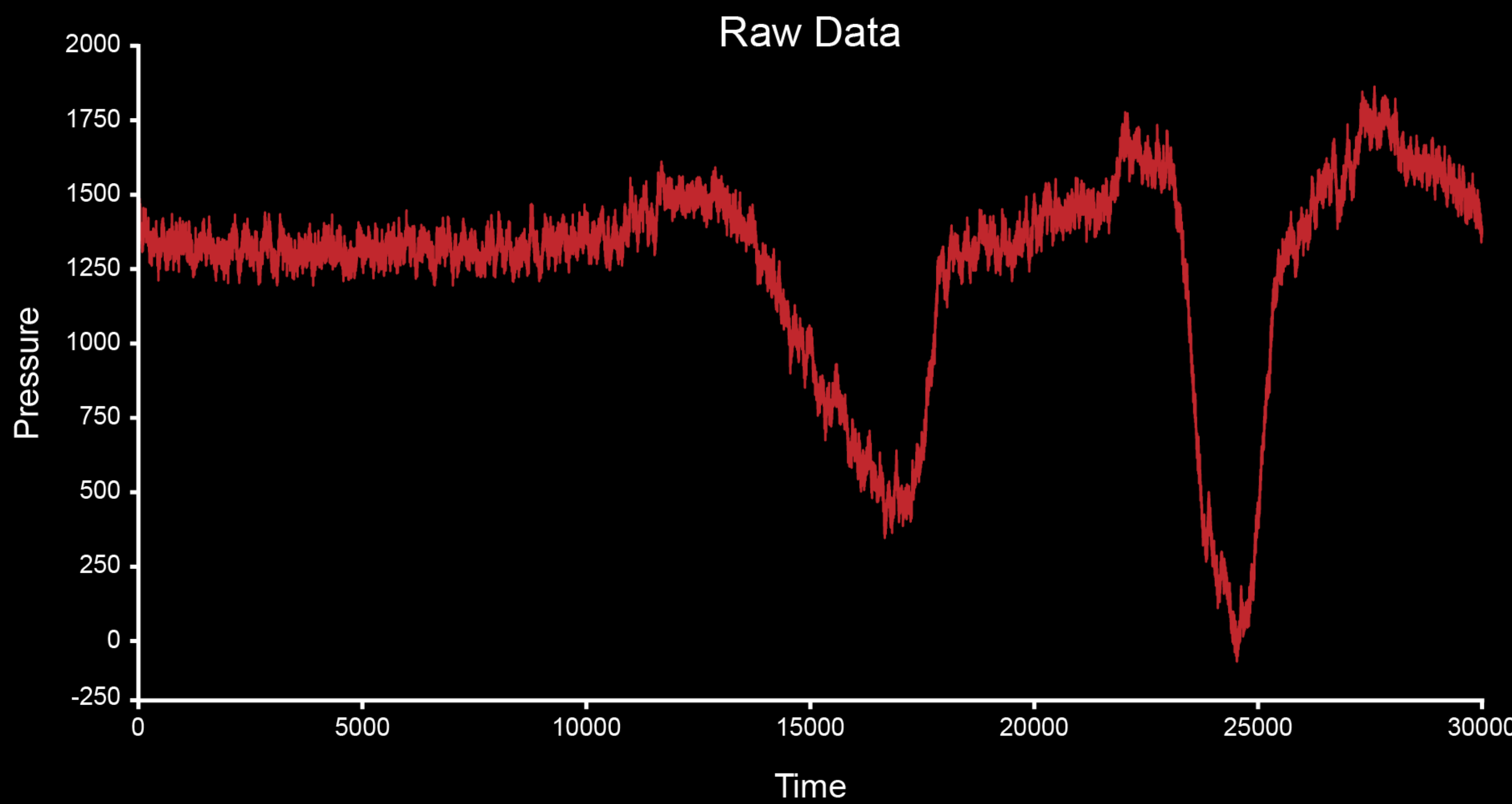
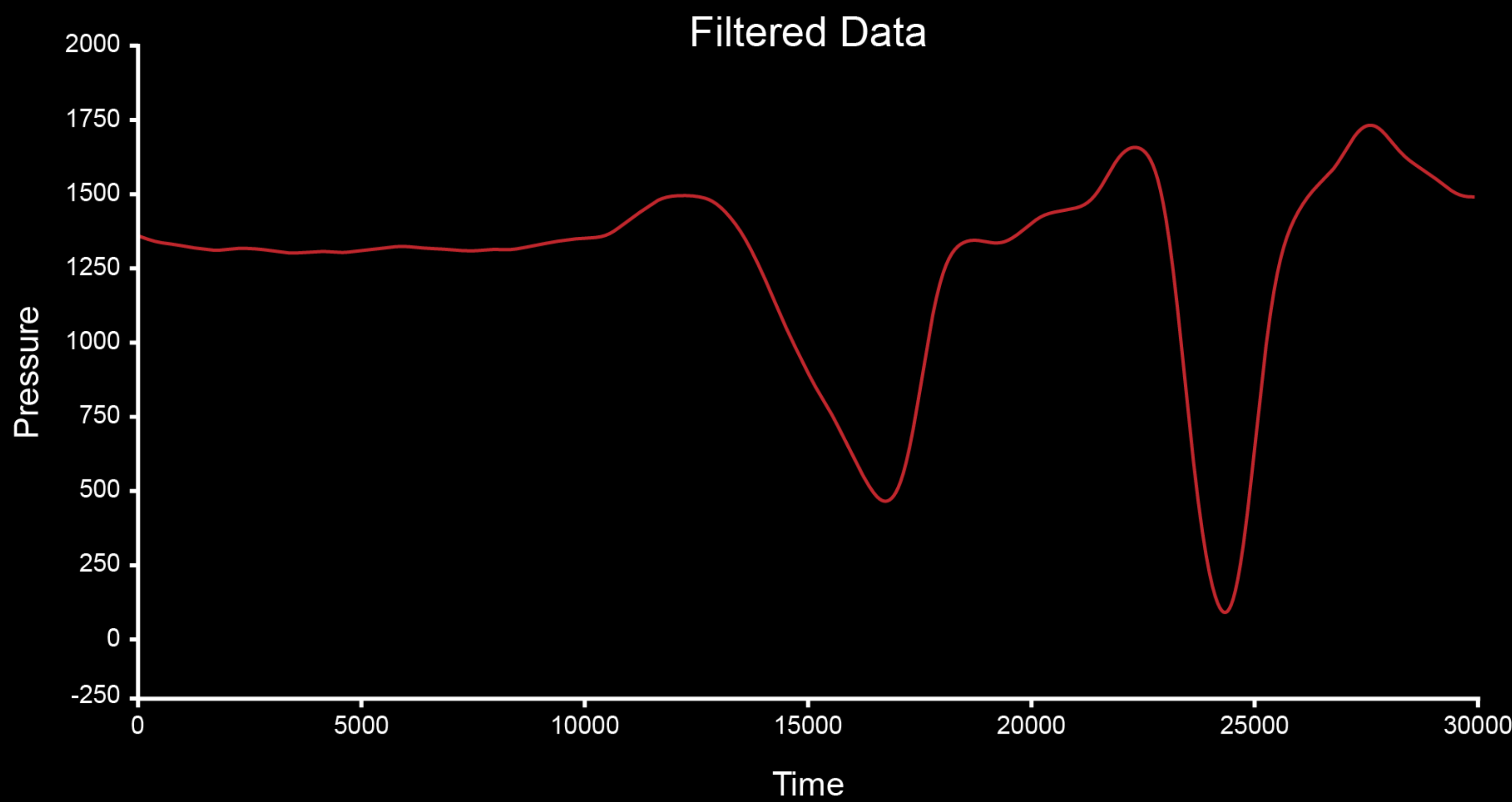
Lab VI: Fourier Transform

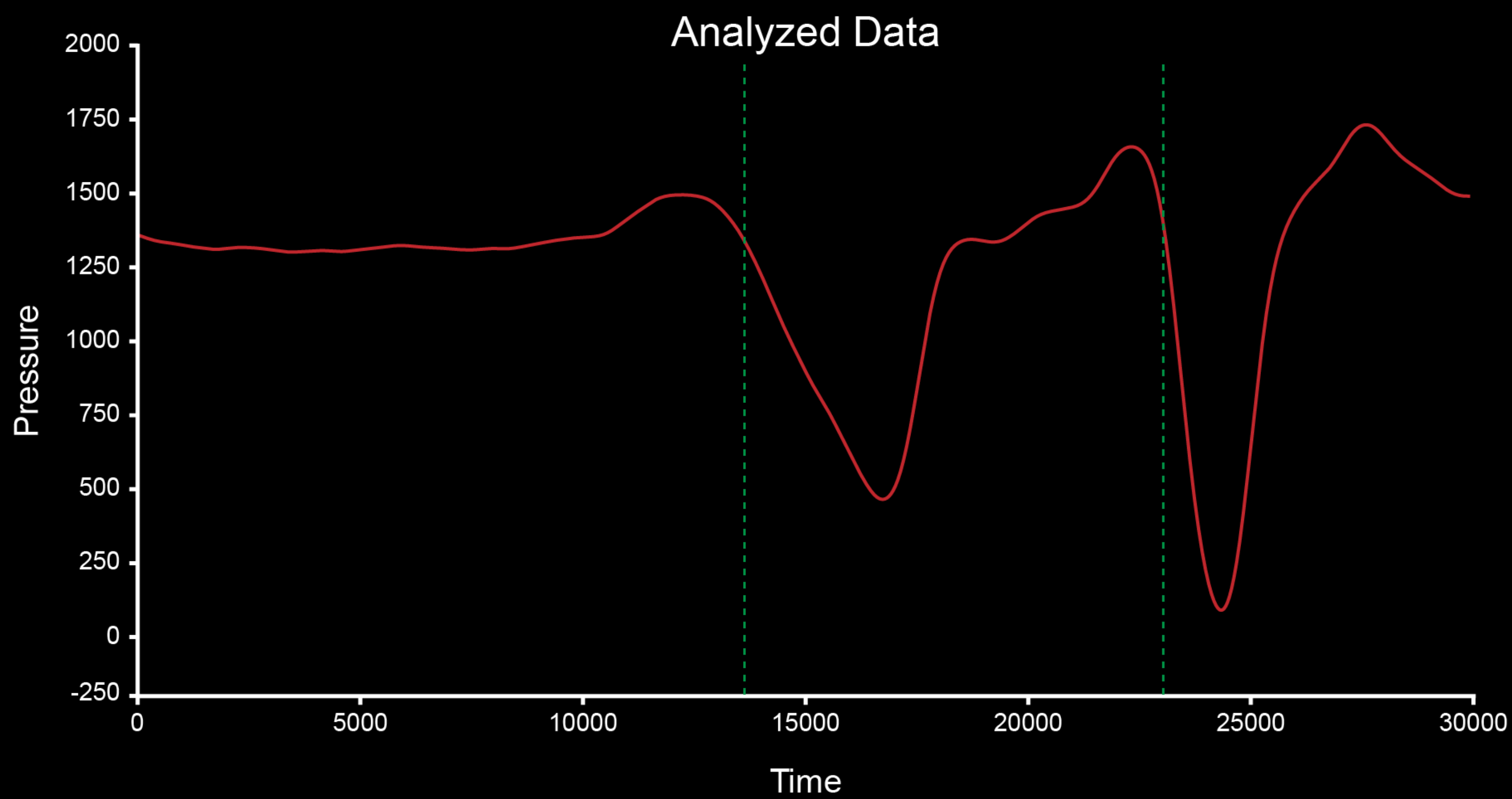
Motivating Example: removing noise



Motivating Example: removing noise



Motivating Example: removing noise



How do we process signals?

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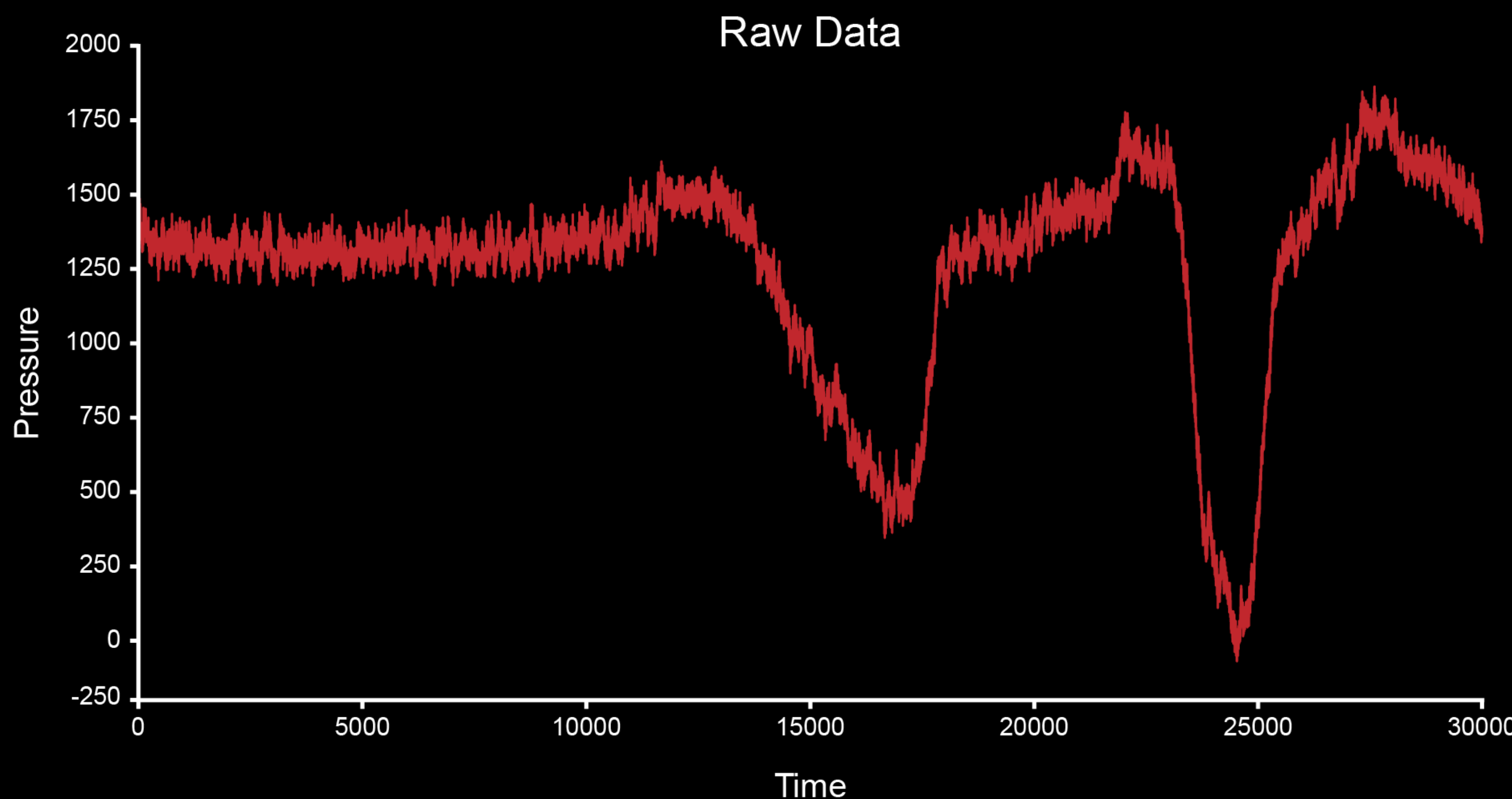
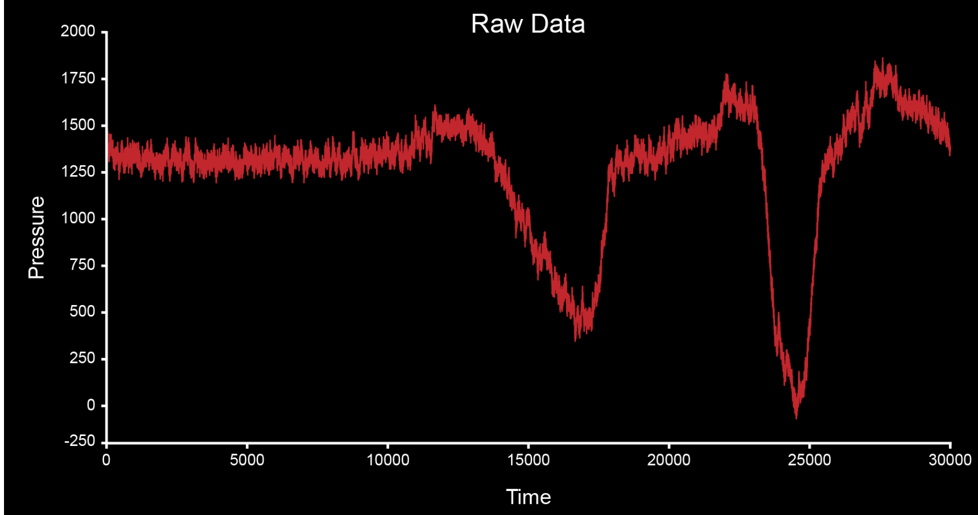
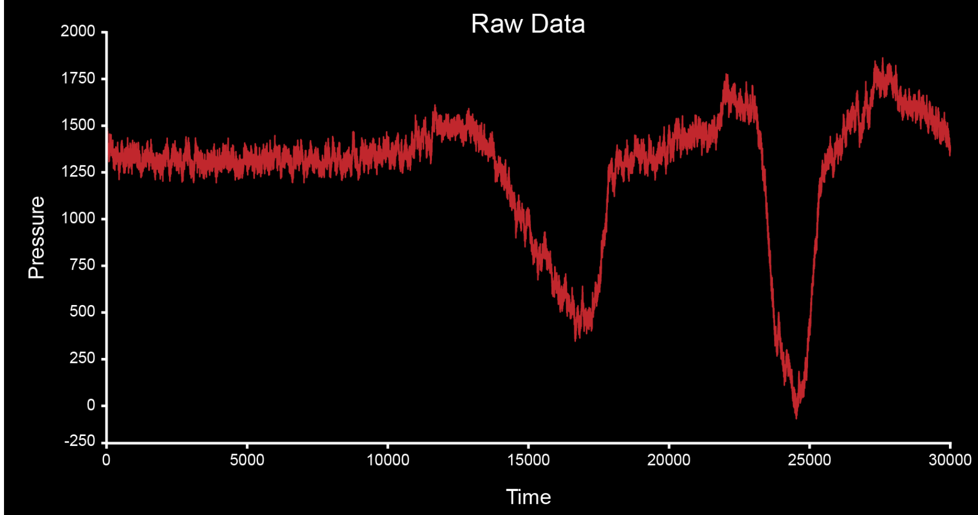


