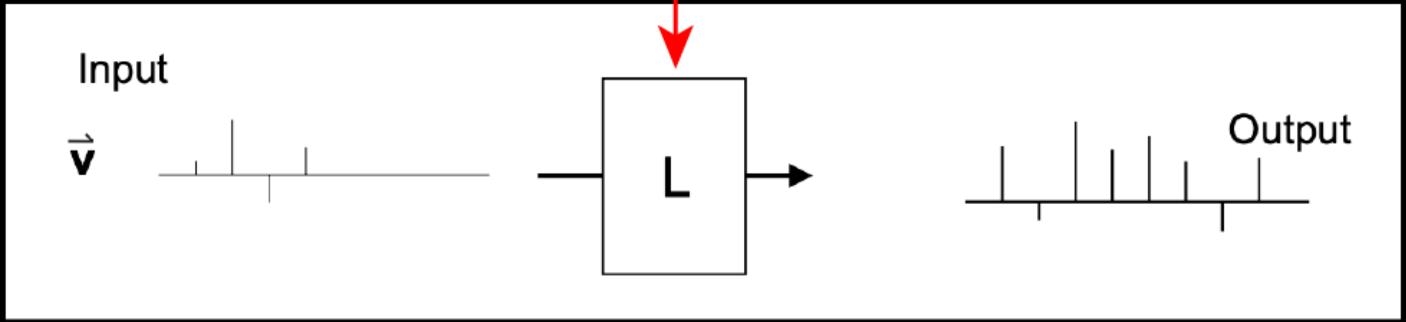
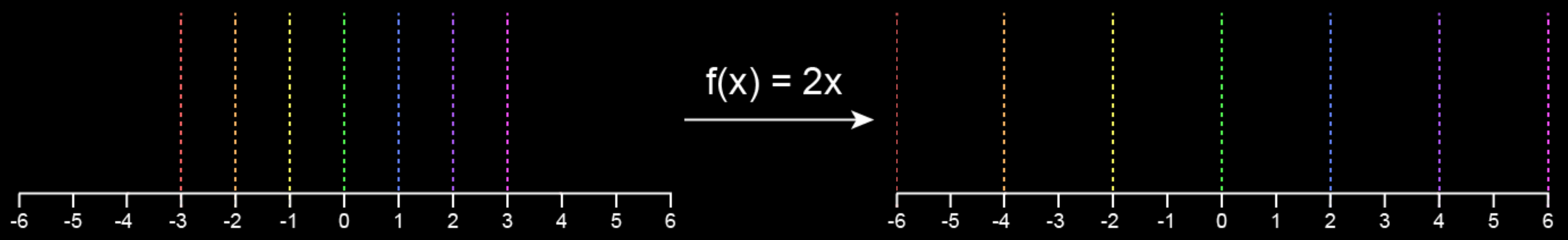
What is a linear transformation?

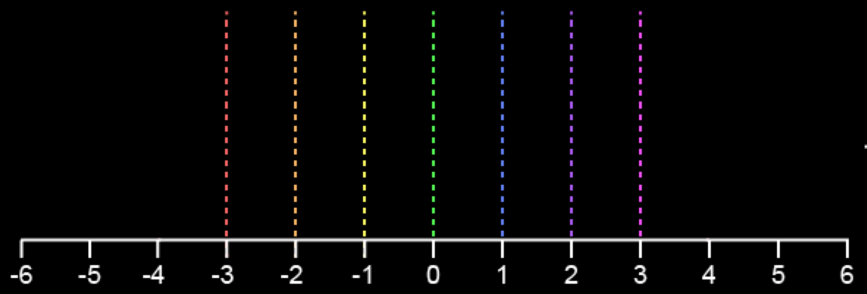
What is this magical box?



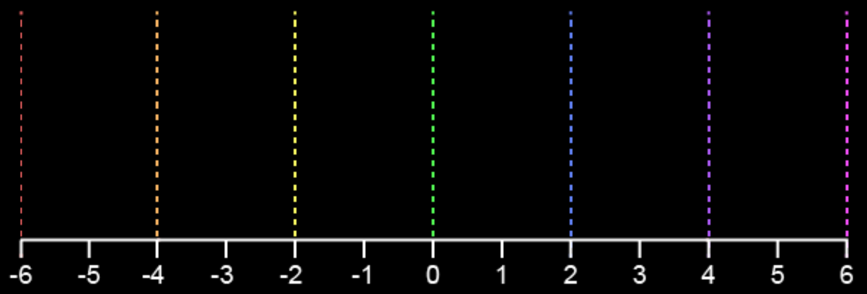
Does the transformation of input to output through L have any useful properties?

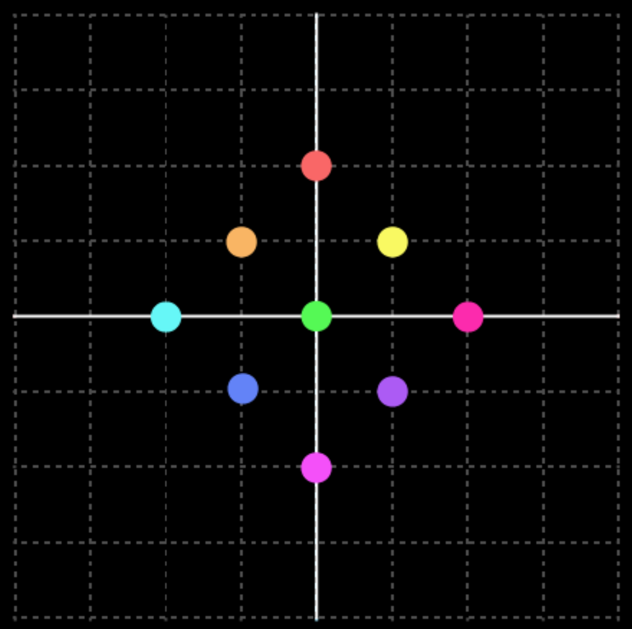


f(x) = 2x
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

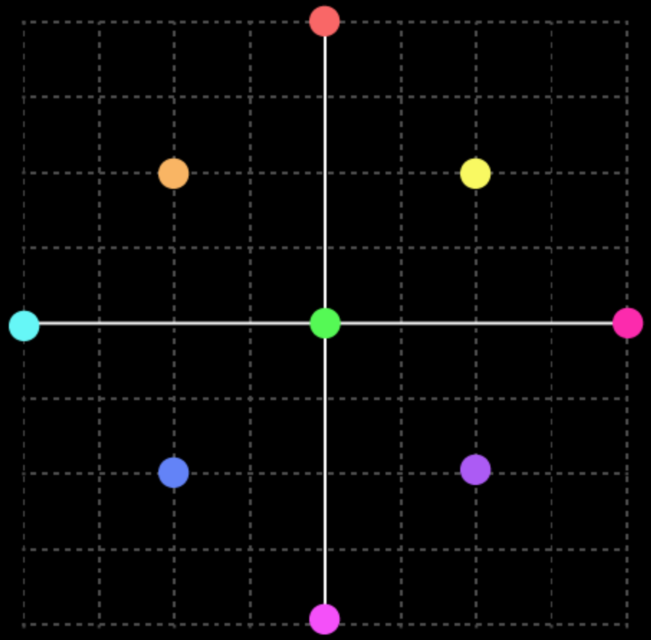


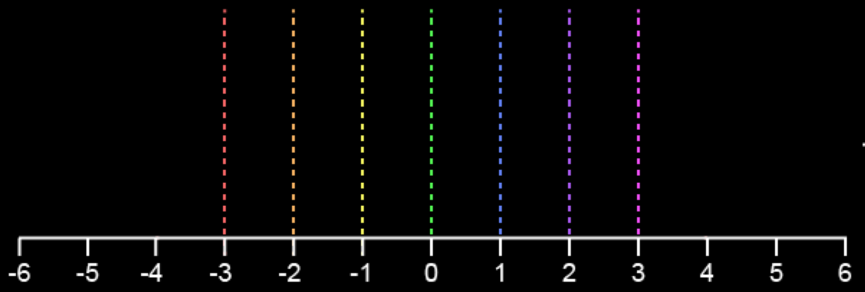
$$f(x) = 2x$$



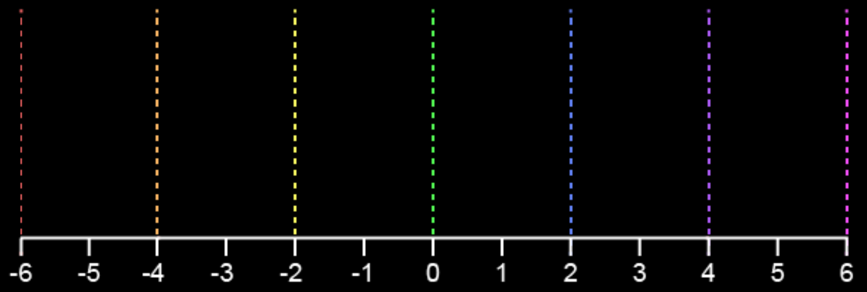


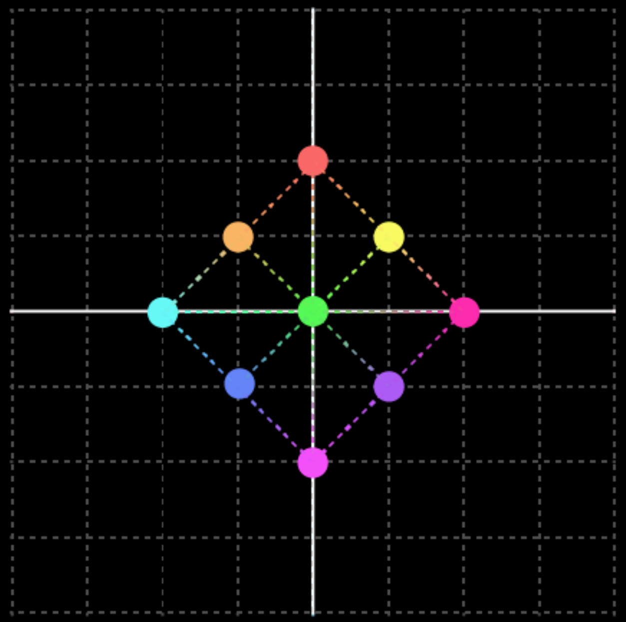
$$L(\vec{x}) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \vec{x}$$



