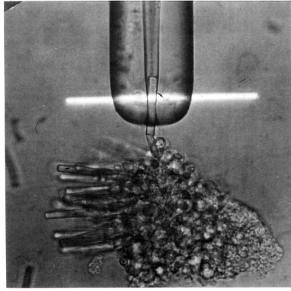## Summary:

- Perceptual measurements: any light with can be matched with a mixture of 3 primaries

- Theory/model: the visual system projects the wavelength spectra of light onto a 3-dimensional subspace
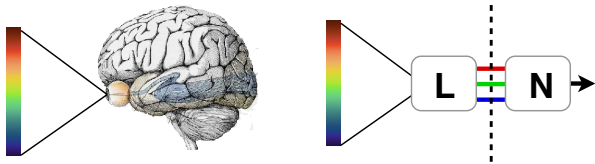
- Prediction/generalization: perceptual "metamers" - lights that appear identical, but have physically distinct wavelength spectra

- Engineering: CIE standards for color representation (1931).

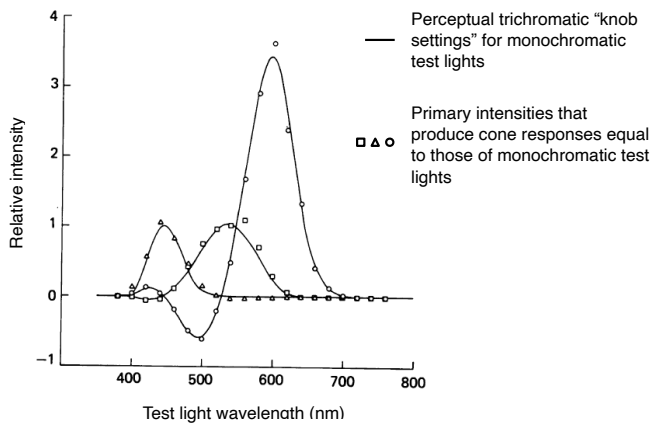- Underlying physiological mechanism (cones), verified in 1987!

The underlying mechanism...

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[Baylor, Nunn & Schnapf, 1987]

Confirmation of prediction (100+ years later!):

— Perceptual trichromatic "knob settings" for monochromatic test lights

□ △ ○ Primary intensities that produce cone responses equal to those of monochromatic test lights

[Baylor, Nunn & Schnapf, '87]

Dimensionality caveats

- Normal human vision, at high ("photopic") light levels is tri-chromatic (cones)

- At low ("scotopic") light levels, we are all monochromats (rods only)

- At intermediate ("mesopic") light levels, we are quadrachromats (rods&cones)!

- Common genetic forms of color blindness are due to lack of one or two cone types