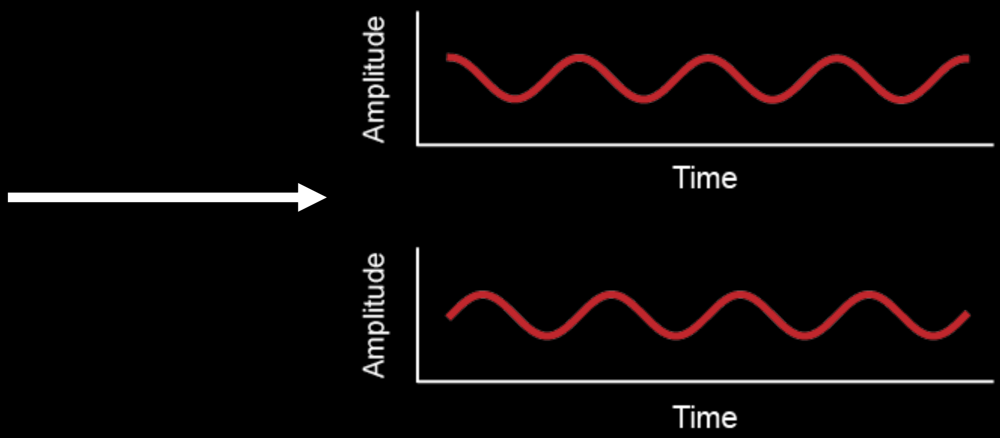
How do we process signals?



Simpler signals that are easier to process

How do we process signals?





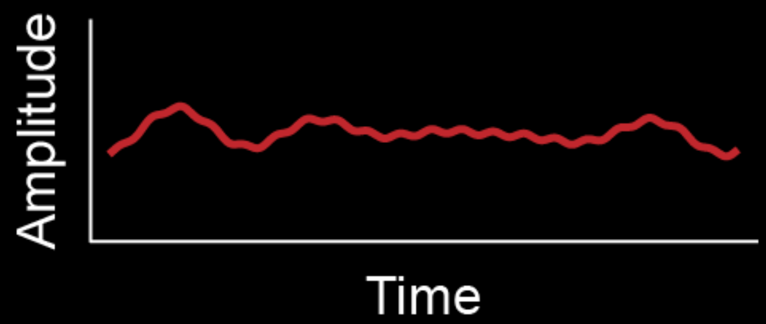
Signals are combinations of signals…

- We can create any signal using a sum of sines and cosines
- Trust me
- Or trust 3blue1brown:

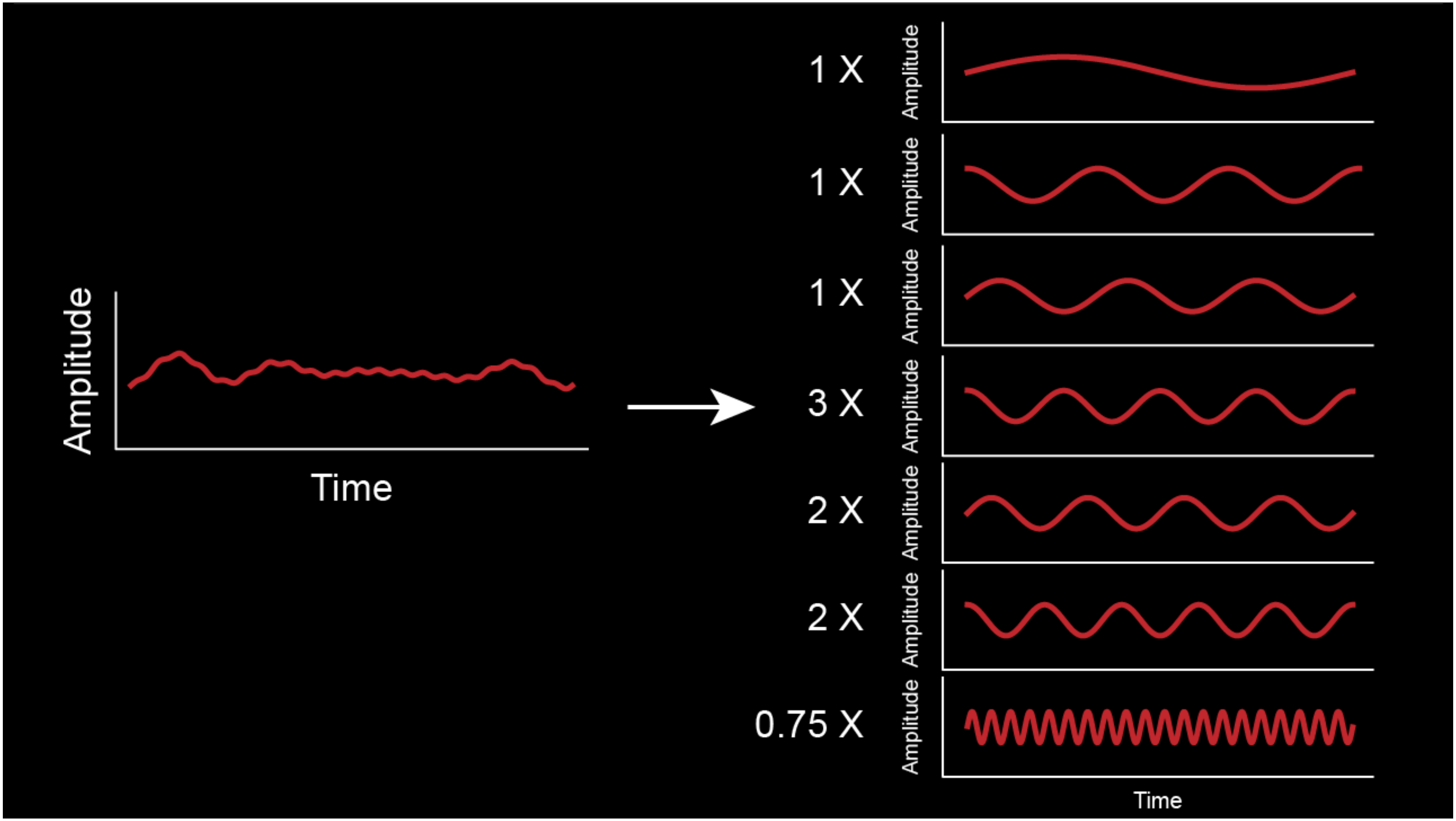
Signals are combinations of signals…

https://www.youtube.com/watch?v=r6sGWTCMz2k

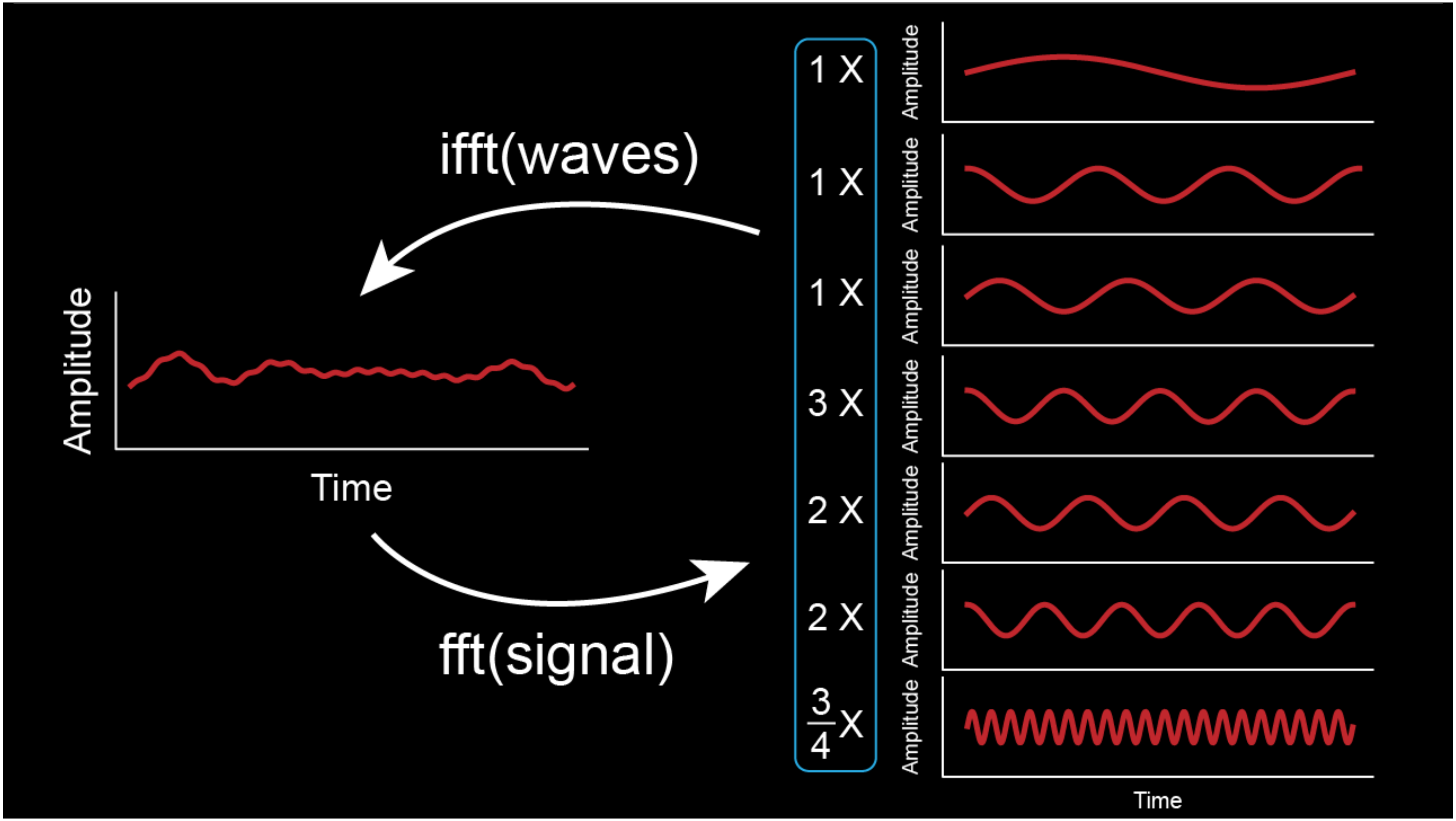
- 5:11-5:50
- 0:34-1:36

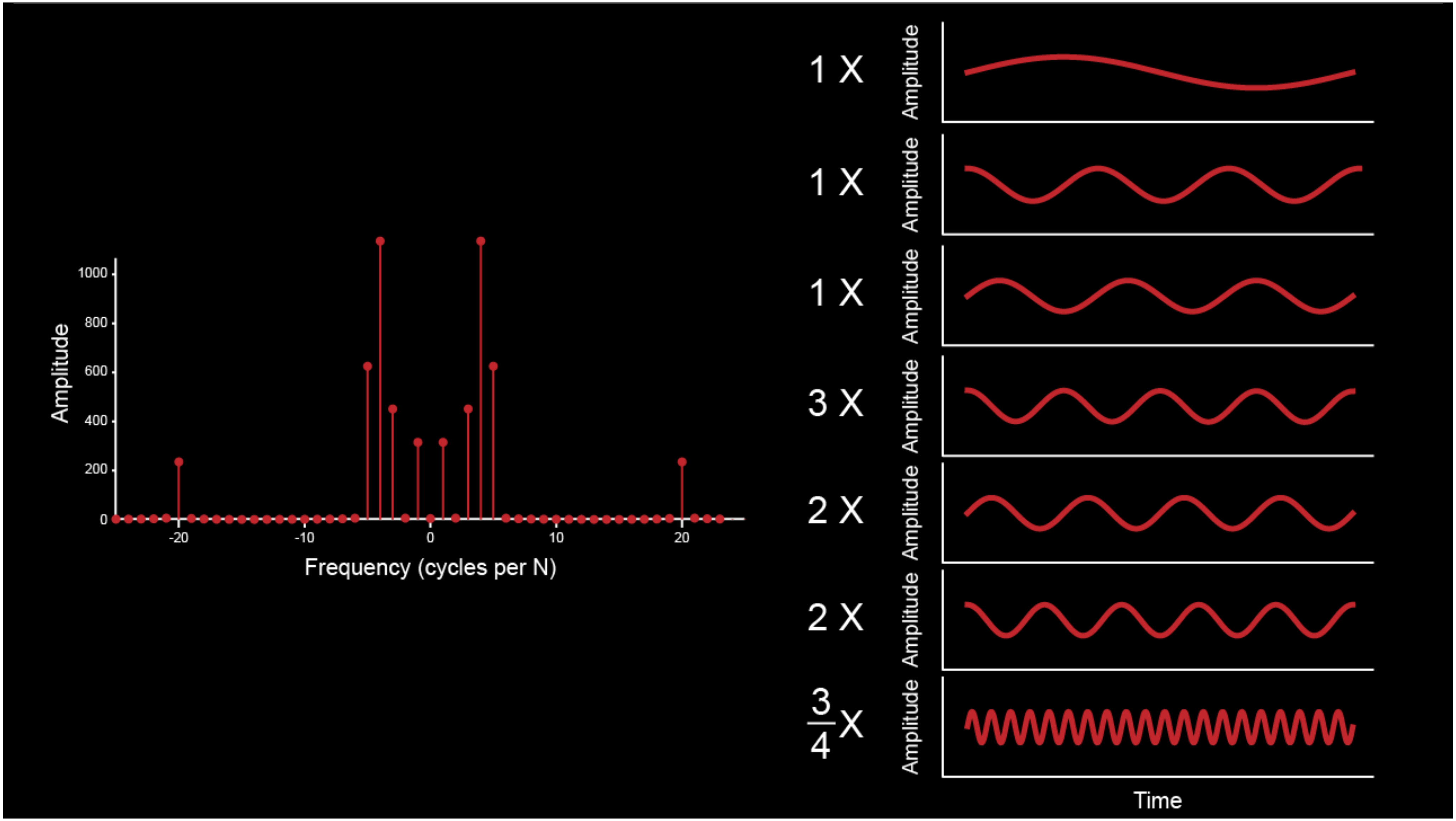


Amplitude
Time

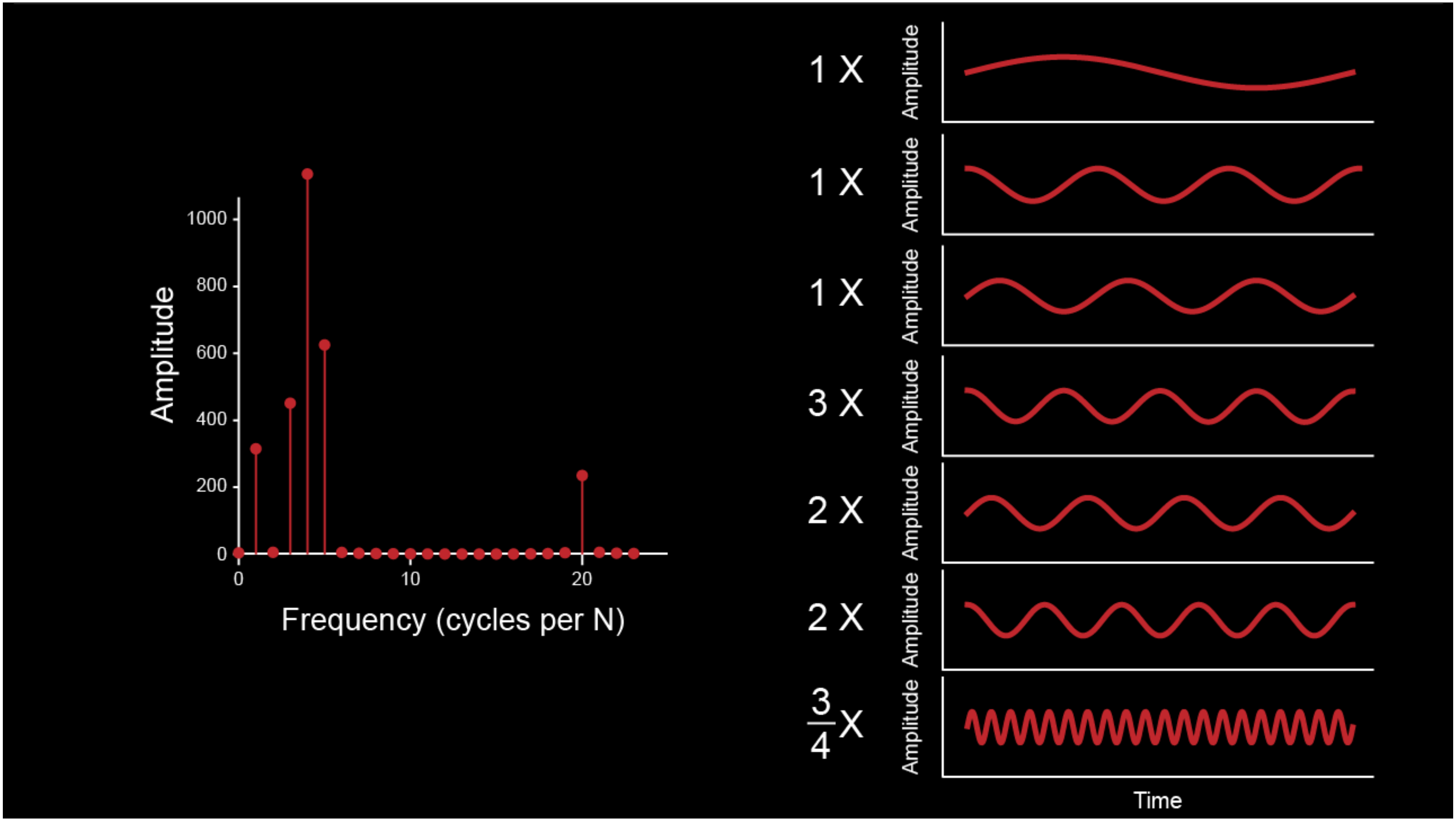


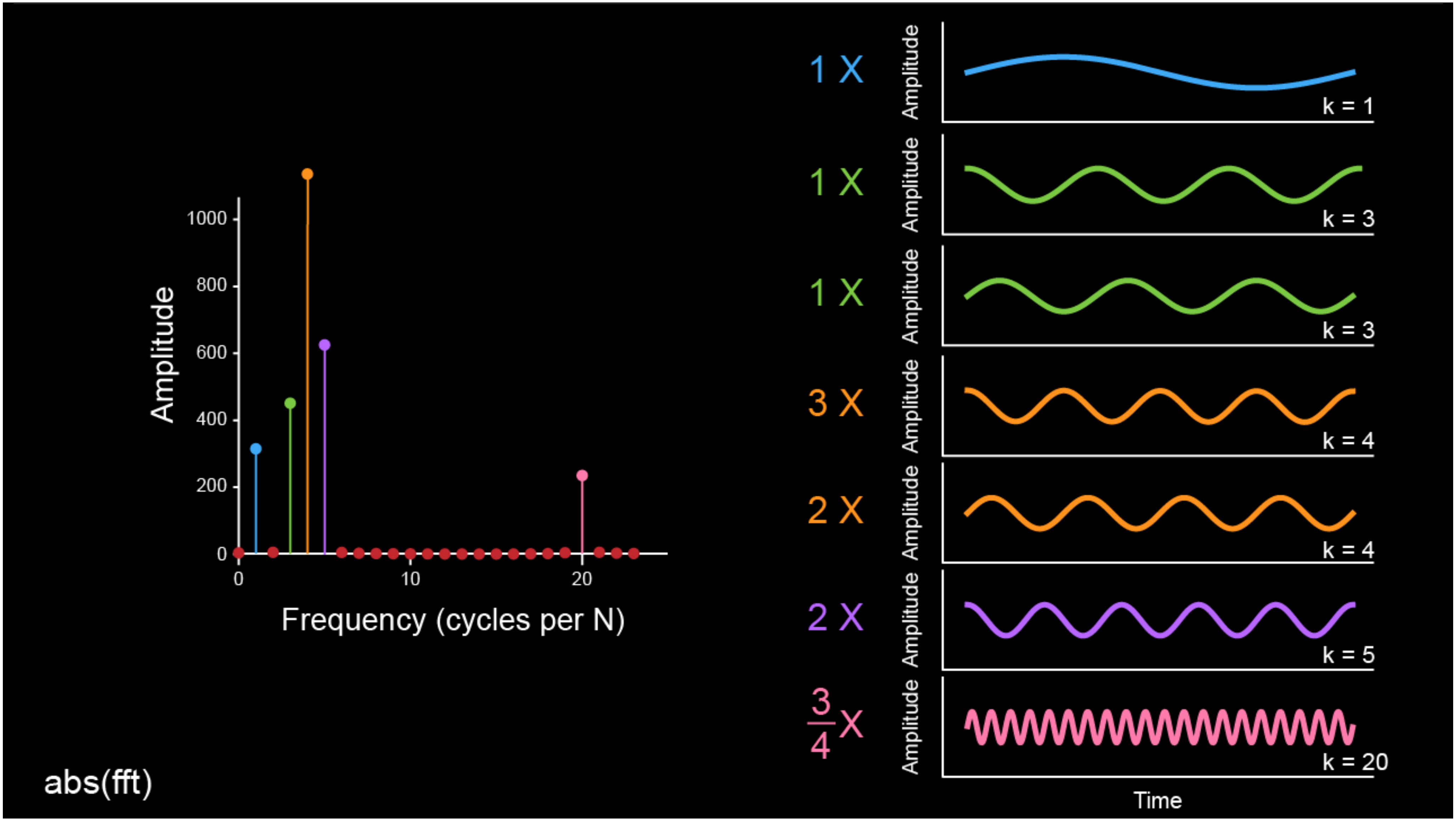
Amplitude
Time
1 X
1 X
1 X
3 X
2 X
2 X
0.75 X
Amplitude
Time


ifft(waves)
Amplitude
Time
fft(signal)
1 X
1 X
1 X
3 X
2 X
2 X
$\frac{3}{4}$X
Amplitude
Amplitude
Amplitude
Amplitude
Amplitude
Amplitude
Amplitude
Time

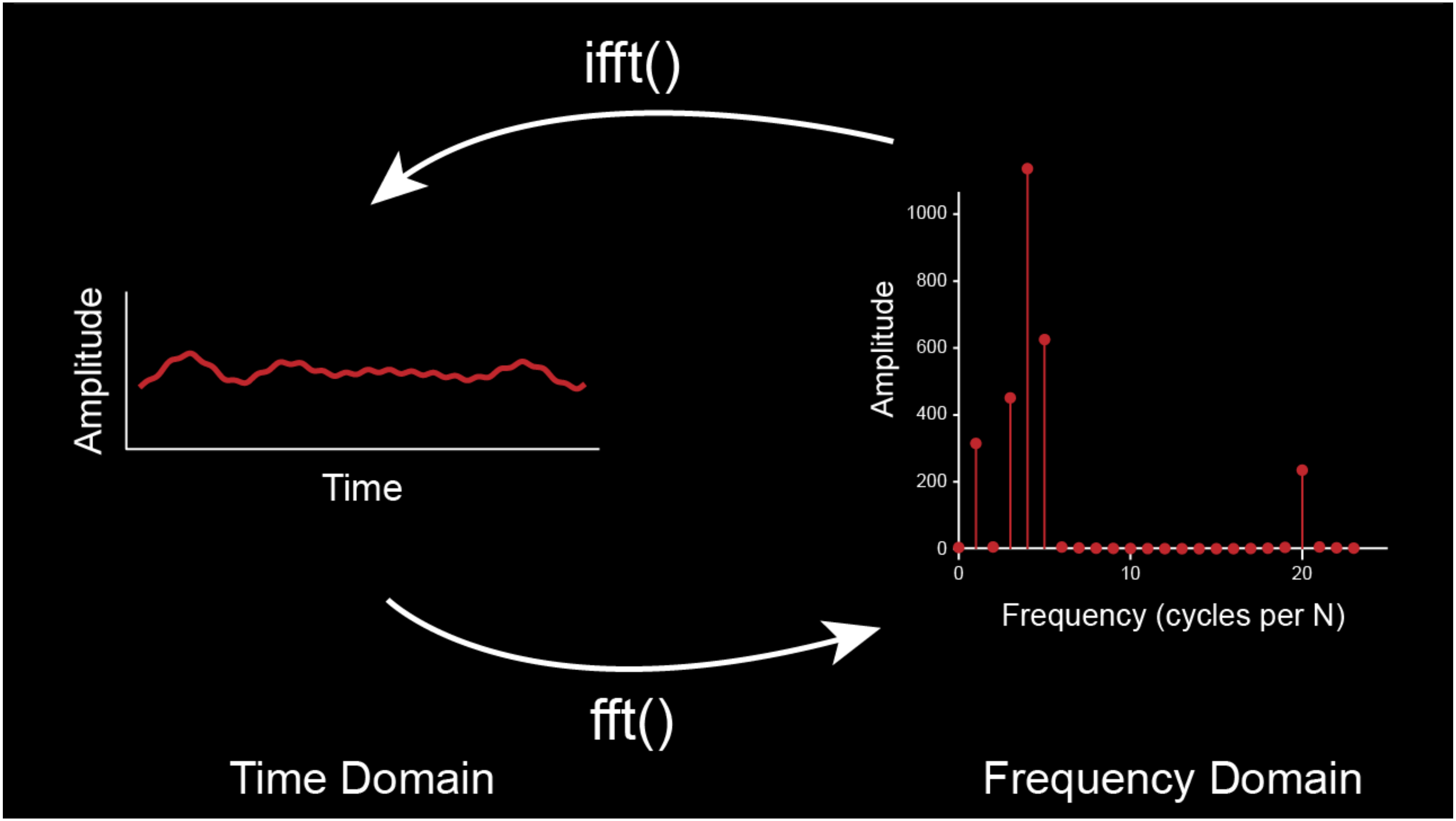


Amplitude
1000
800
600
400
200
0
-20
-10
0
10
20
Frequency (cycles per N)
1 X
1 X
1 X
3 X
2 X
2 X
$\frac{3}{4}$X
Amplitude
Time


Amplitude
1000
800
600
400
200
0
0
10
20
Frequency (cycles per N)
1 X
1 X
1 X
3 X
2 X
2 X
$\frac{3}{4}$X
Amplitude
Time



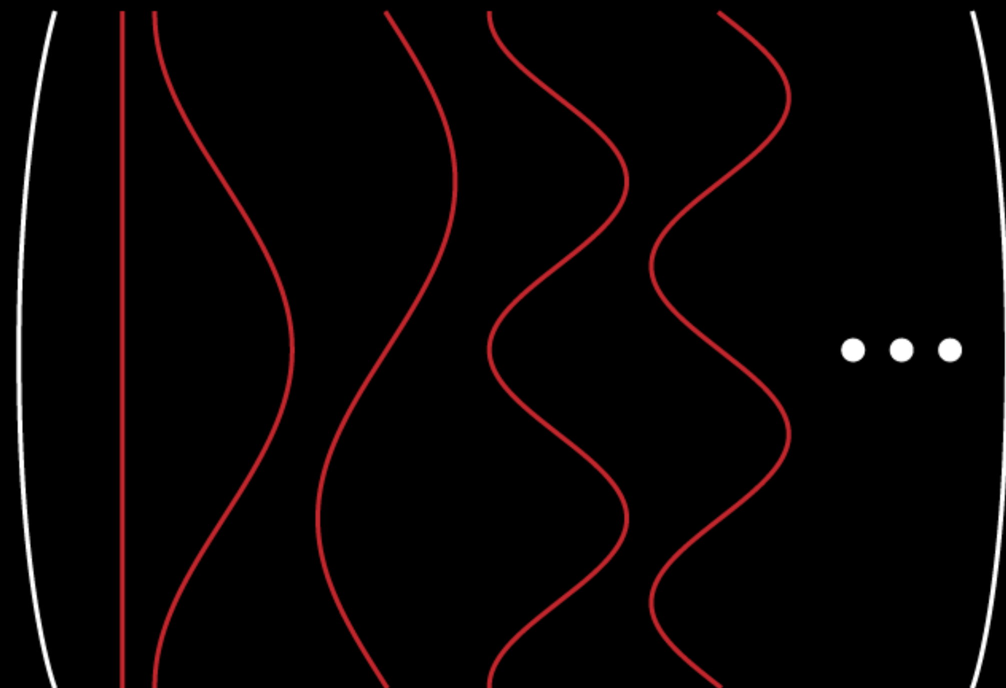
Amplitude
1000
800
600
400
200
0
0
10
20
Frequency (cycles per N)
abs(fft)
1 X
1 X
1 X
3 X
2 X
2 X
3/4 X
Amplitude
k = 1
k = 3
k = 3
k = 4
k = 4
k = 5
k = 20
Time


ifft()
Amplitude
Time
Amplitude
1000
800
600
400
200
0
0
10
20
Frequency (cycles per N)
fft()
Time Domain
Frequency Domain

How do you perform the fft or ifft?