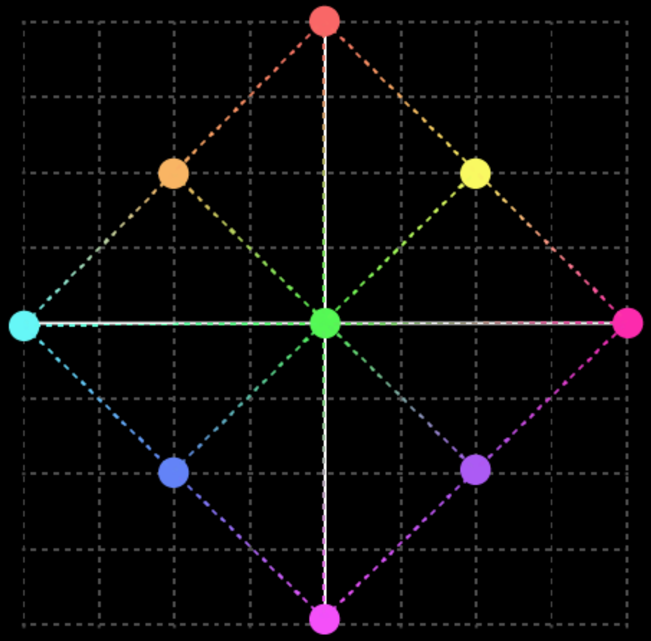


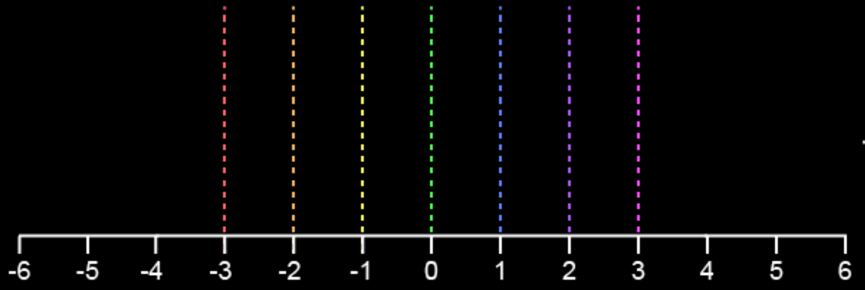
$$f(x) = 2x$$



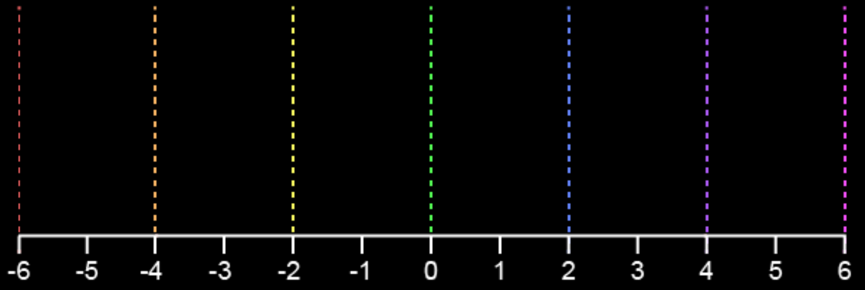


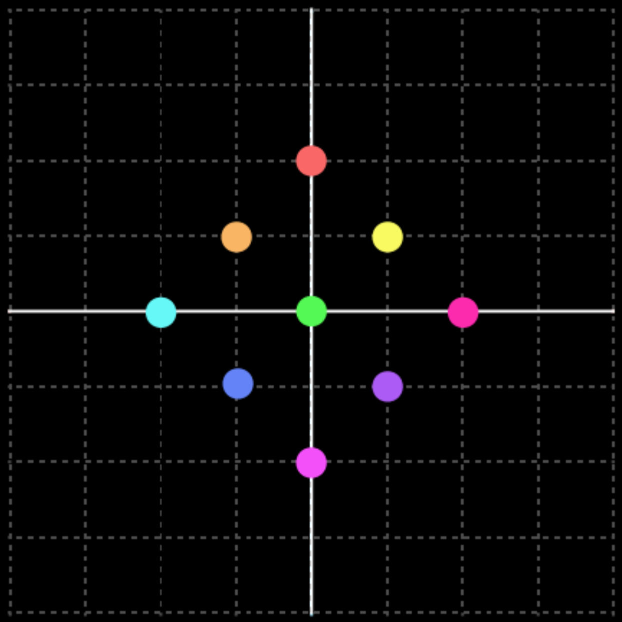
$$L(\vec{x}) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\vec{x}$$