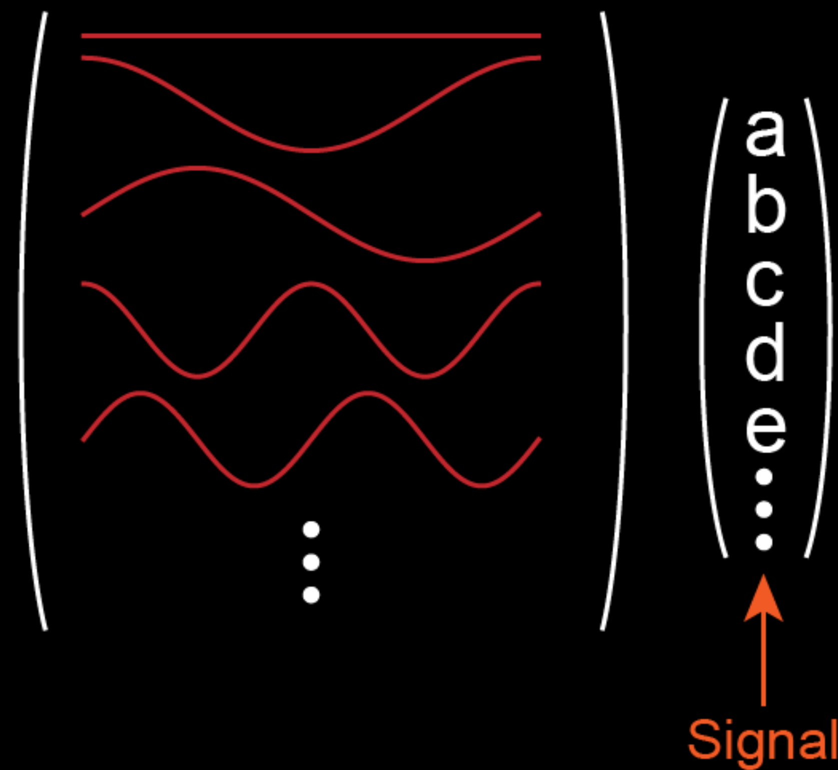
How do you perform the fft or ifft?

$F =$



How do you perform the fft or ifft?

$$\text{fft}(\vec{r}) = F^T * \vec{r} =$$



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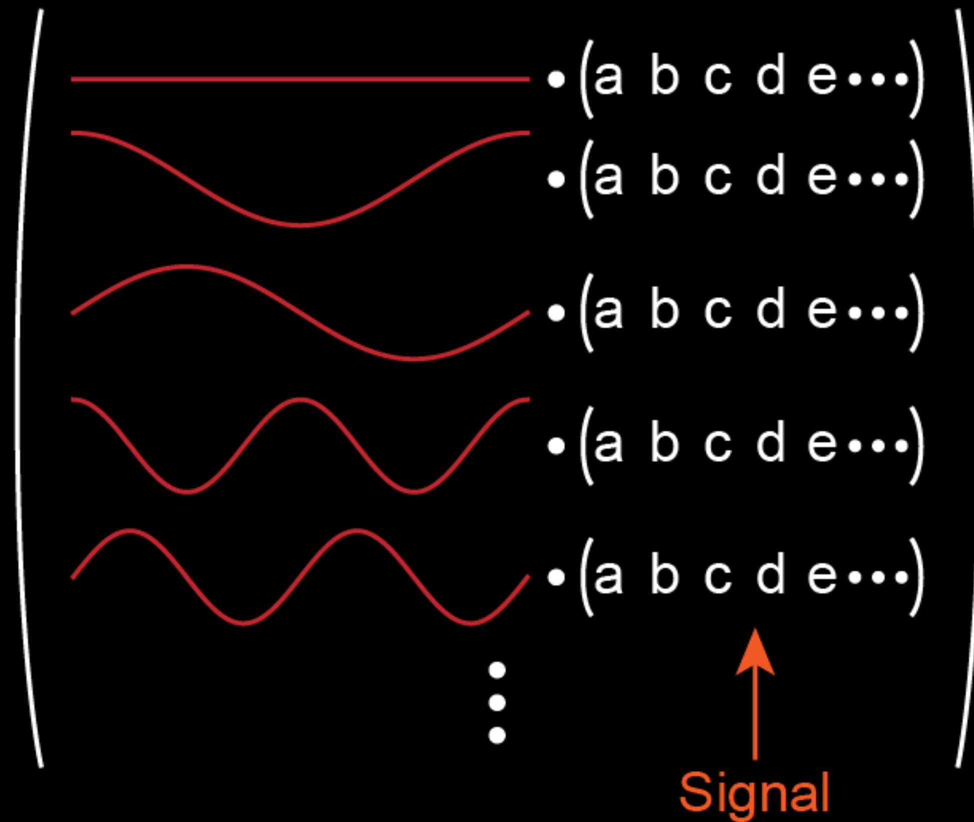

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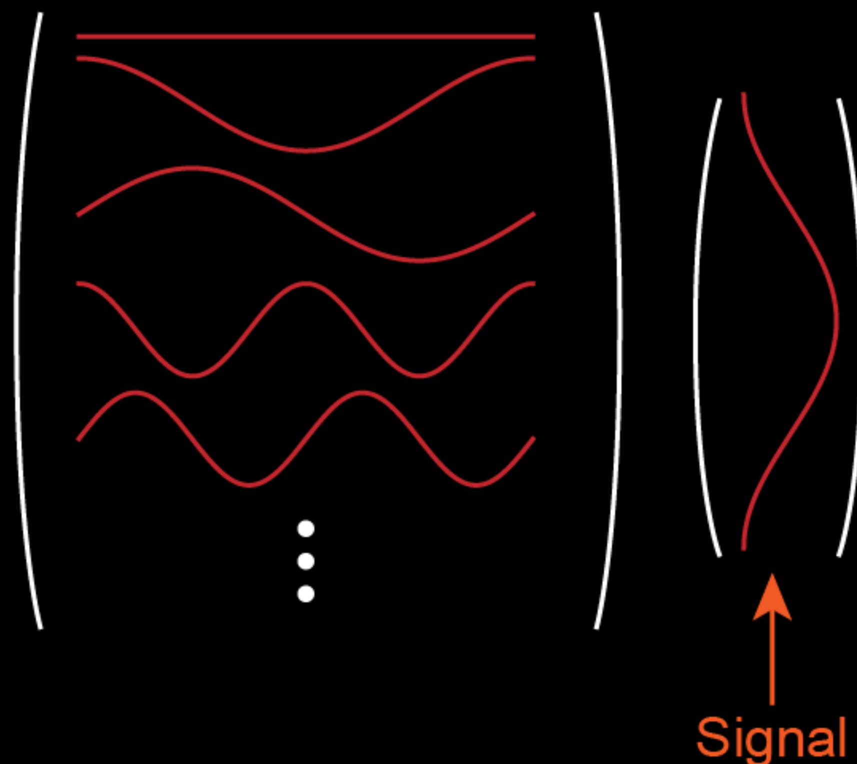
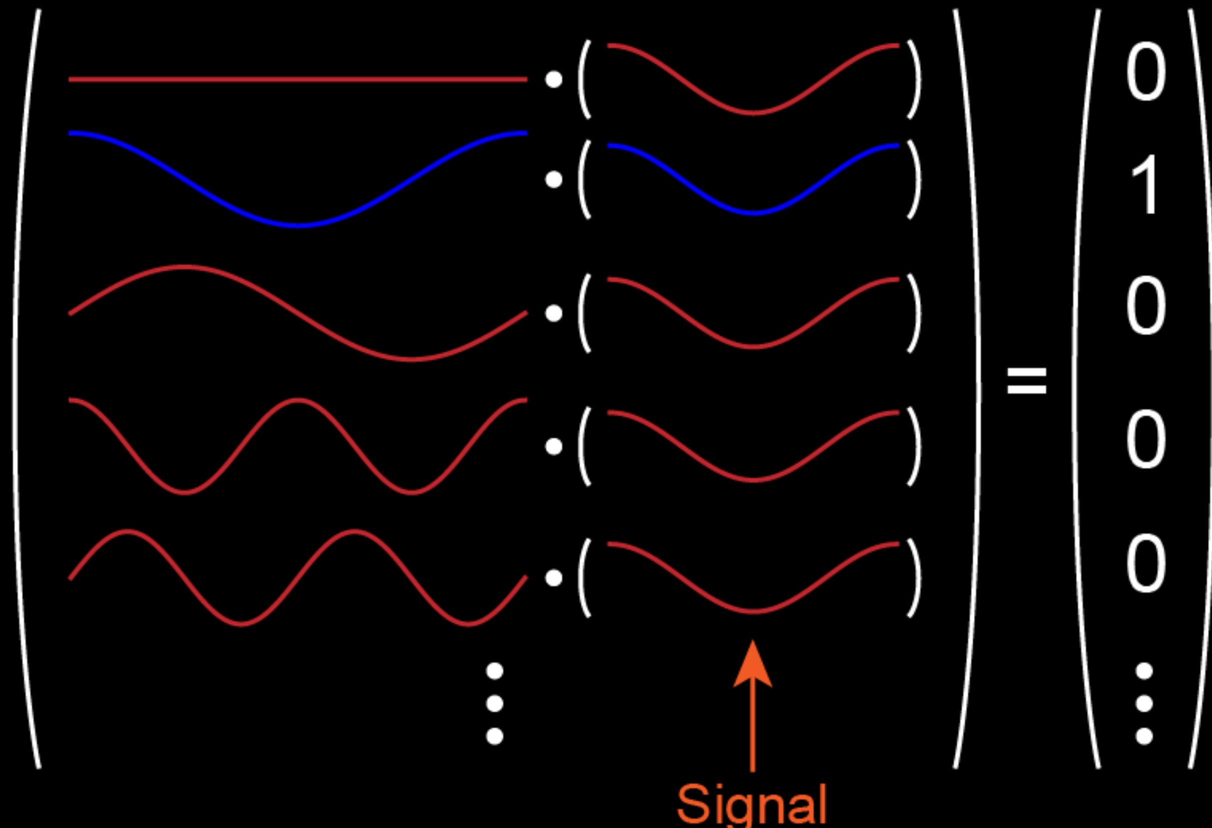


How do you perform the fft or ifft?

$$\mathrm{fft}(\vec{r}) = F^T * \vec{r} =$$



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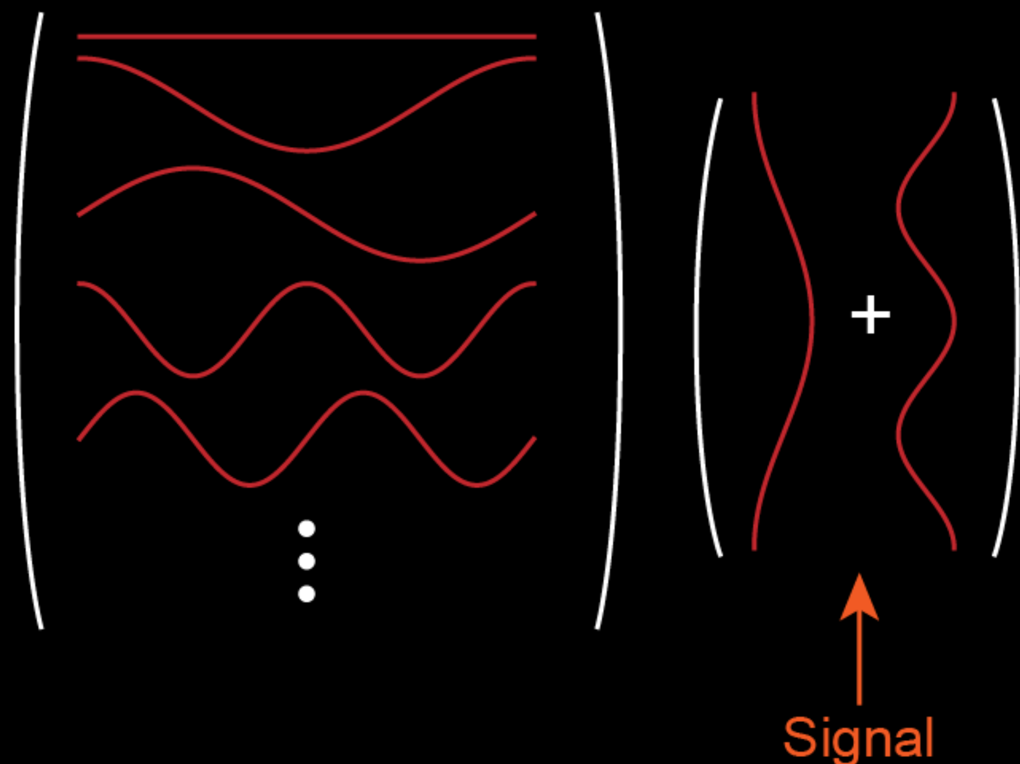
$$\mathrm{fft}(\vec{r}) = F^{T} * \vec{r} =$$