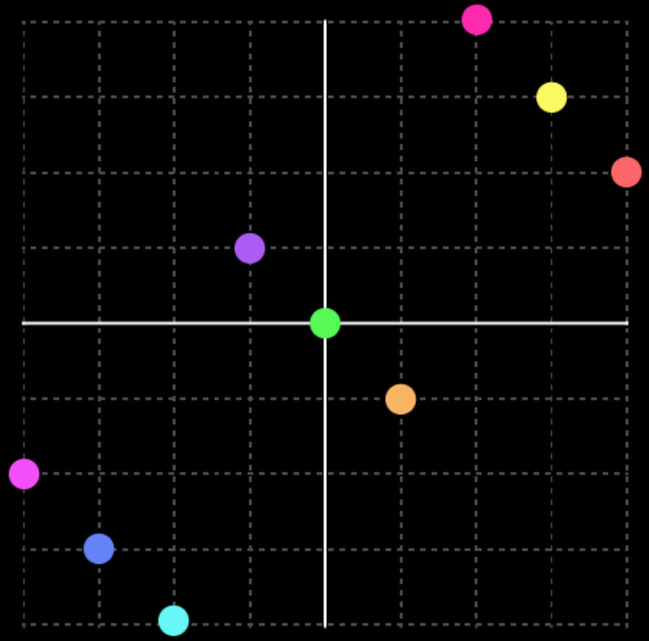


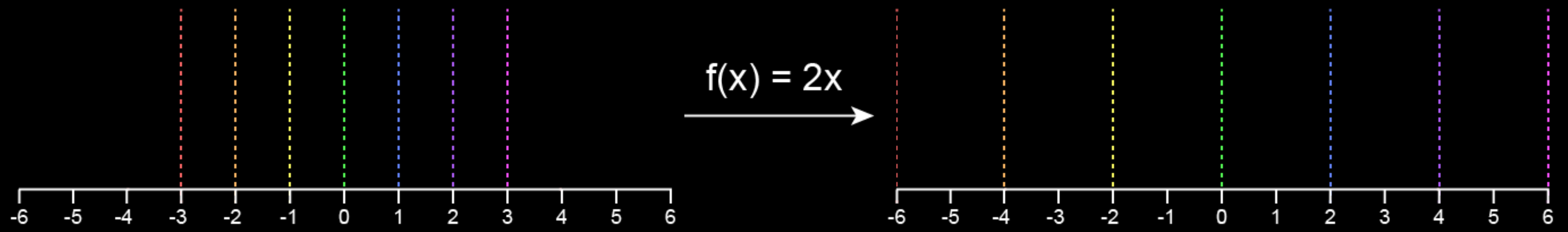
$$f(x) = 2x$$

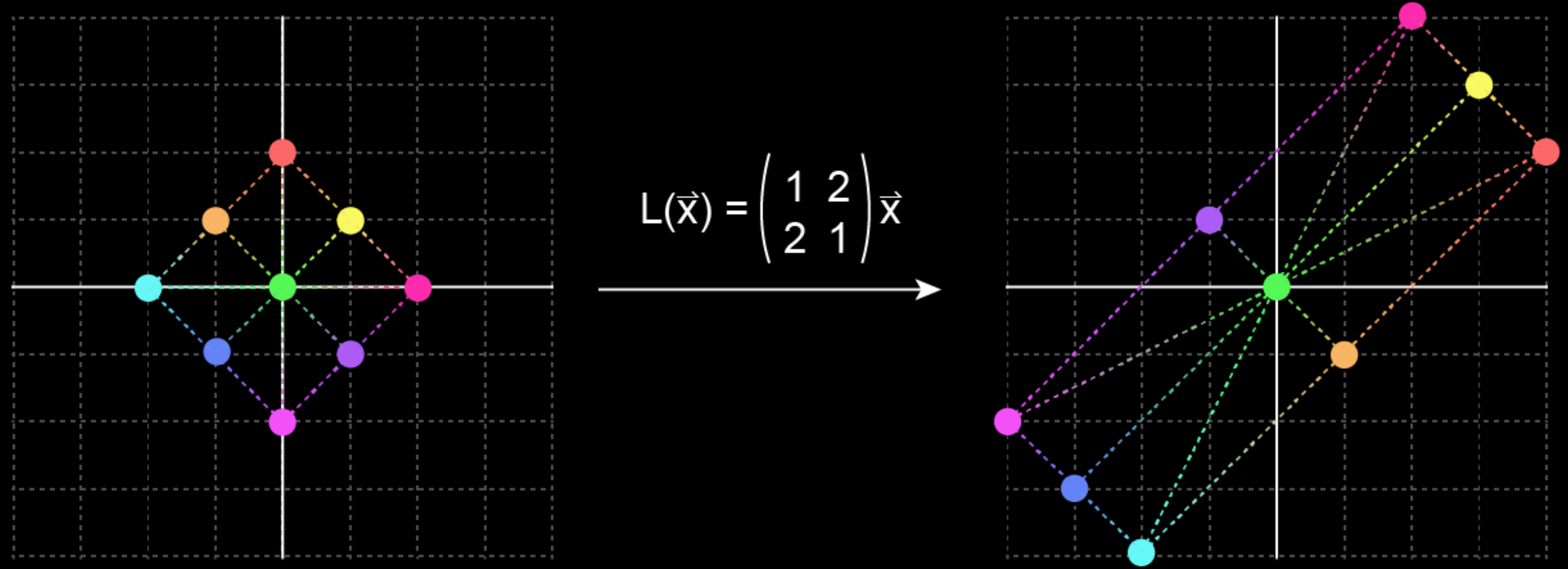


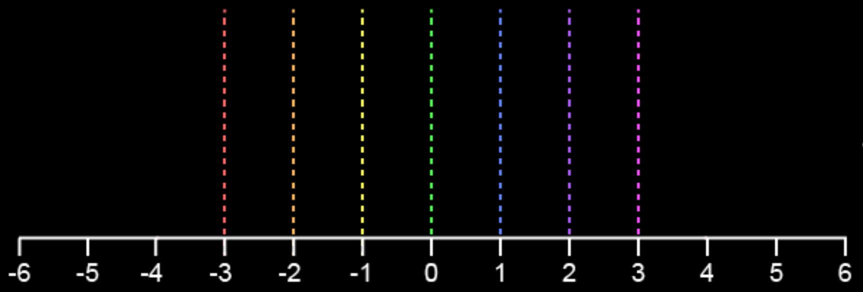


$$L(\vec{x}) = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \vec{x}$$

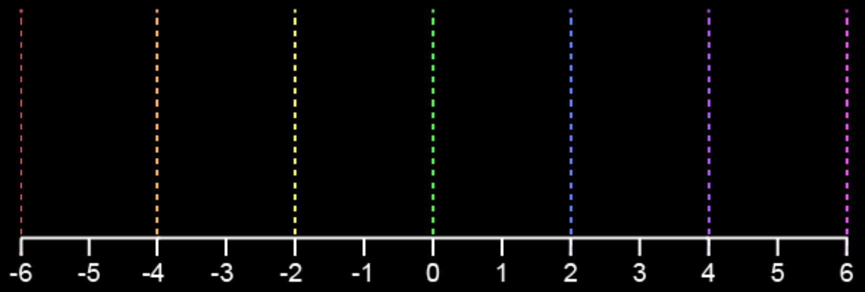


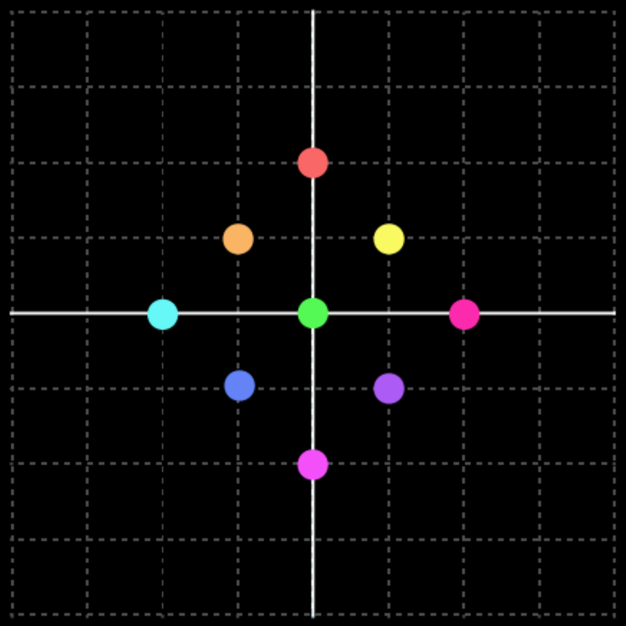

f(x) = 2x
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6


$L(\vec{x}) = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \vec{x}$

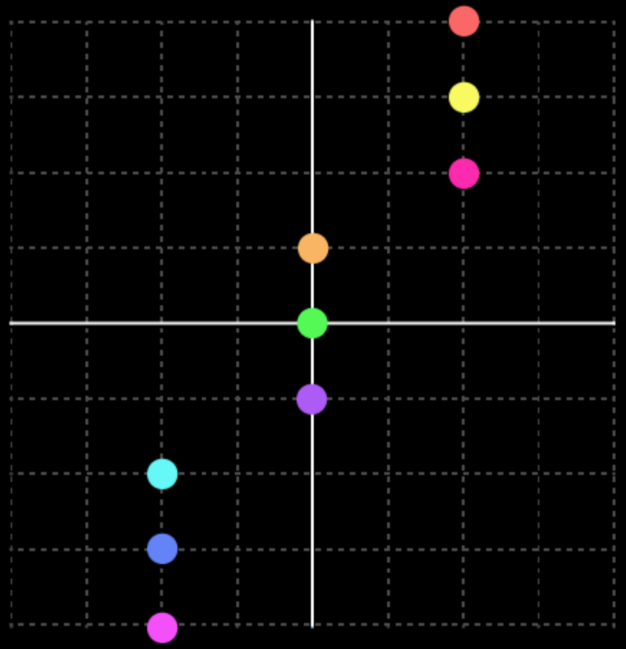


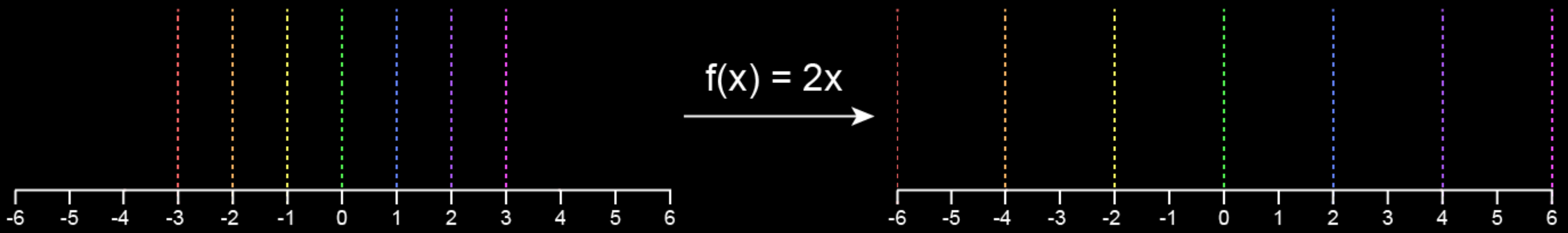
$$f(x) = 2x$$

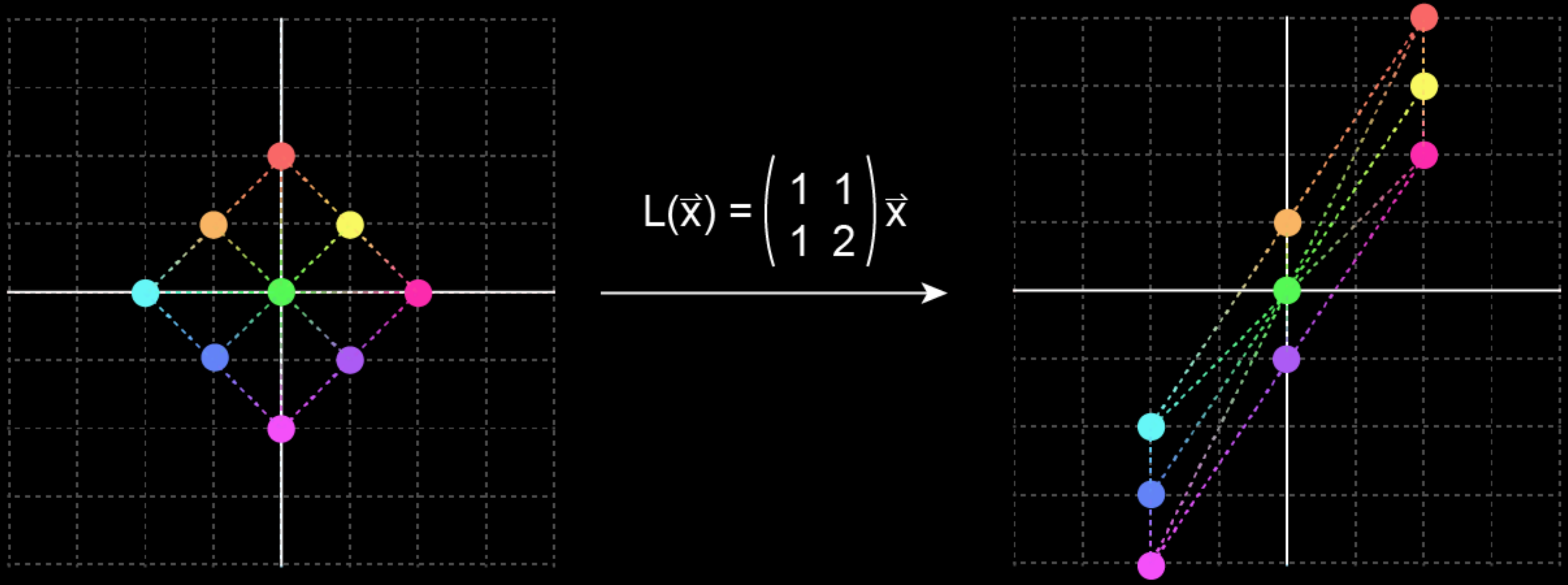


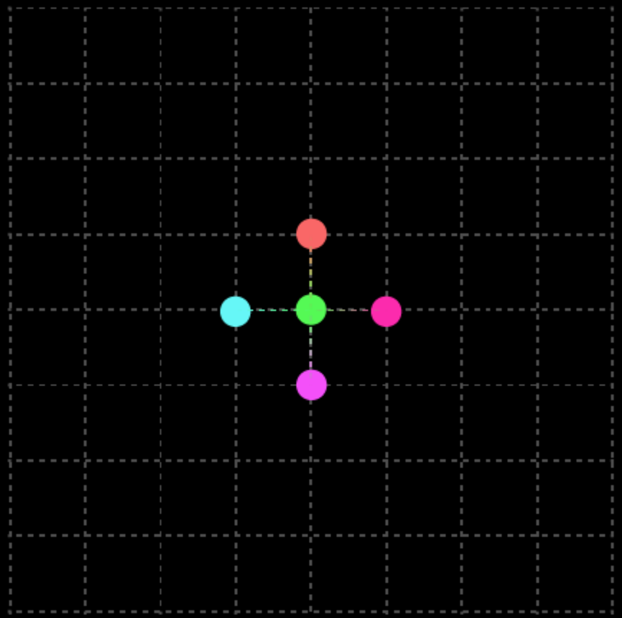


$$L(\vec{x}) = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \vec{x}$$

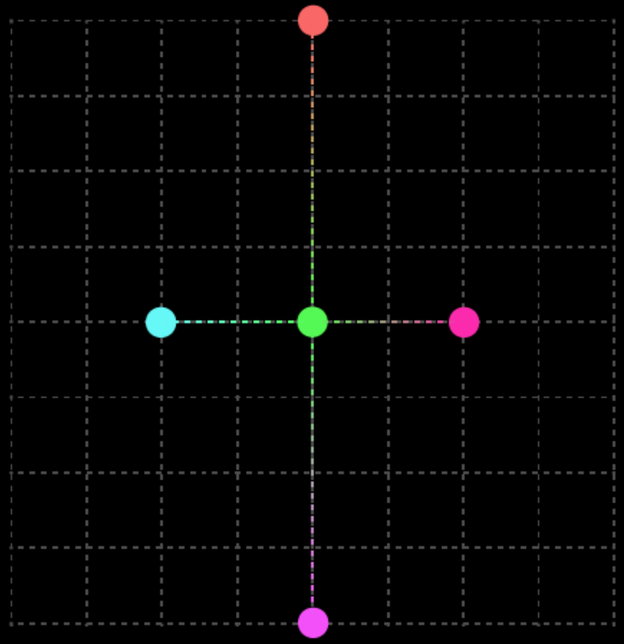


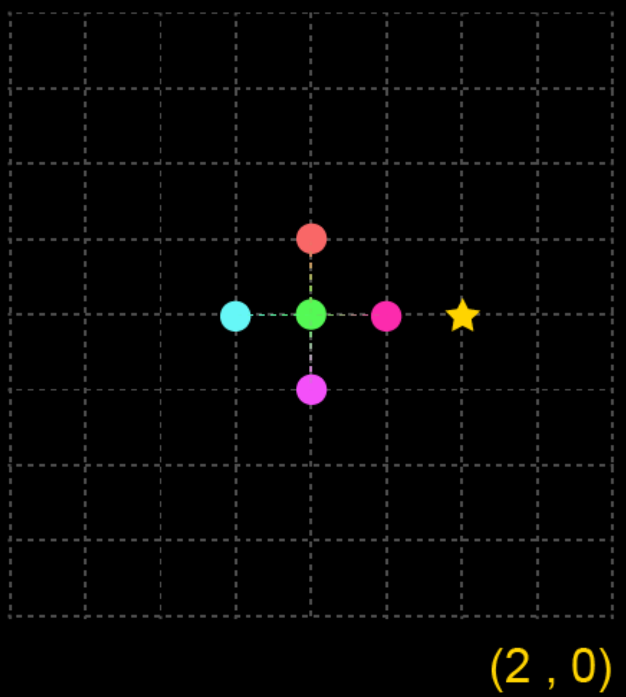

f(x) = 2x
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6


$L(\vec{x}) = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \vec{x}$

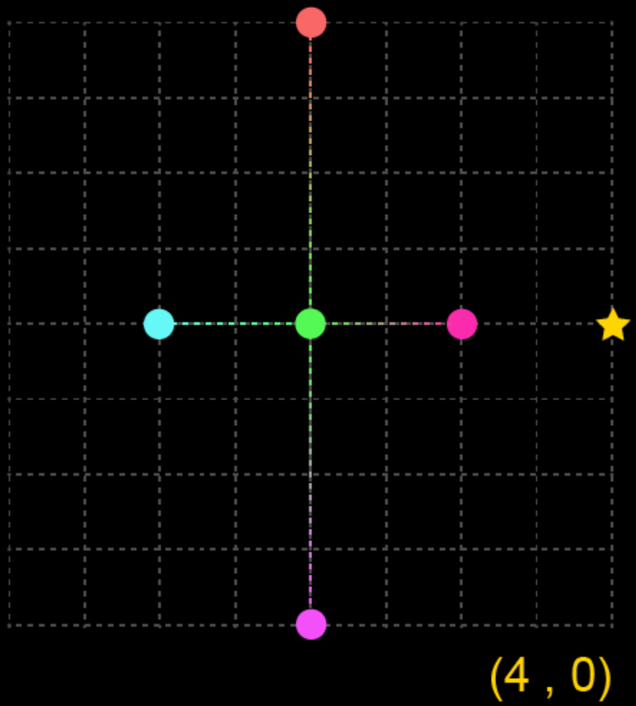


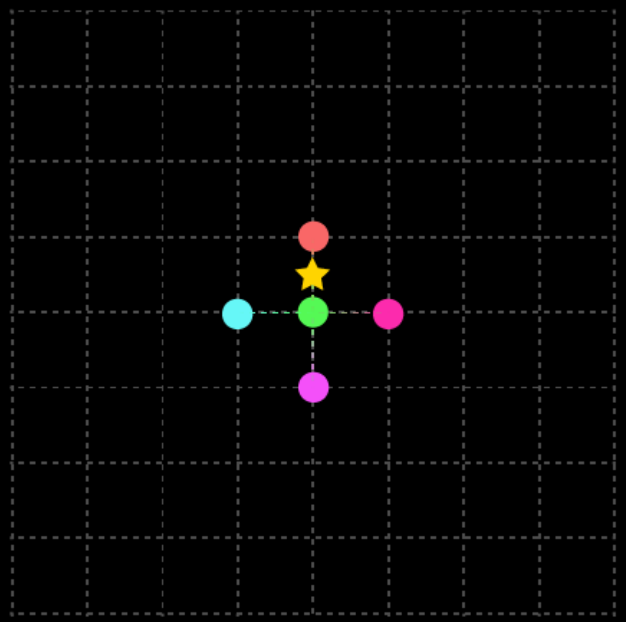
$$L(\vec{x}) = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix} \vec{x}$$





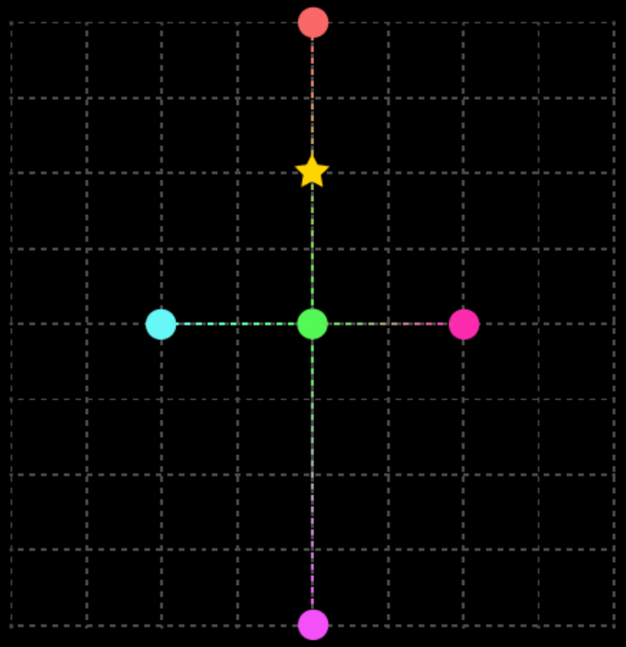
$$L(\vec{x}) = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix} \vec{x}$$



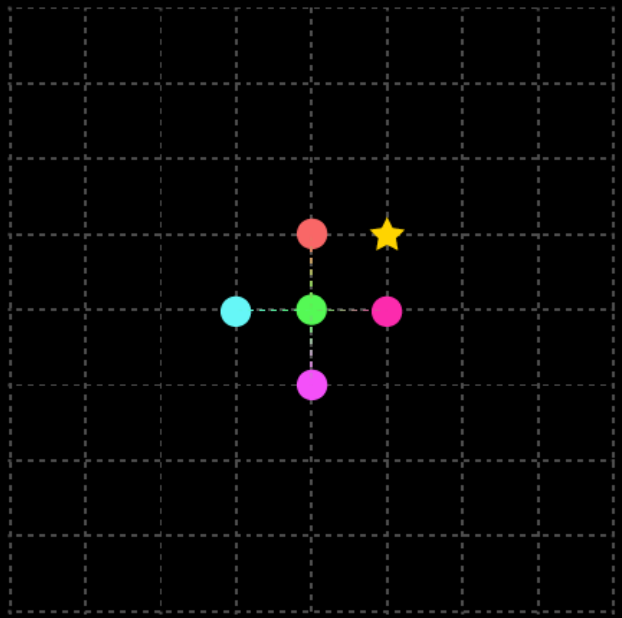


(1/2 , 0)

$$L(\vec{x}) = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix} \vec{x}$$

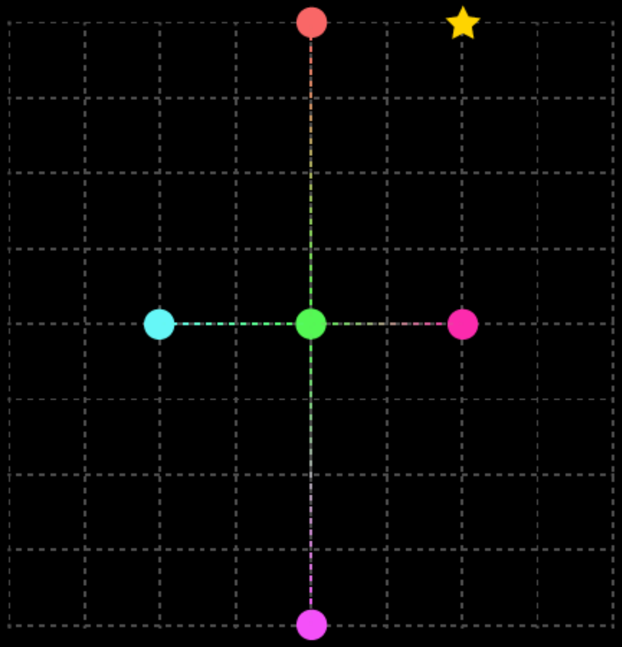


(2 , 0)

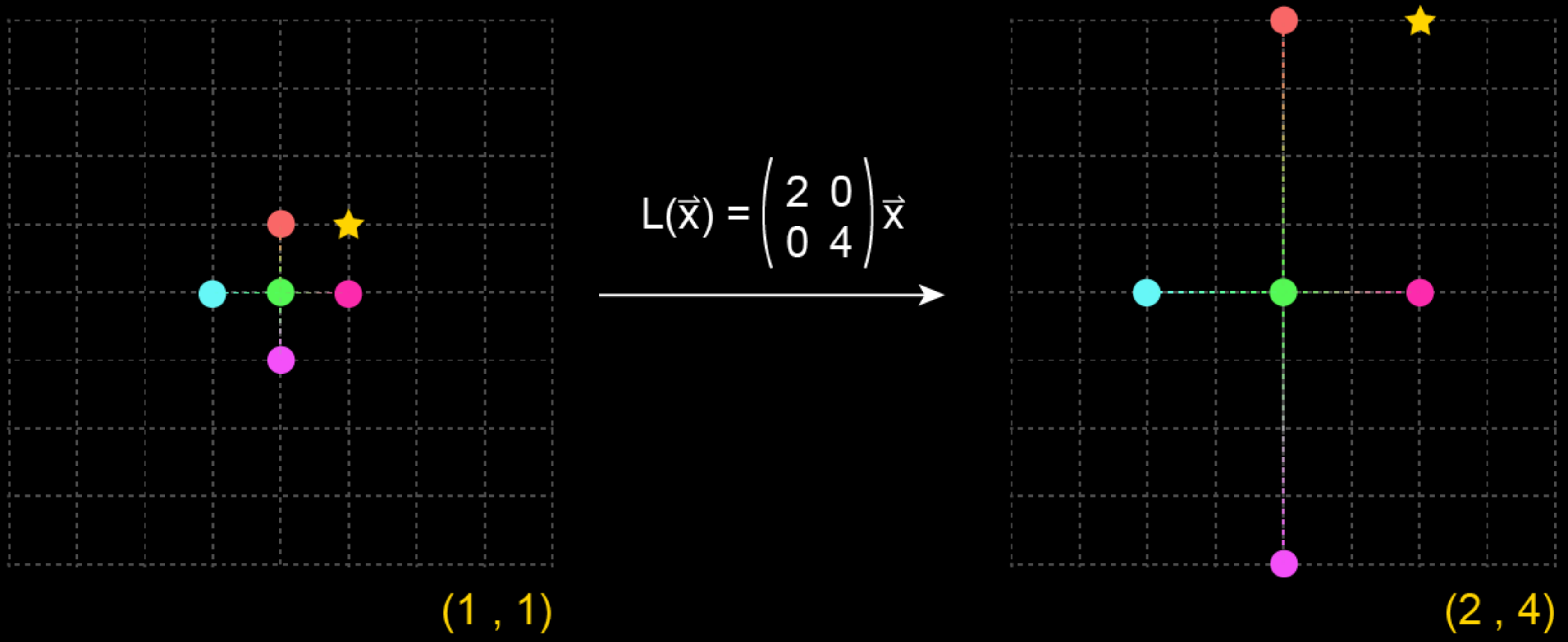


(1 , 1)

$$L(\vec{x}) = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix} \vec{x}$$



(2 , 4)