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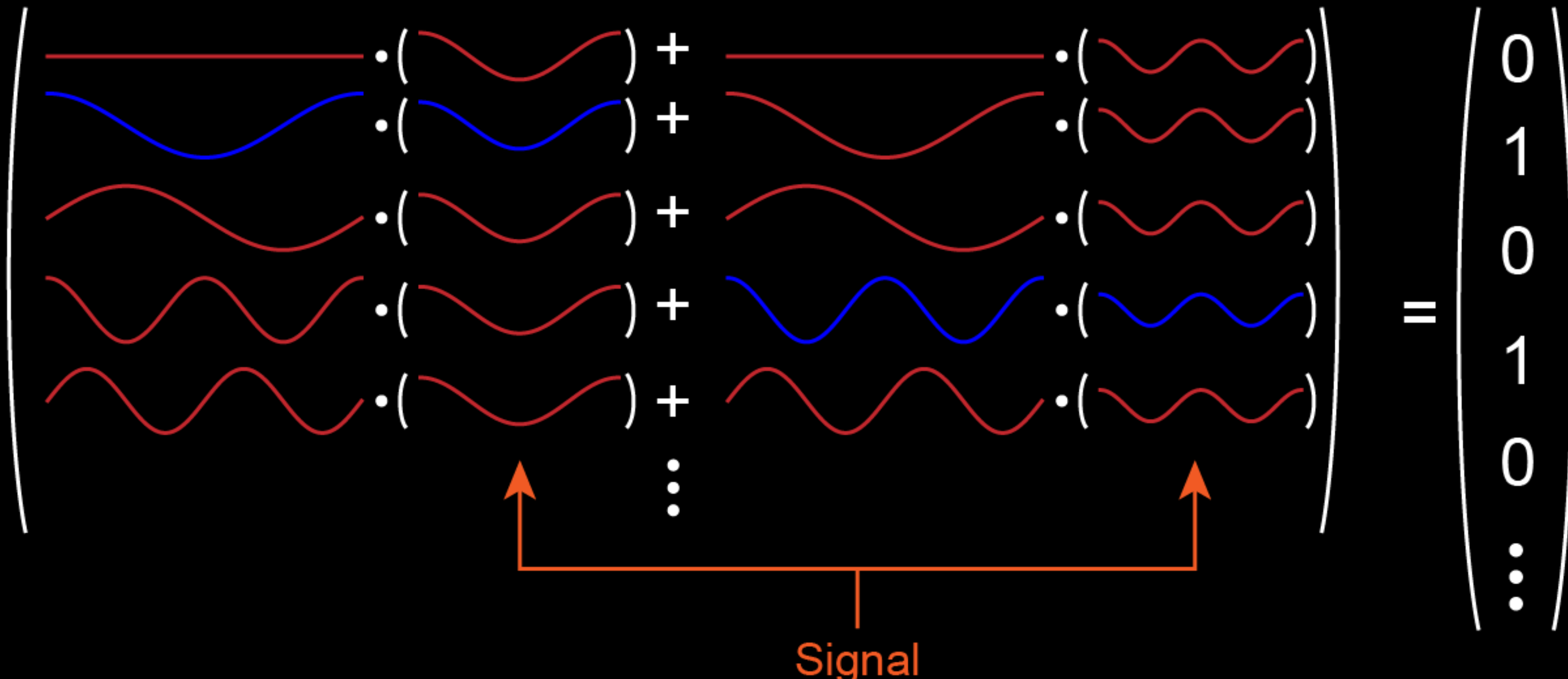
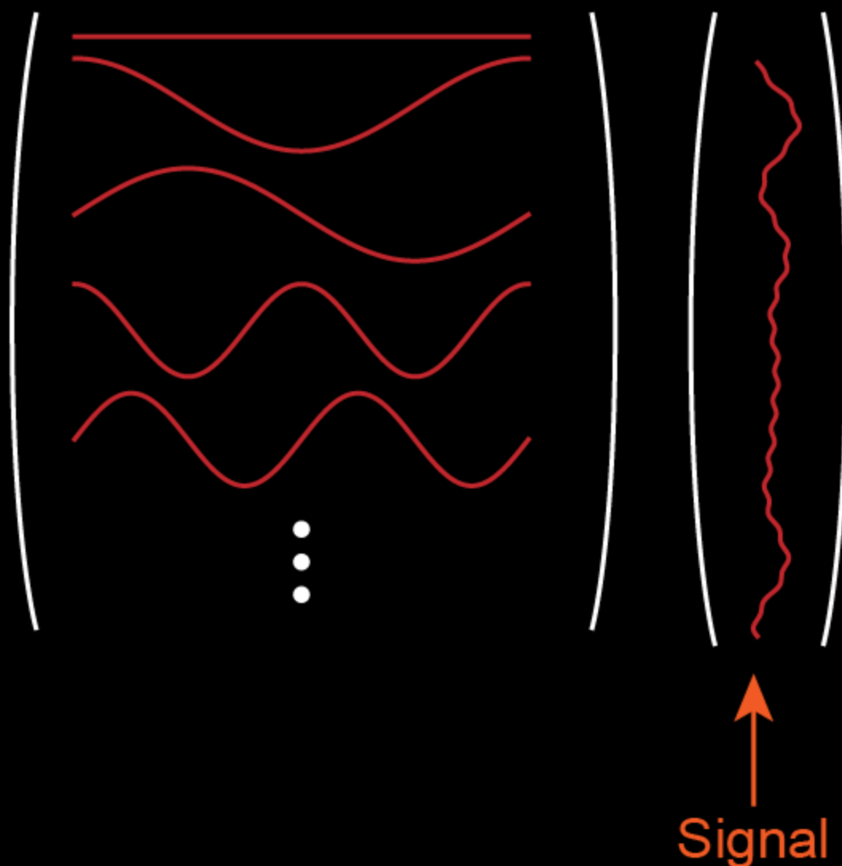
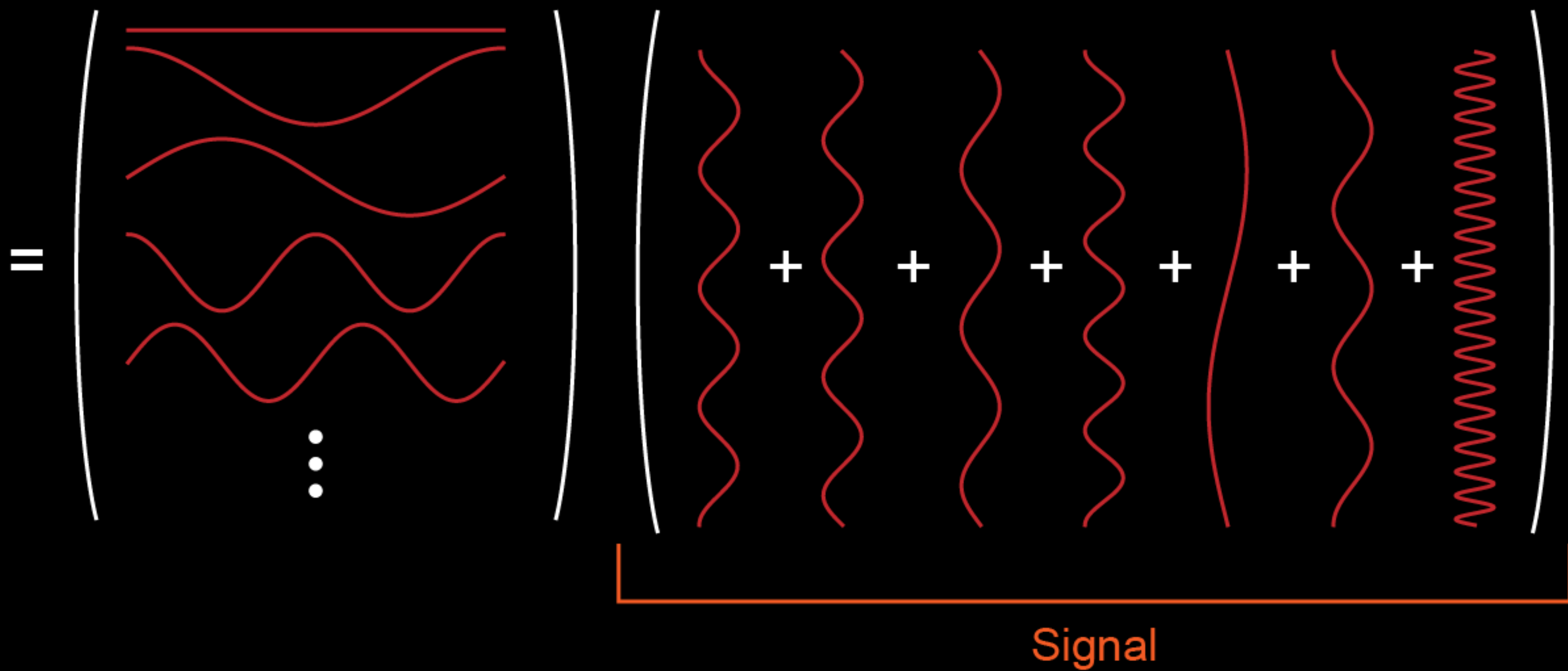


How do you perform the fft or ifft?

$\mathrm{fft}(\vec{r}) = F^T * \vec{r} =$



How do you perform the fft or ifft?



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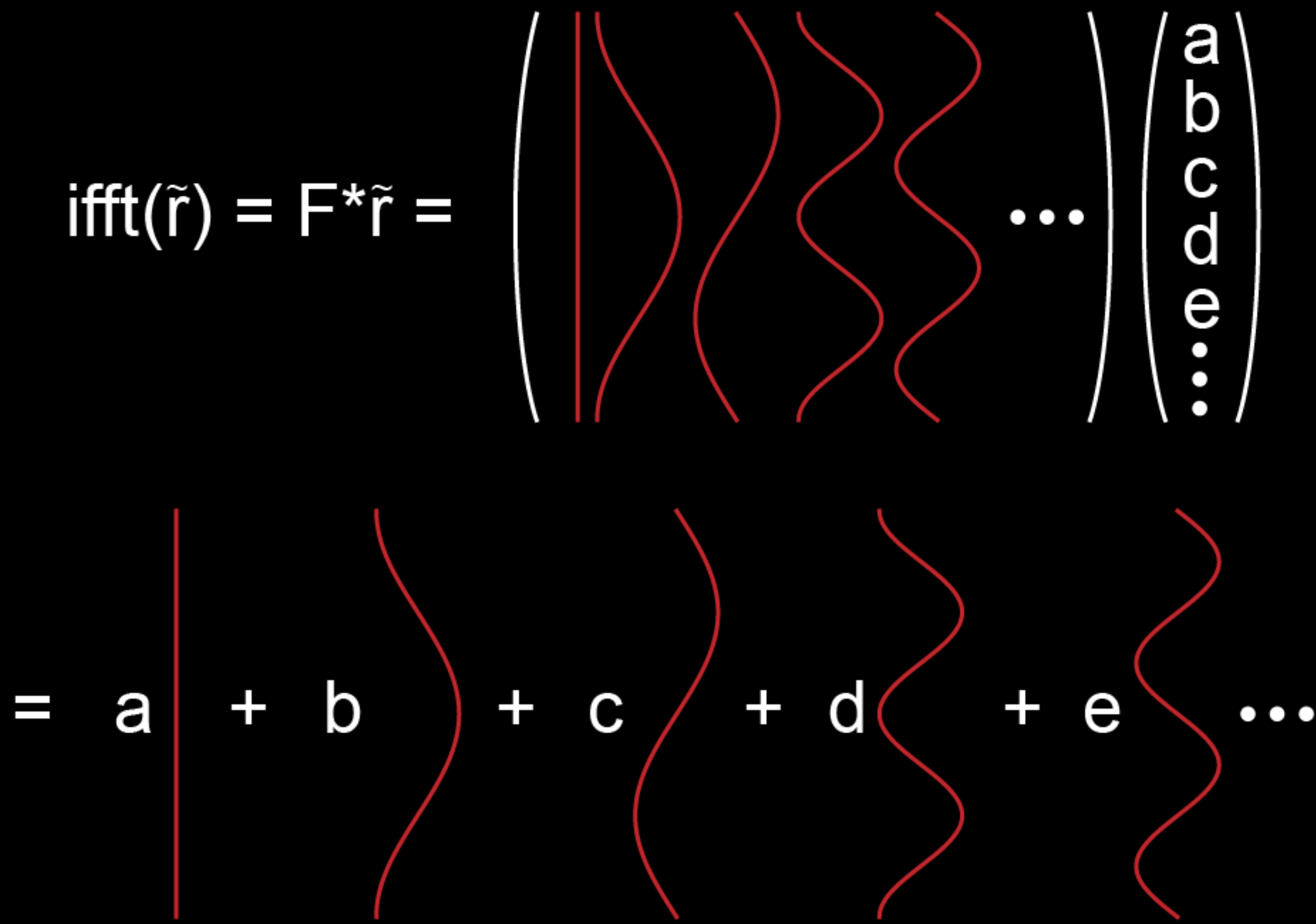

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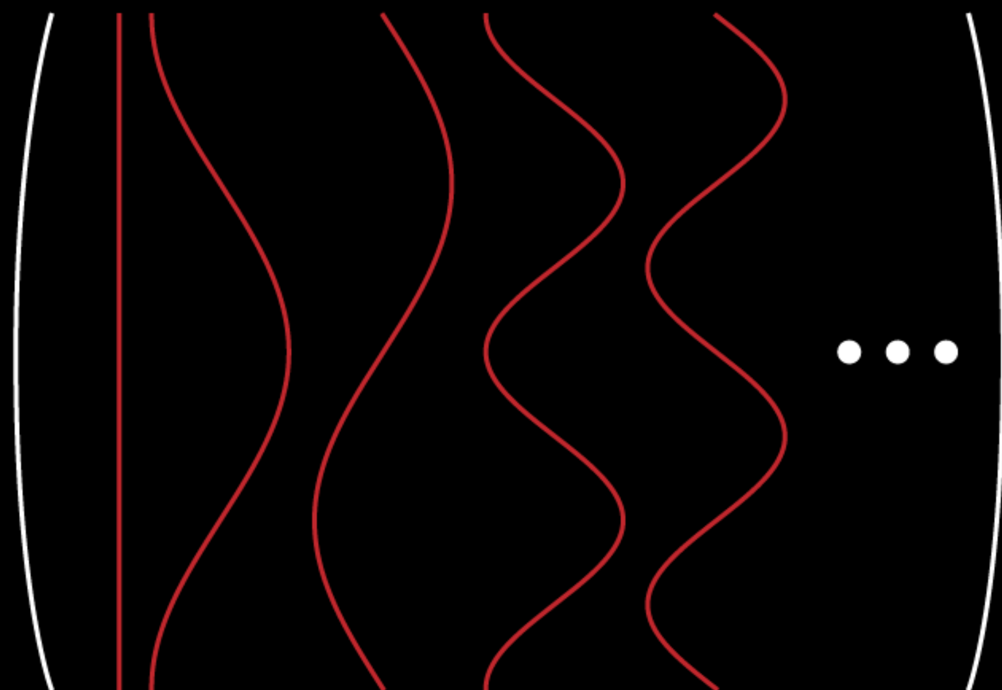
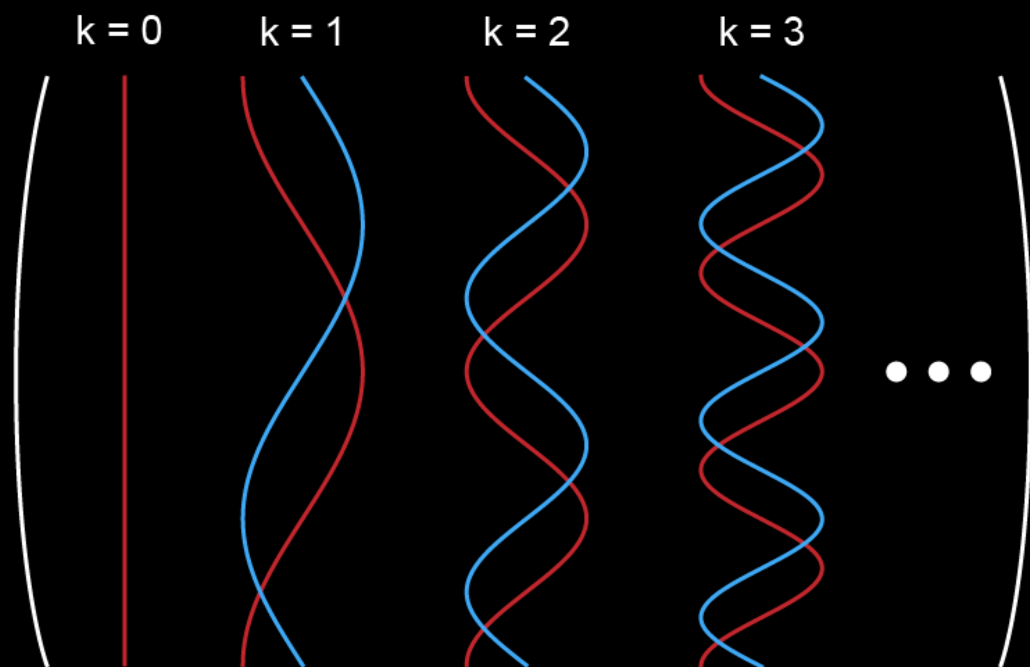
F =



How do you perform the fft or ifft?

$F =$



$$e^{\frac{i * 2\pi k * n}{N}} = \cos\left(\frac{2\pi k}{N} n\right) + i*\sin\left(\frac{2\pi k}{N} n\right)$$

How do you *actually* perform the fft?

Signal (N x 1)

1) $\tilde{r} = \mathrm{fft}(\vec{r})$

Frequency components (N x 1, complex)

1) $\tilde{r} = \text{fft}(\vec{r})$

(i) $\tilde{r} = \text{real}(\text{fft}(\vec{r}))$

(ii) $\tilde{r} = \text{imag}(\text{fft}(\vec{r}))$

(iii) $\tilde{r} = \text{abs}(\text{fft}(\vec{r}))$

(iv) $\tilde{r} = \text{angle}(\text{fft}(\vec{r}))$

$$e^{\frac{i * 2\pi k * n}{N}} = \cos\left(\frac{2\pi k}{N} n\right) + i*\sin\left(\frac{2\pi k}{N} n\right)$$

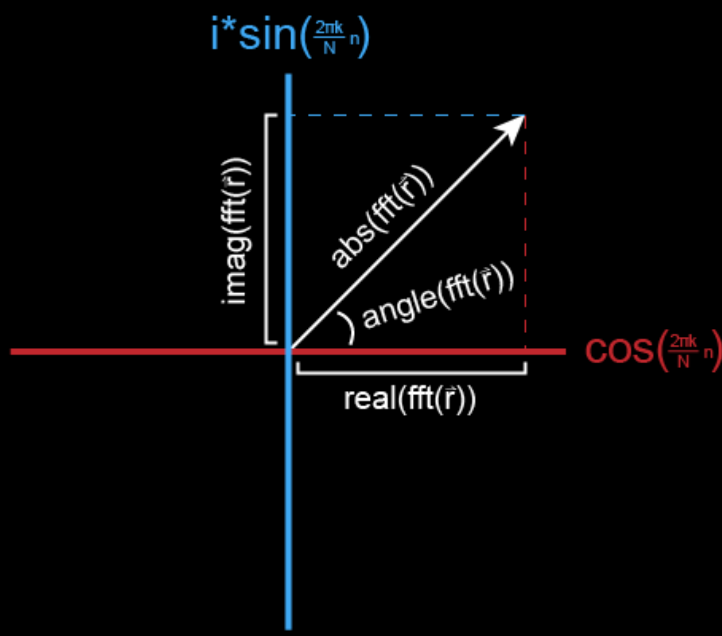
1) $\tilde{r} = \mathrm{fft}(\vec{r})$

   (i) $\tilde{r} = \mathrm{real}(\mathrm{fft}(\vec{r}))$

   (ii) $\tilde{r} = \mathrm{imag}(\mathrm{fft}(\vec{r}))$

   (iii) $\tilde{r} = \mathrm{abs}(\mathrm{fft}(\vec{r}))$

   (iv) $\tilde{r} = \mathrm{angle}(\mathrm{fft}(\vec{r}))$

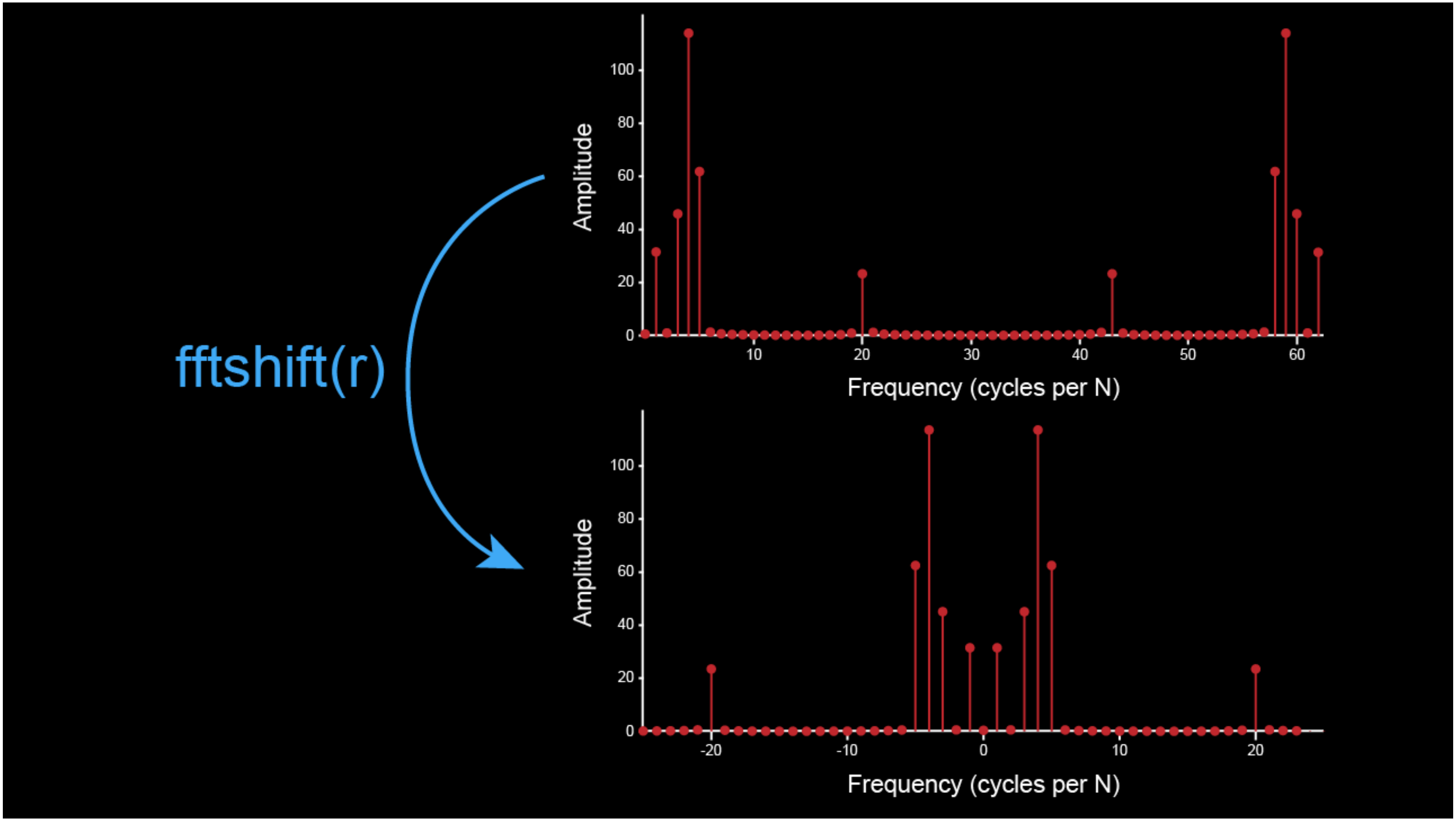


Frequency components (N x 1, complex)

1) $\tilde{r} = \mathrm{fft}(\vec{r})$

2) $\tilde{r}_{\mathrm{shifted}} = \mathrm{fftshift}(\tilde{r})$

Frequency components (N x 1, complex)
Rearranged to make interpretation easier


fftshift(r)
Amplitude
Frequency (cycles per N)
Amplitude
Frequency (cycles per N)

1) $\tilde{r} = \mathrm{fft}(\vec{r})$

2) $\tilde{r}_{shifted} = \mathrm{fftshift}(\tilde{r})$

3) $\mathrm{stem}(\, x,\ \tilde{r}_{shifted}\ )$

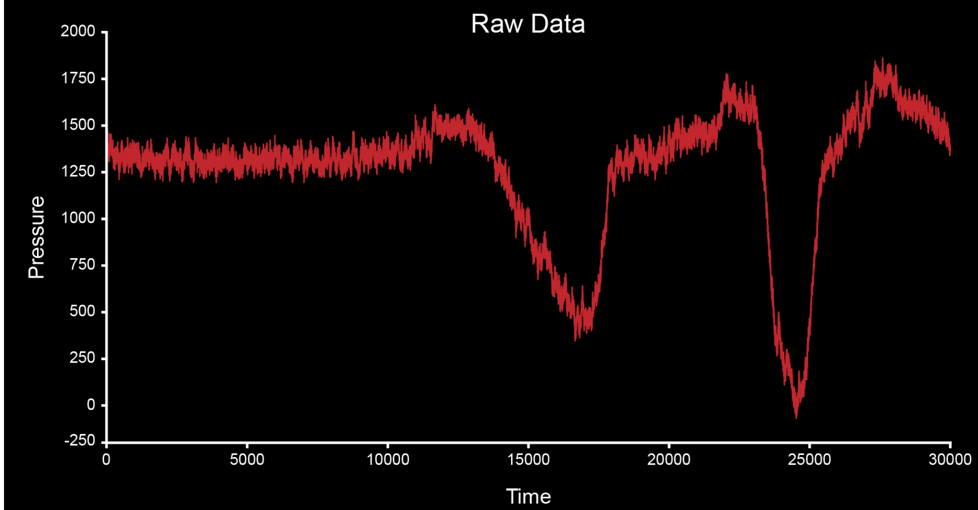
Frequencies
-N/2 -> N/2

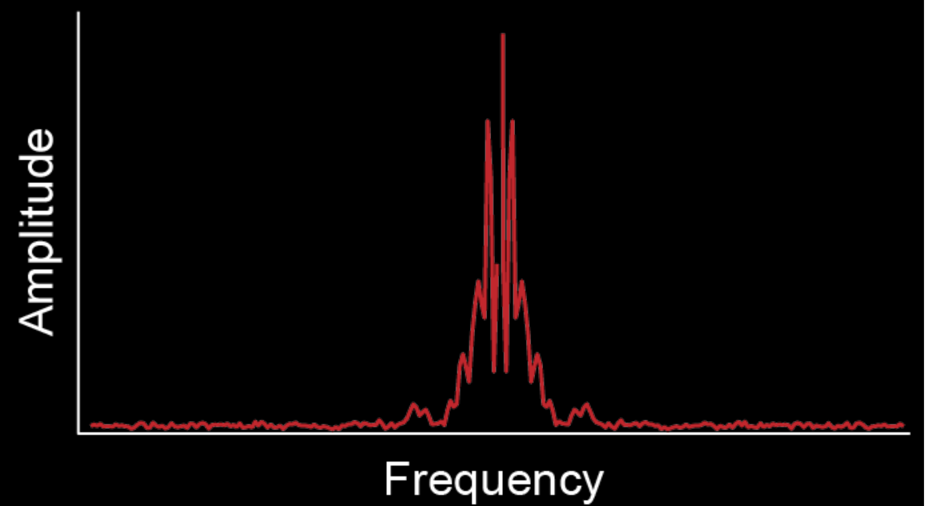
Frequency components (N x 1, complex)
Rearranged to make interpretation easier

Exercise #1

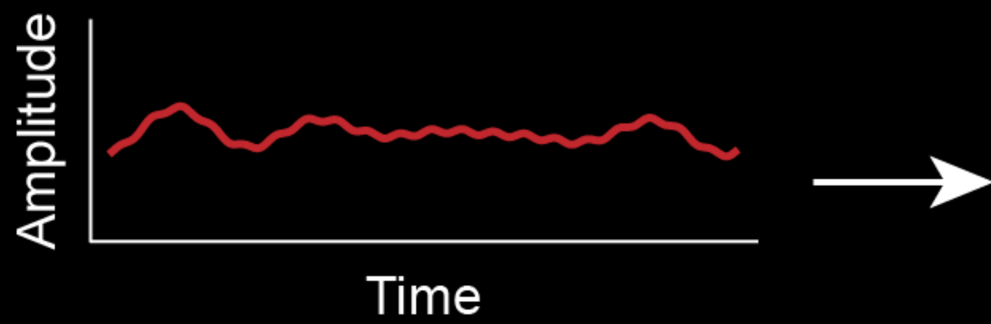
Fourier Transform and Shift-Invariant Systems

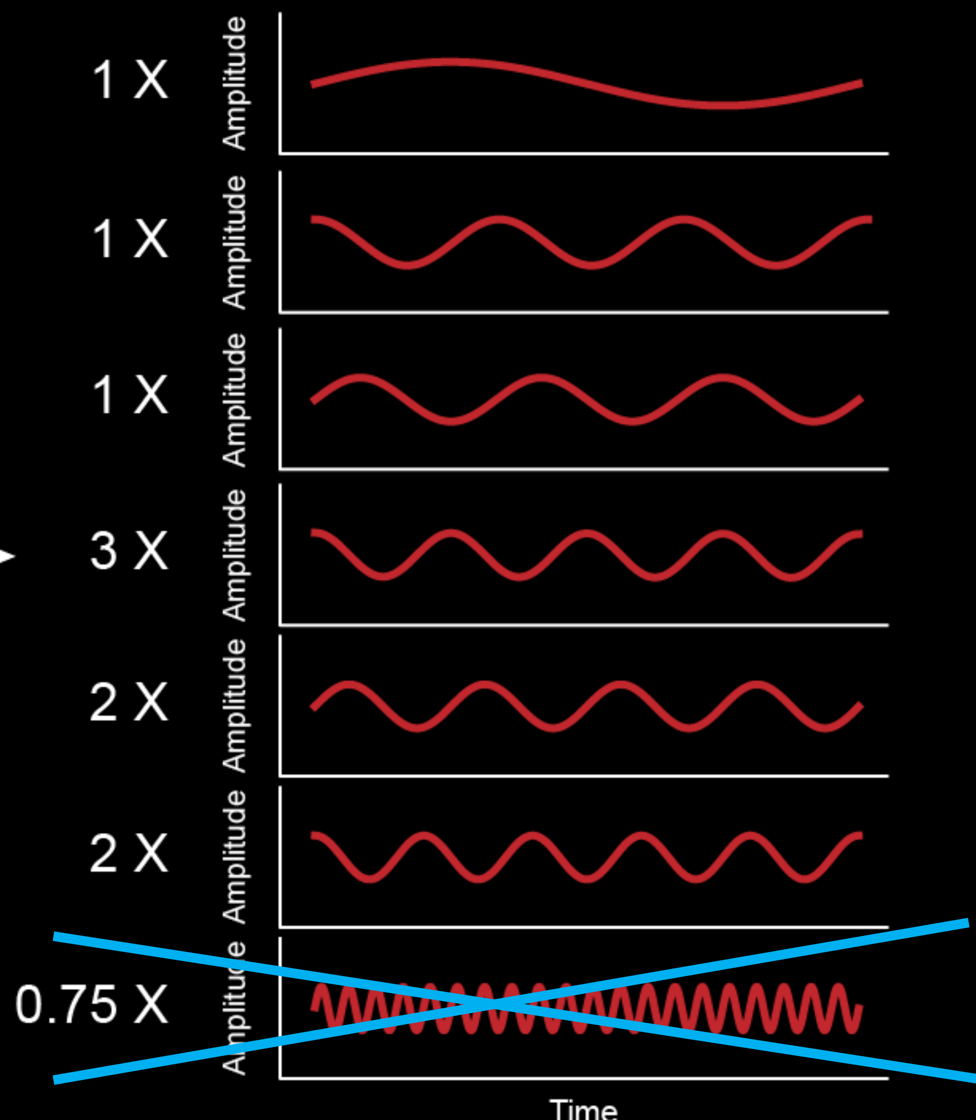
What now?