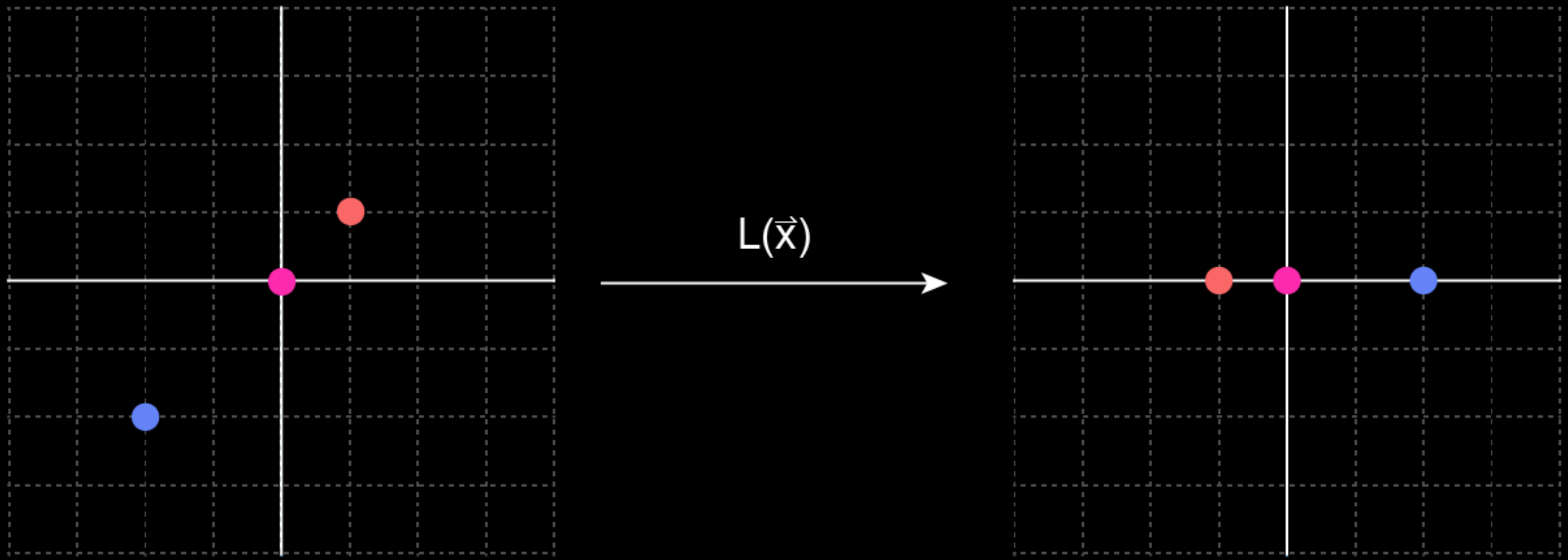
$$L(\vec{x}) = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix} \vec{x}$$

$$S(a\vec{x} + b\vec{y}) = aS(\vec{x}) + bS(\vec{y})$$

Exercises

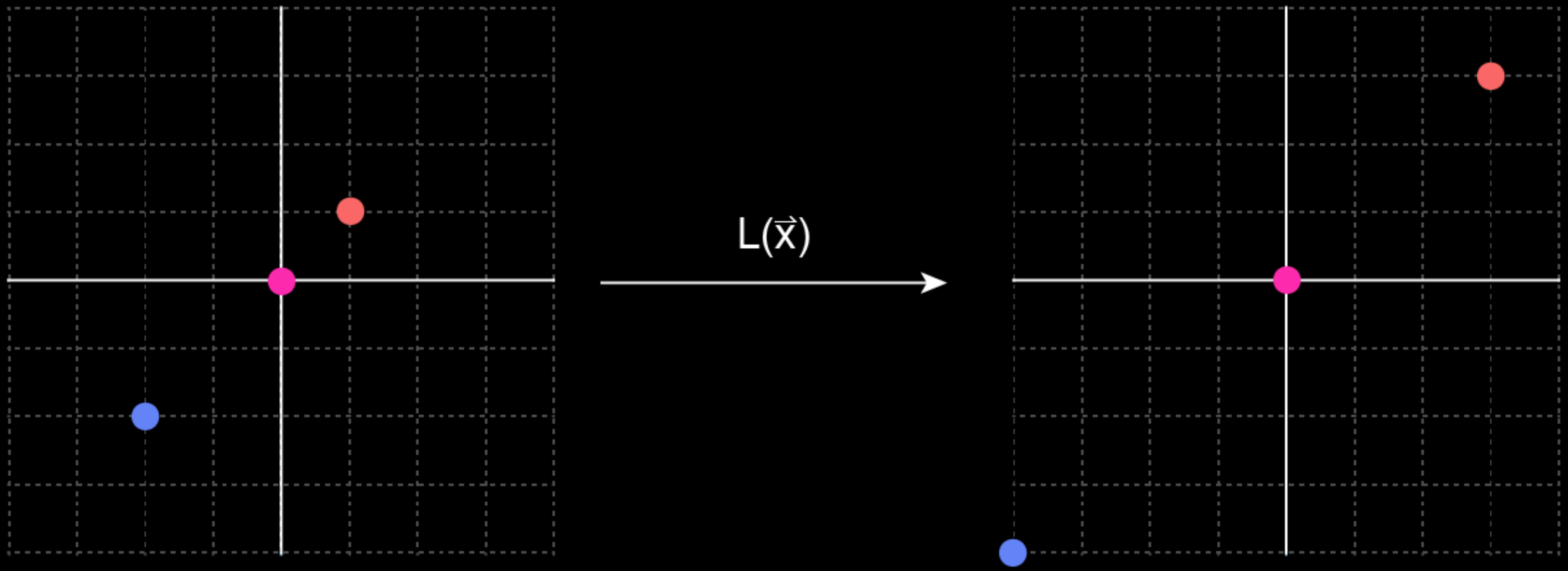
Determine if any of the following are non-linear transformations

Exercise 1



(-2 , -2)	(2 , 0)
(0 , 0)	(0 , 0)
(1 , 1)	(-1 , 0)

Exercise 2



(-2 , -2)

(0 , 0)

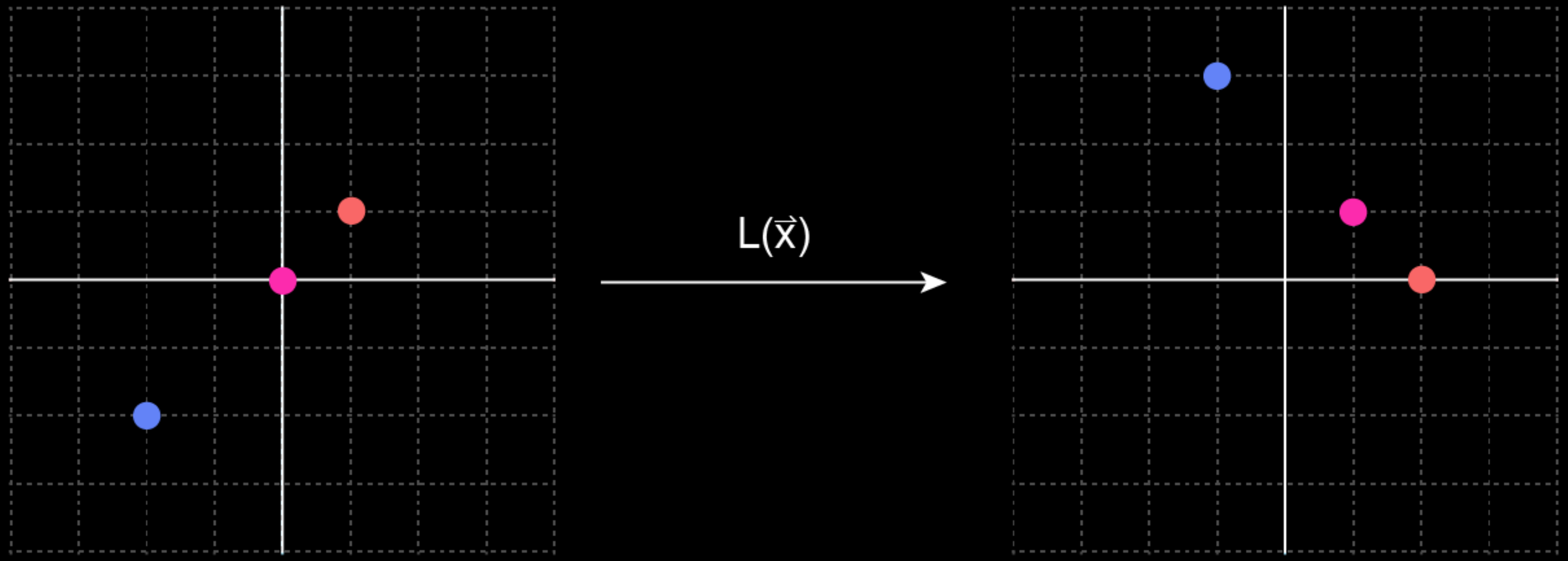
(1 , 1)

(-4 , -4)

(0 , 0)