Raw Data
Pressure
Time
2000
1750
1500
1250
1000
750
500
250
0
-250
0
5000
10000
15000
20000
25000
30000


Amplitude
Frequency

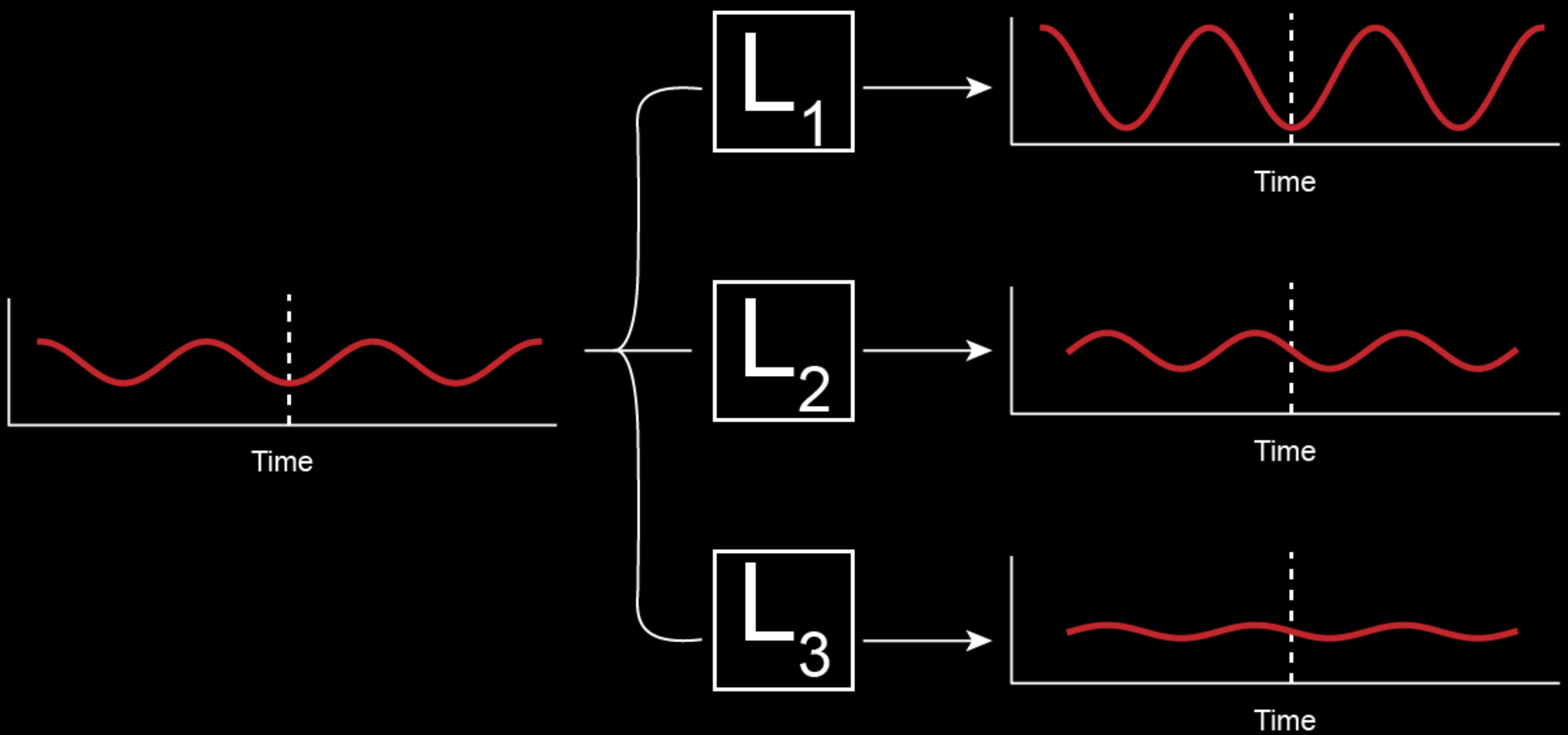
FFT





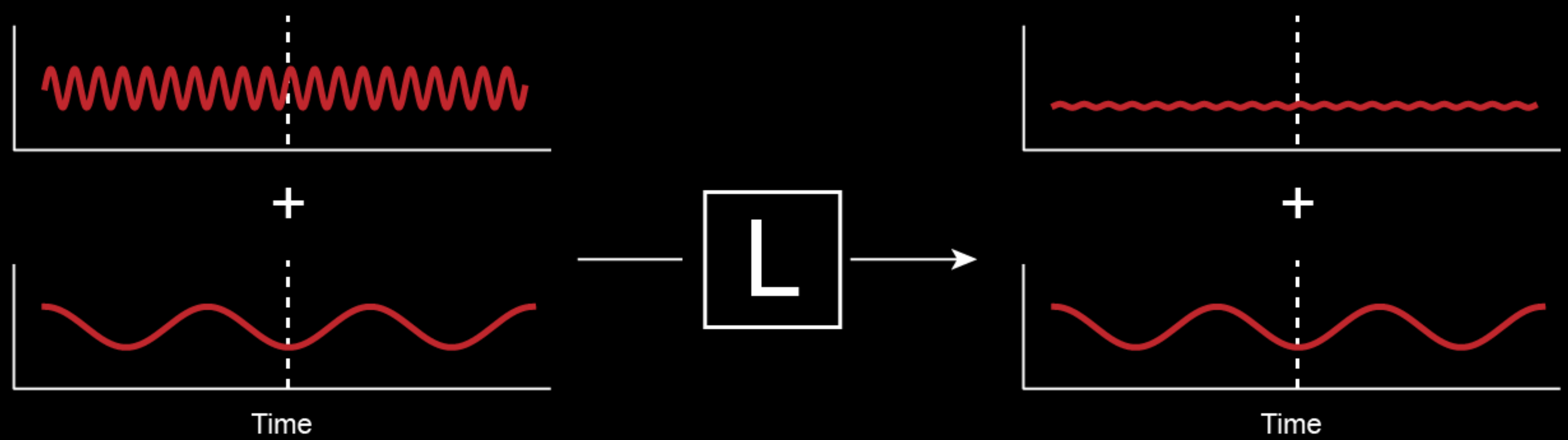
The entire point of this is to process our signals. Is there a way to remove or modify certain waves but not others?

Is there a way to remove or modify certain waves but not others?



Linear shift invariant systems

Is there a way to remove or modify certain waves but not others?



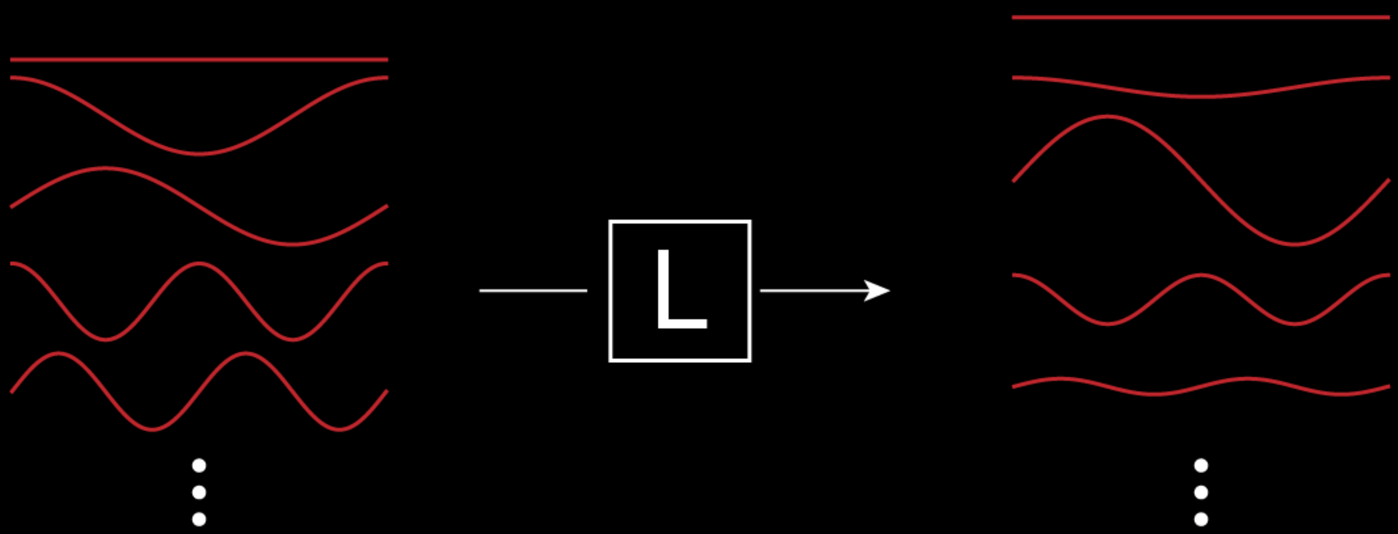
Linear shift invariant system

How do we use linear shift invariant systems to analyze our data?

How do we use linear shift invariant systems to analyze our data?

1. Figure out how the system will alter cosine and sine waves

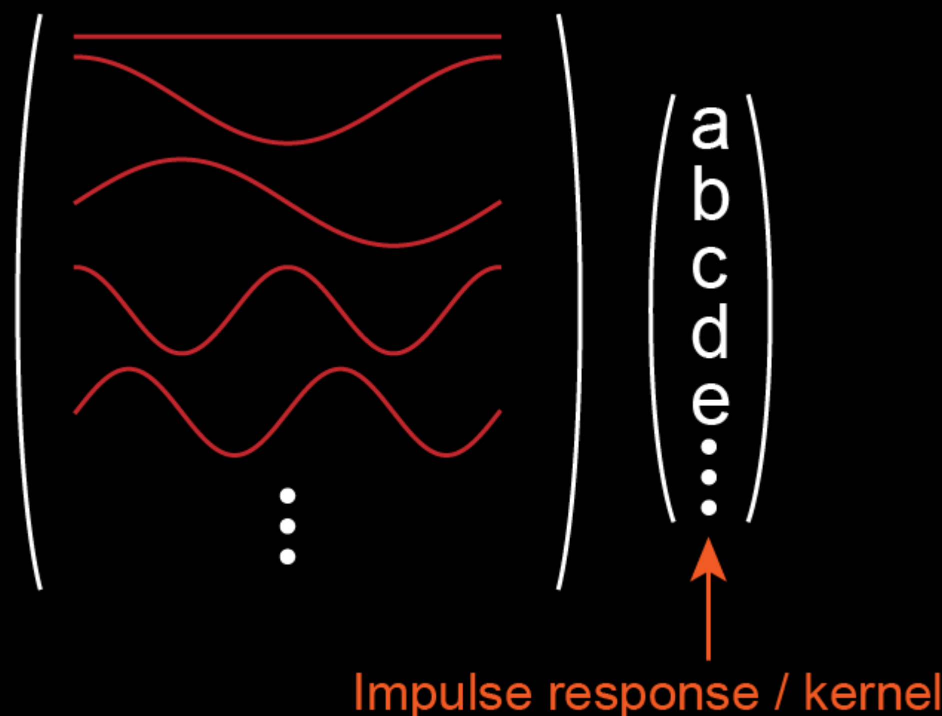
How do we use linear shift invariant systems to analyze our data?



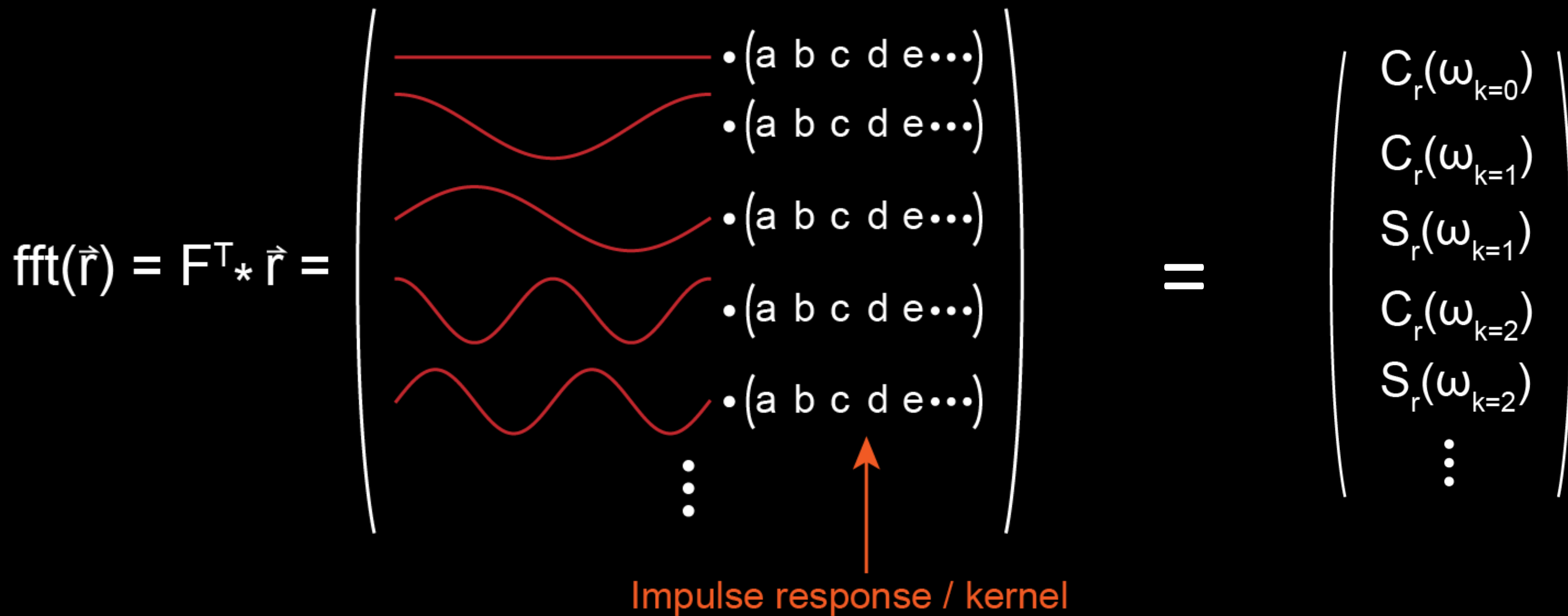
Linear shift invariant system

How do we use linear shift invariant systems to analyze our data?

$$\mathrm{fft}(\vec{r}) = F^{T} * \vec{r} =$$



How do we use linear shift invariant systems to analyze our data?