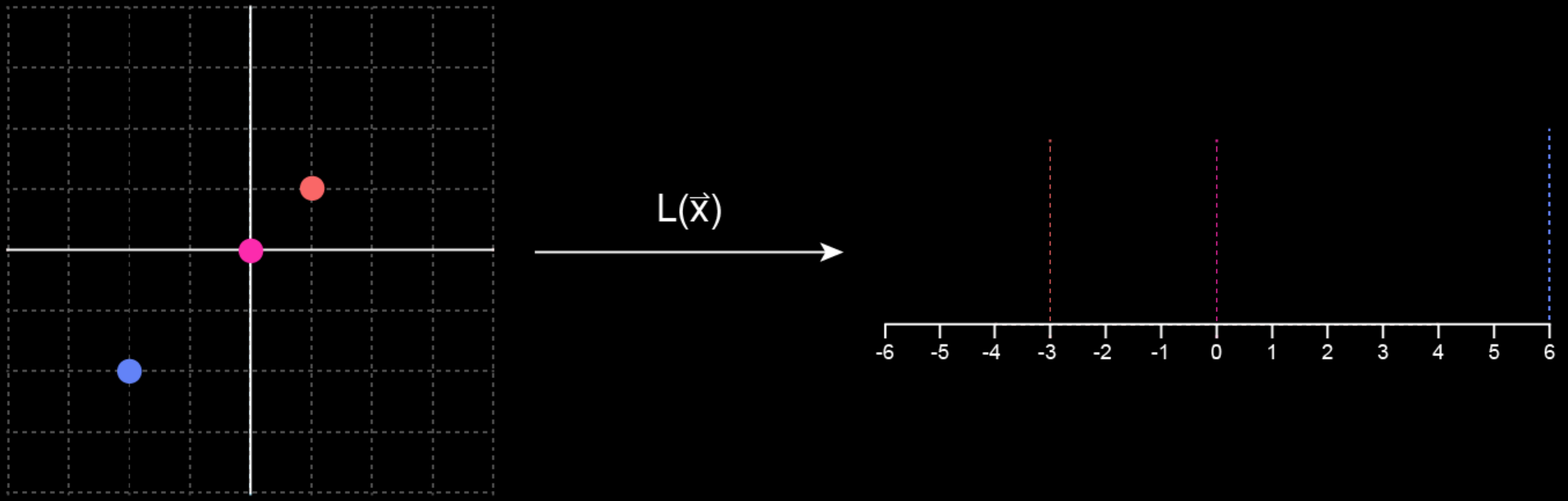
(3 , 3)

Exercise 3



(-2 , -2)	(-1 , 3)
(0 , 0)	(1 , 1)
(1 , 1)	(2 , 0)

Exercise 4



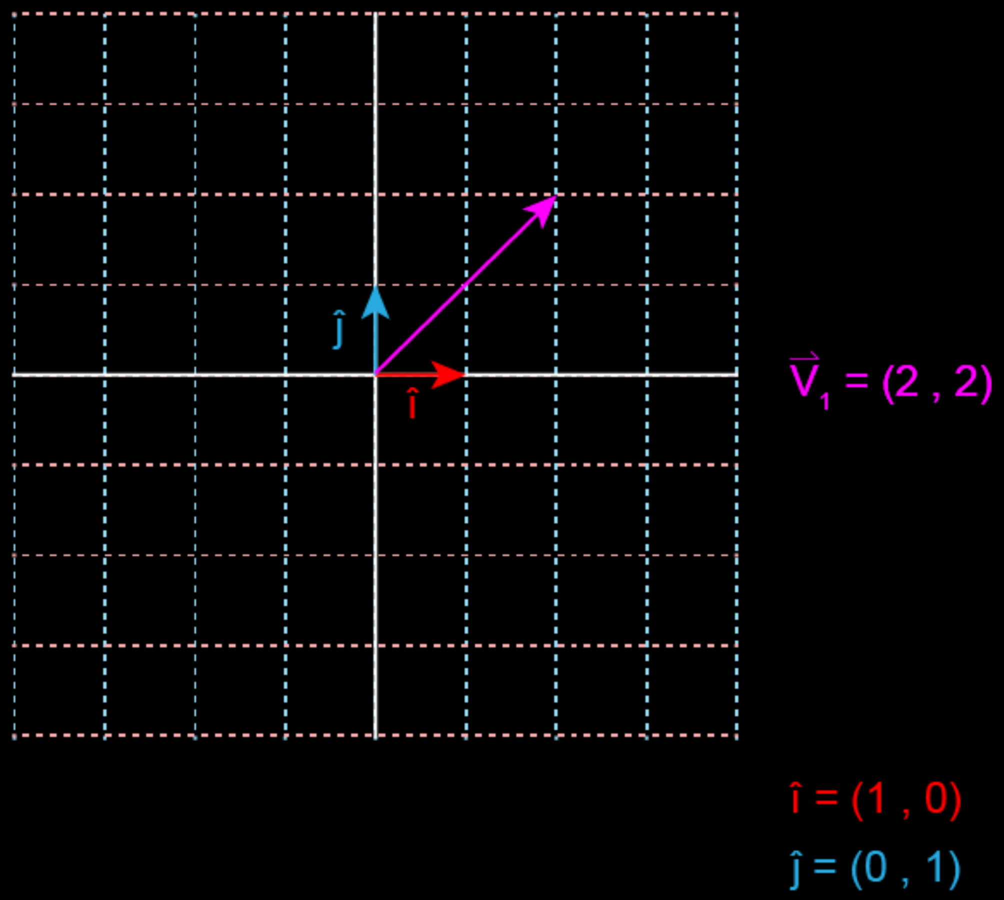
(-2 , -2) → (6)

(0 , 0) → (0)

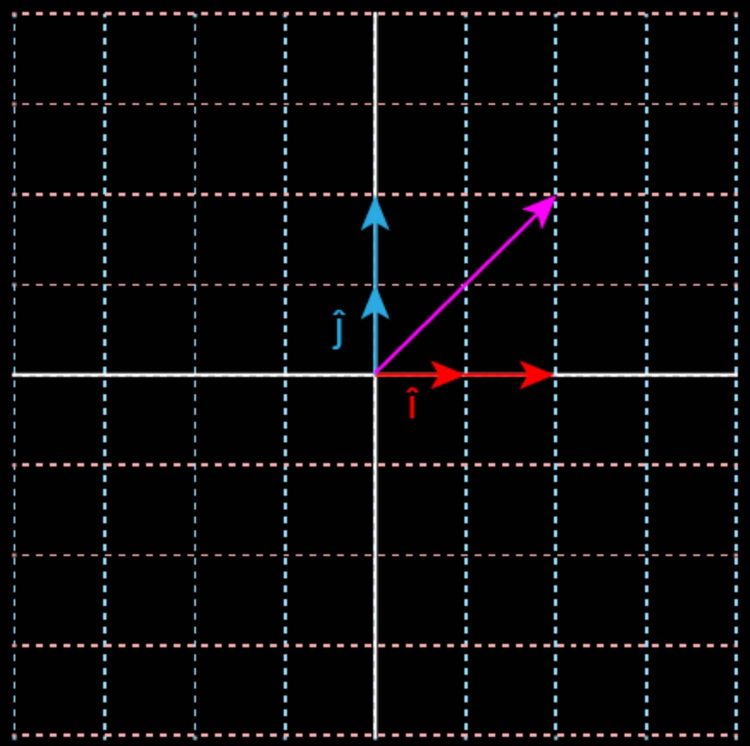
(1 , 1) → (-3)

What is the relationship between matrices and linear transformations?