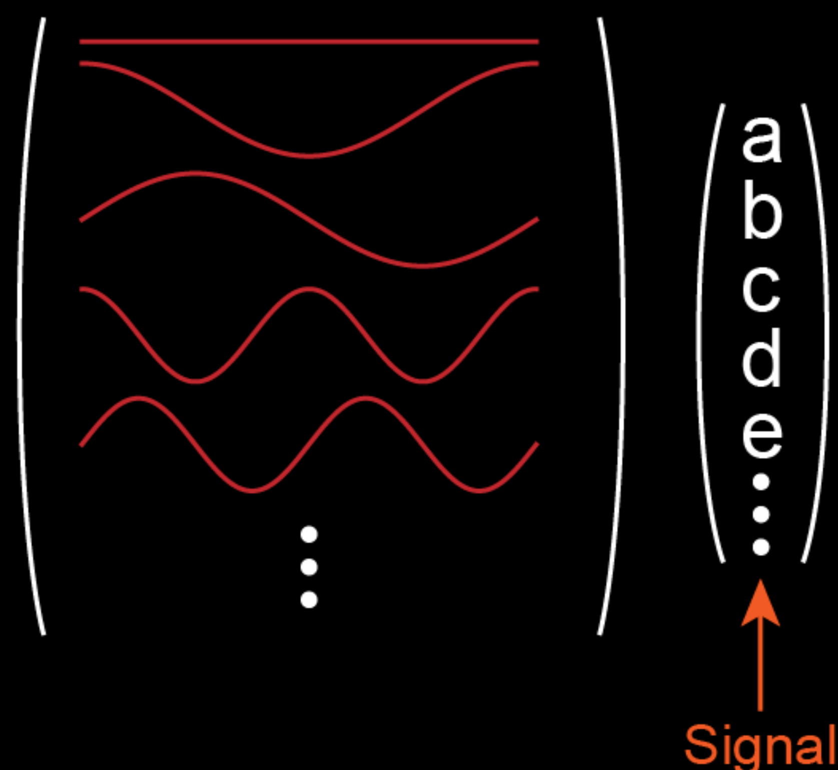
How do we use linear shift invariant systems to analyze our data?

1. Figure out how the system will alter cosine and sine waves
   - Take fft() of impulse response

2. Figure out how your signal of interest is broken down into cosine and sine waves

How do we use linear shift invariant systems to analyze our data?

$$\mathrm{fft}(\vec{x}) = F^T * \vec{x} =$$



How do we use linear shift invariant systems to analyze our data?

1. Figure out how the system will alter cosine and sine waves
   - Take fft() of impulse response

2. Figure out how your signal of interest is broken down into cosine and sine waves
   - Take fft() of signal

3. Use #1 and #2 to figure out how the system alters your signal

How do we use linear shift invariant systems to analyze our data?

$$\tilde{R} \qquad\qquad \mathrm{fft}(\vec{x})$$

$$\begin{pmatrix} C_r(\omega_{k=0}) & 0 & 0 & 0 & 0 \\ 0 & C_r(\omega_{k=1}) & 0 & 0 & 0 \\ 0 & 0 & S_r(\omega_{k=1}) & 0 & 0 \\ 0 & 0 & 0 & C_r(\omega_{k=2}) & 0 \\ 0 & 0 & 0 & 0 & S_r(\omega_{k=2}) \cdots \\ & & & & \vdots \end{pmatrix} \begin{pmatrix} \\ C_x(\omega_{k=1}) \\ S_x(\omega_{k=1}) \\ C_x(\omega_{k=2}) \\ S_x(\omega_{k=2}) \\ \vdots \end{pmatrix}$$

What happens to each cosine and sine wave in the form of a diagonal matrix

Amount of each cosine and sine wave in signal

How do we use linear shift invariant systems to analyze our data?

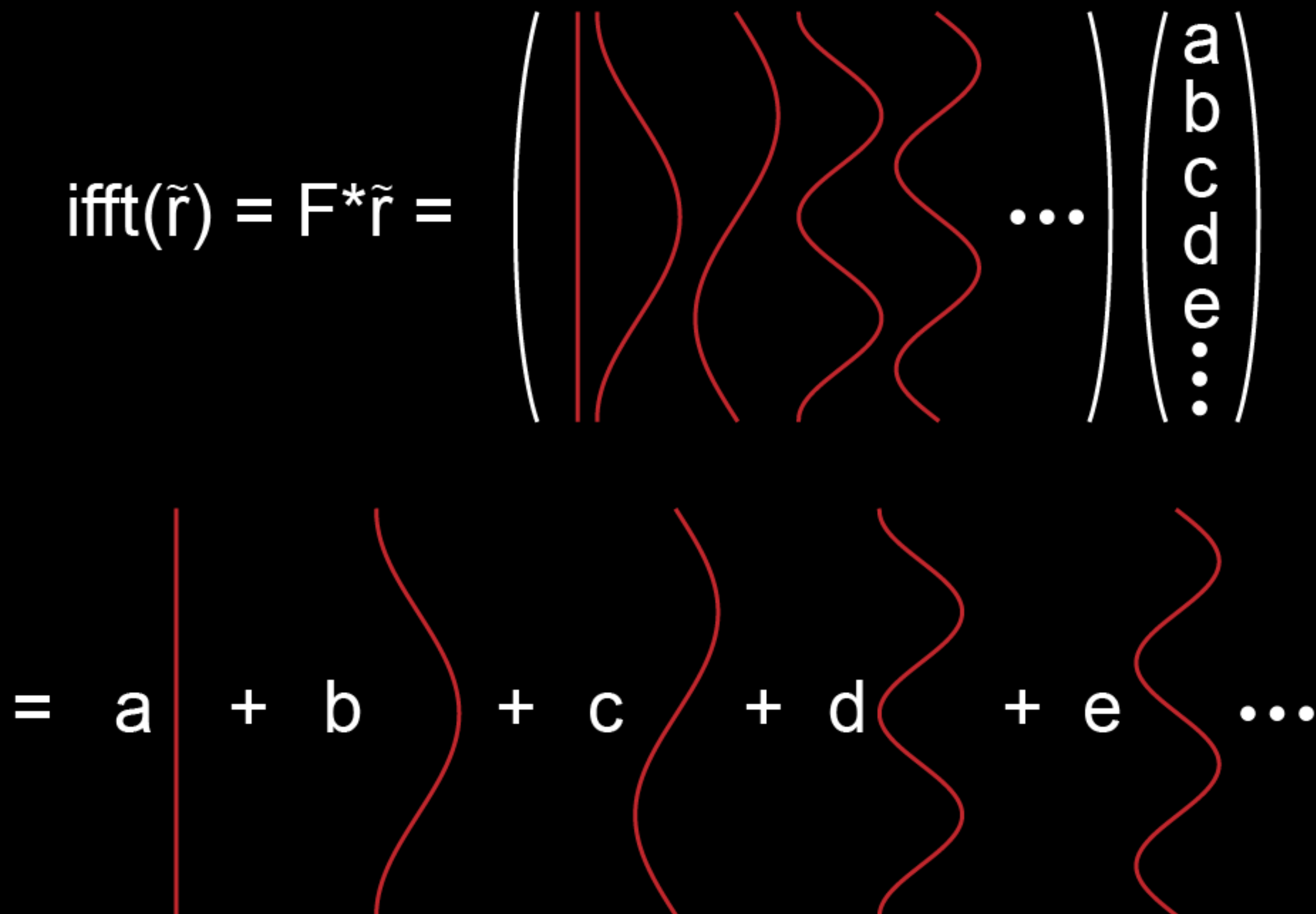
1. Figure out how the system will alter cosine and sine waves
   - Take fft() of impulse response

2. Figure out how your signal of interest is broken down into cosine and sine waves
   - Take fft() of signal

3. Use #1 and #2 to figure out how the system alters your signal
   - R * fft(x)

4. Convert back into a signal

How do you perform the fft or ifft?



How do we use linear shift invariant systems to analyze our data?

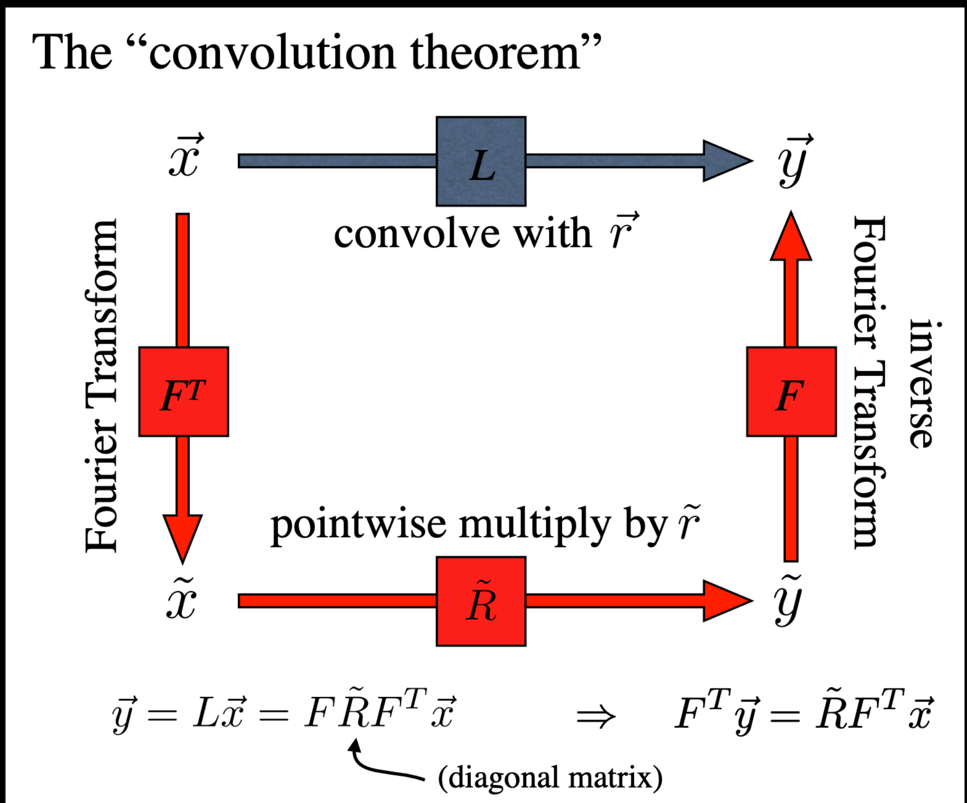
1. Figure out how the system will alter cosine and sine waves
   - Take fft() of impulse response

2. Figure out how your signal of interest is broken down into cosine and sine waves
   - Take fft() of signal

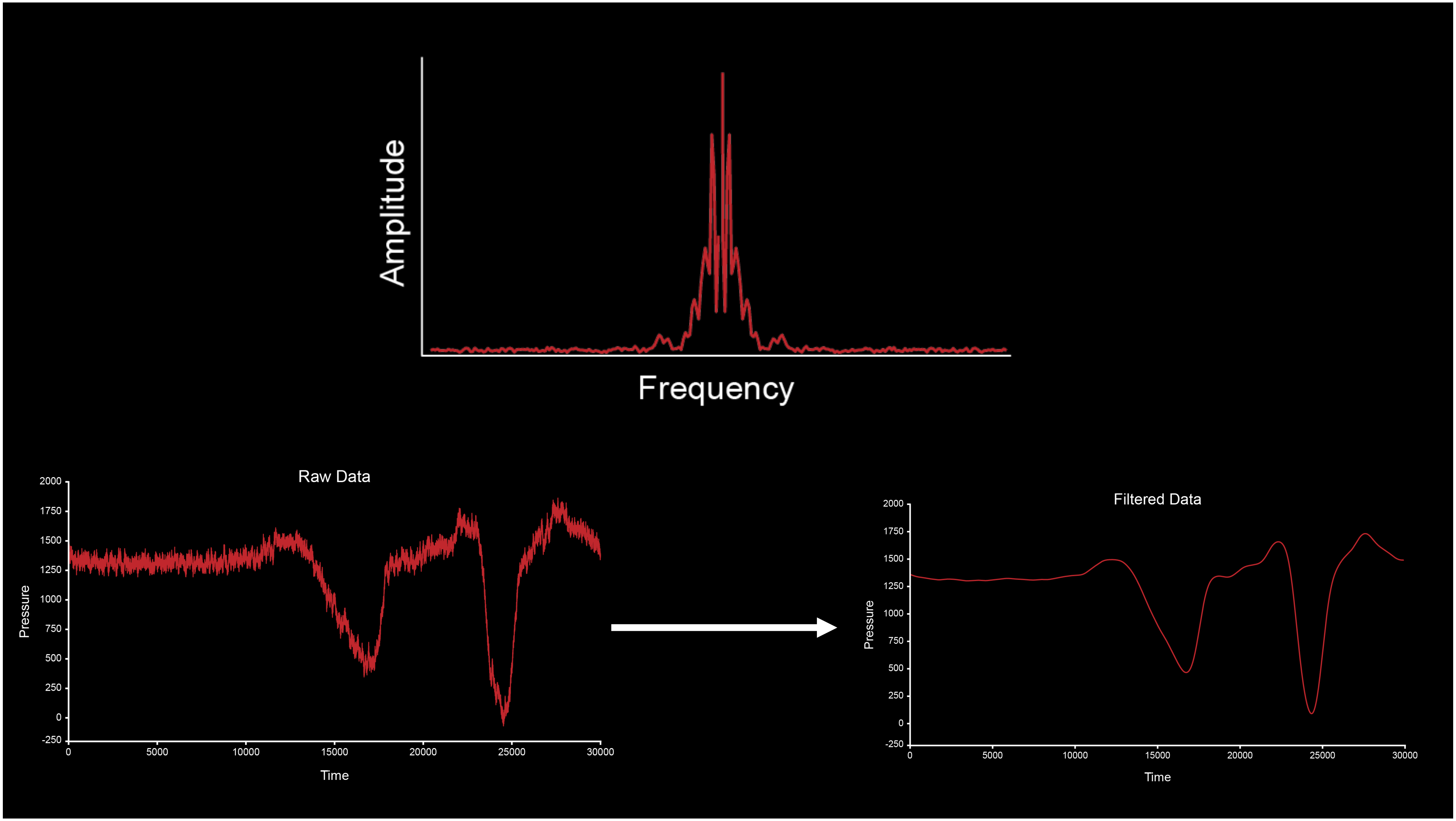
3. Use #1 and #2 to figure out how the system alters your signal
   - R * fft(x)

4. Convert back into a signal
   - ifft(R * fft(x))

How do we use linear shift invariant systems to analyze our data?



Exercise #2



Amplitude
Frequency
Raw Data
2000
1750
1500
1250
1000
750
500
250
0
-250
Pressure
0
5000
10000
15000
20000
25000
30000
Time
Filtered Data
2000
1750
1500
1250
1000
750
500
250
0
-250
Pressure
0
5000
10000
15000
20000
25000
30000
Time

Questions?