Any vector can be created through a linear combination of impulses



Any vector can be created through a linear combination of impulses



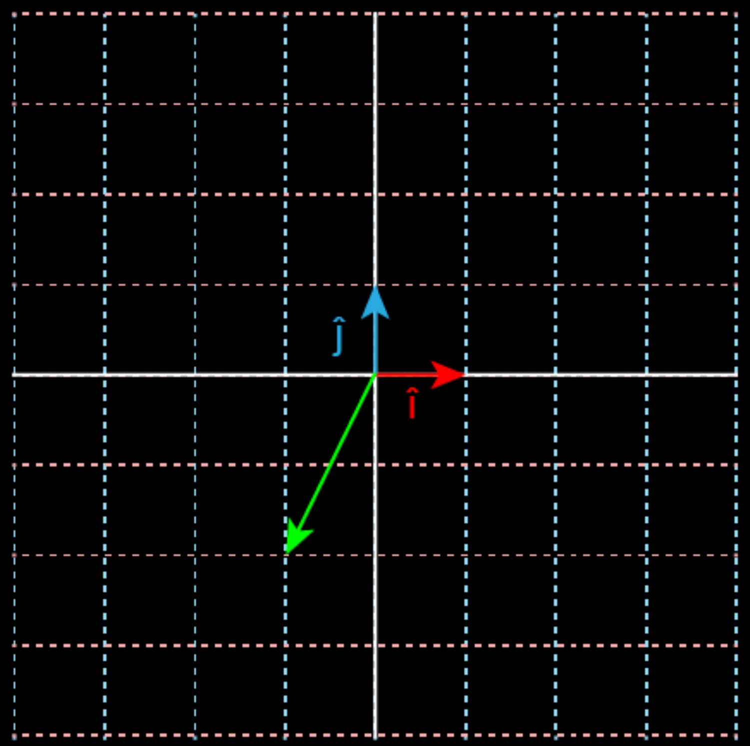
$\vec{V}_1 = (2, 2)$

$\vec{V}_1 = 2\hat{\imath} + 2\hat{\jmath}$

$\hat{\imath} = (1, 0)$

$\hat{\jmath} = (0, 1)$

Any vector can be created through a linear combination of impulses

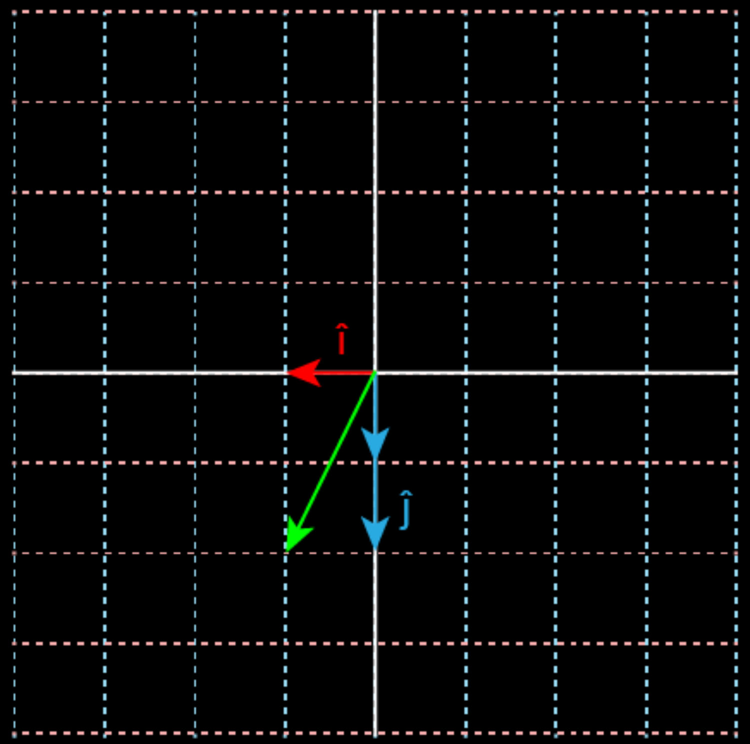


$\vec{V}_2 = (-1 , -2)$

$\hat{\imath} = (1 , 0)$

$\hat{\jmath} = (0 , 1)$

Any vector can be created through a linear combination of impulses



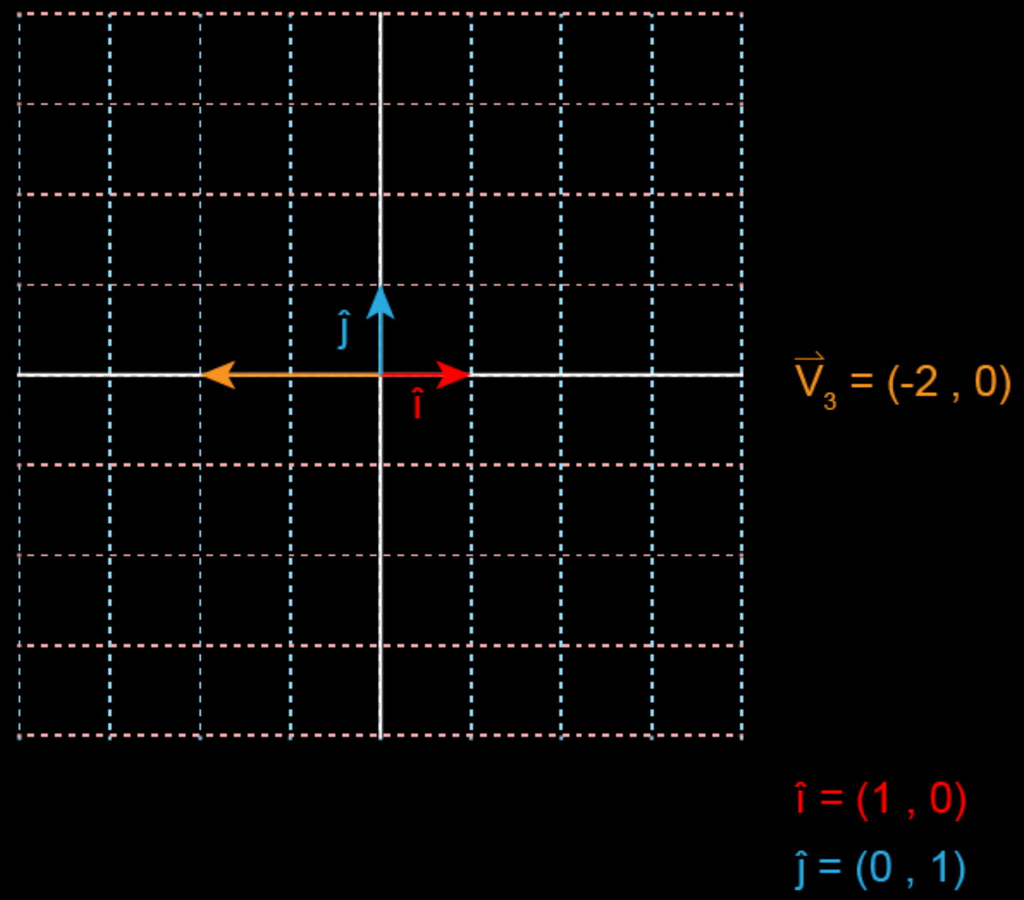
$\vec{V}_2 = (-1\ ,\ -2)$

$\vec{V}_2 = -1\hat{\imath} + -2\hat{\jmath}$

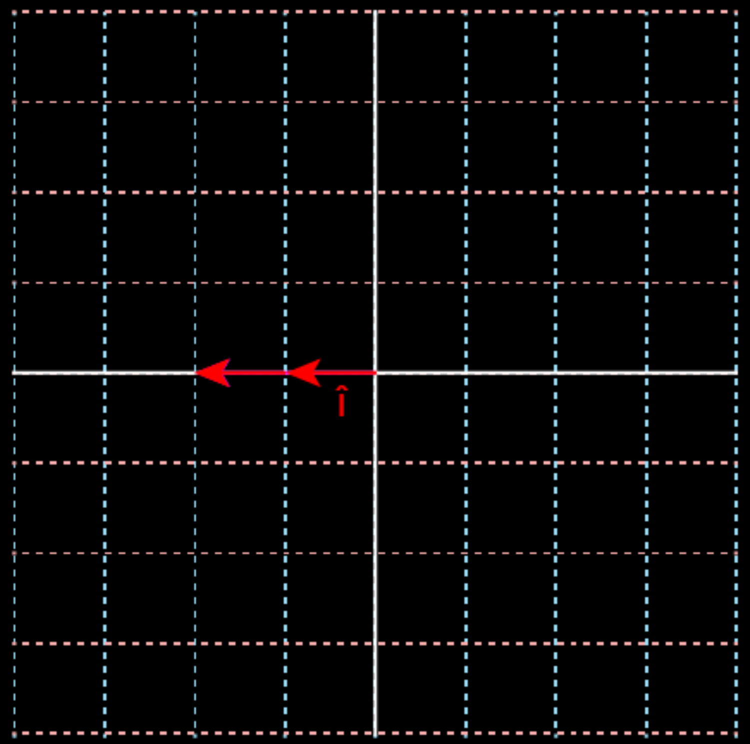
$\hat{\imath} = (1\ ,\ 0)$

$\hat{\jmath} = (0\ ,\ 1)$

Any vector can be created through a linear combination of impulses



Any vector can be created through a linear combination of impulses



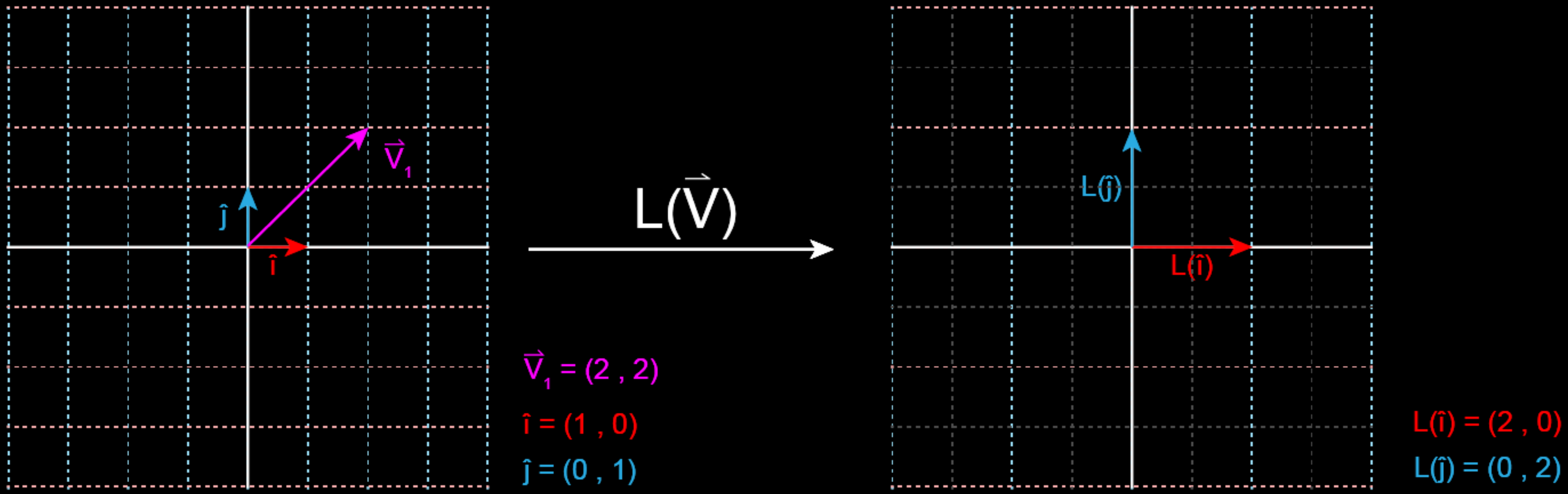
$\vec{V}_3 = (-2\ ,\ 0)$

$\vec{V}_3 = -2\hat{\imath} + 0\hat{\jmath}$

$\hat{\imath} = (1\ ,\ 0)$

$\hat{\jmath} = (0\ ,\ 1)$

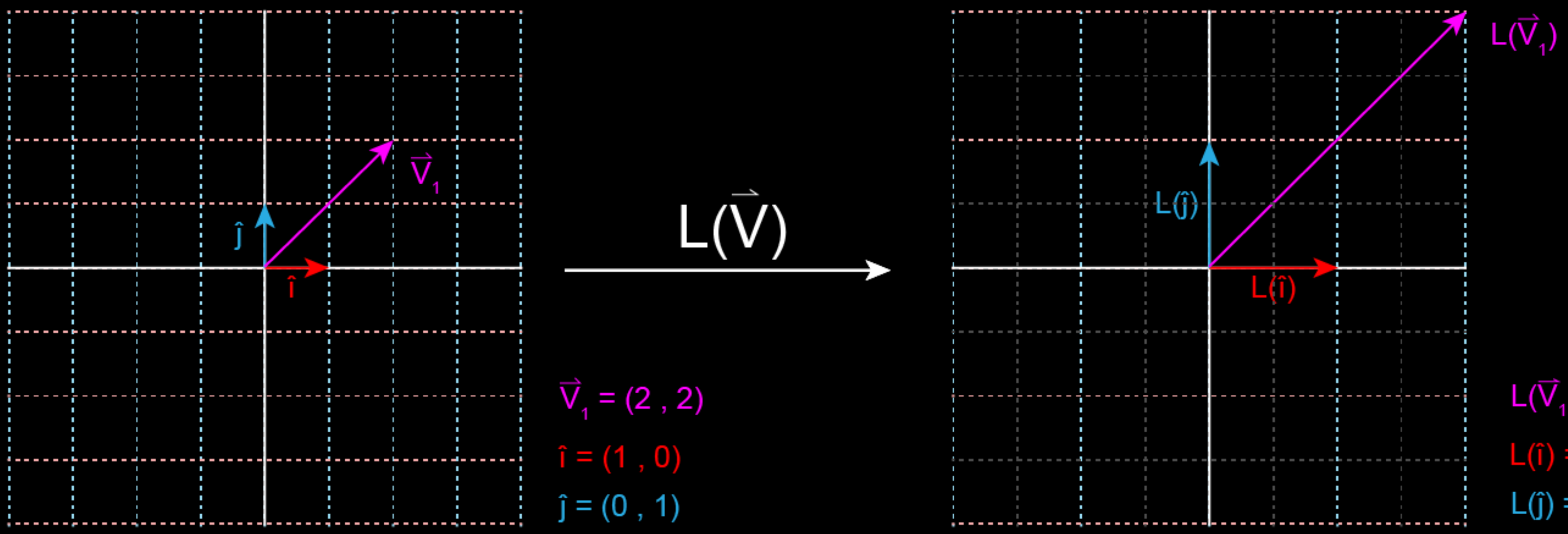
Matrices describe linear transformations



$$\vec{V}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$S(a\vec{x} + b\vec{y}) = aS(\vec{x}) + bS(\vec{y})$$

Matrices describe linear transformations



$\vec{V}_1 = (2\ ,\ 2)$

$\hat{\imath} = (1\ ,\ 0)$

$\hat{j} = (0\ ,\ 1)$

$L(\vec{V}_1) = (4\ ,\ 4)$

$L(\hat{\imath}) = (2\ ,\ 0)$

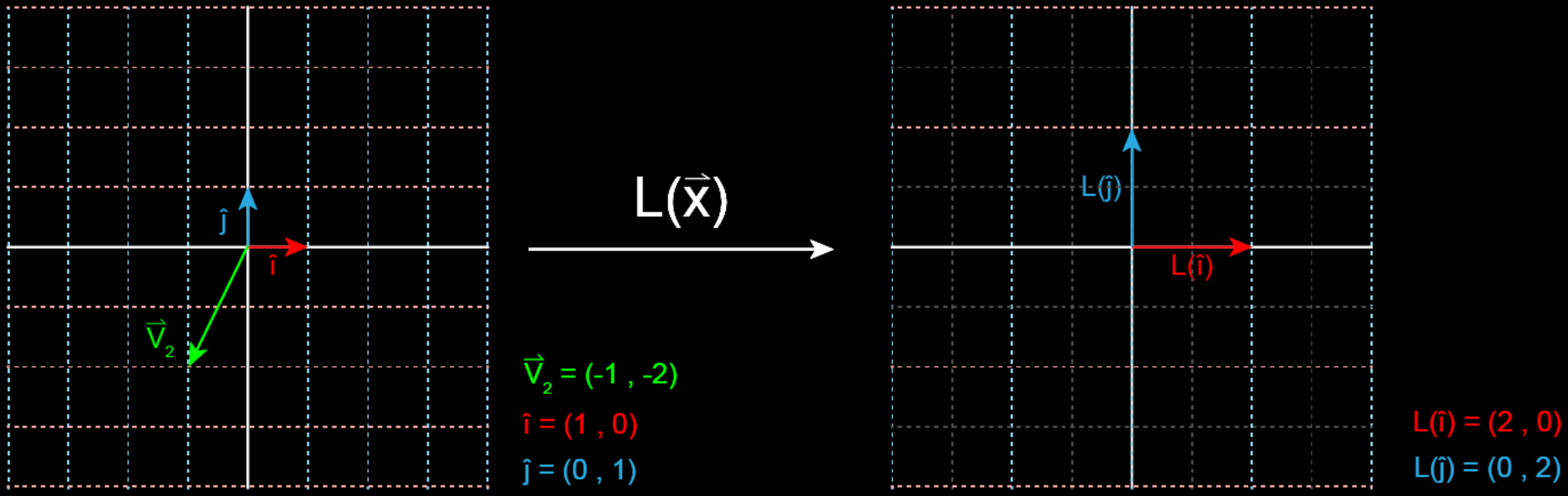
$L(\hat{j}) = (0\ ,\ 2)$

$$\vec{V}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$L(\vec{V}_1) = \begin{pmatrix} 4 \\ 4 \end{pmatrix} = 2\begin{pmatrix} 2 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$S(a\vec{x} + b\vec{y}) = aS(\vec{x}) + bS(\vec{y})$$

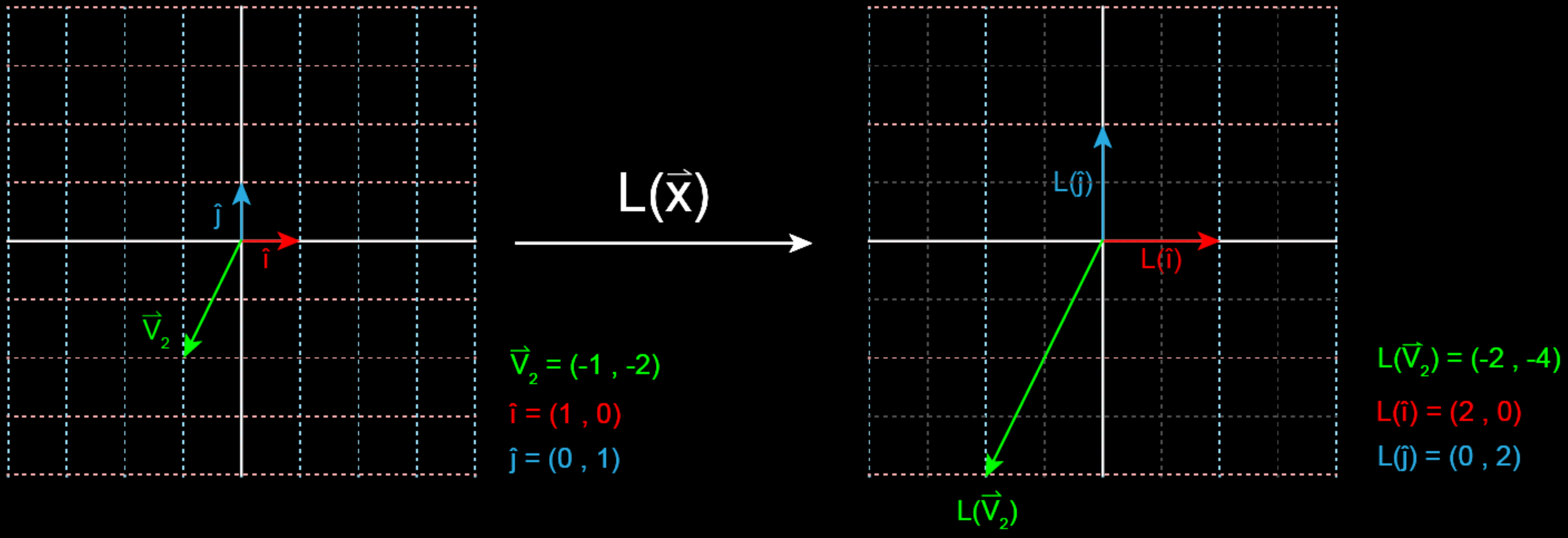
Matrices describe linear transformations



$$\vec{V}_2 = \begin{pmatrix} -1 \\ -2 \end{pmatrix} = -1\begin{pmatrix} 1 \\ 0 \end{pmatrix} + -2\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$S(a\vec{x} + b\vec{y}) = aS(\vec{x}) + bS(\vec{y})$$

Matrices describe linear transformations

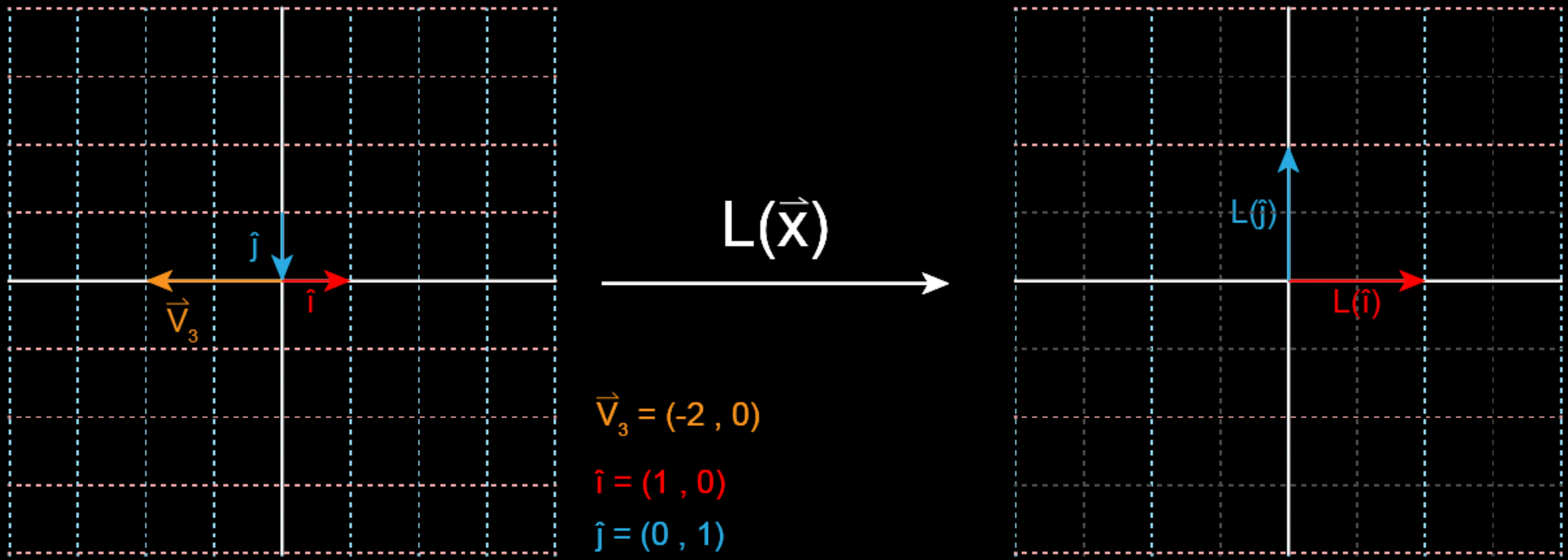


$$\vec{V}_2 = \begin{pmatrix}-1\\-2\end{pmatrix} = -1\begin{pmatrix}1\\0\end{pmatrix} + -2\begin{pmatrix}0\\1\end{pmatrix}$$

$$L(\vec{V}_2) = \begin{pmatrix}-2\\-4\end{pmatrix} = -1\begin{pmatrix}2\\0\end{pmatrix} + -2\begin{pmatrix}0\\2\end{pmatrix}$$

$$\boxed{S(a\vec{x} + b\vec{y}) = aS(\vec{x}) + bS(\vec{y})}$$

Matrices describe linear transformations



$\vec{V}_3 = (-2\ ,\ 0)$

$\hat{\imath} = (1\ ,\ 0)$

$\hat{\jmath} = (0\ ,\ 1)$

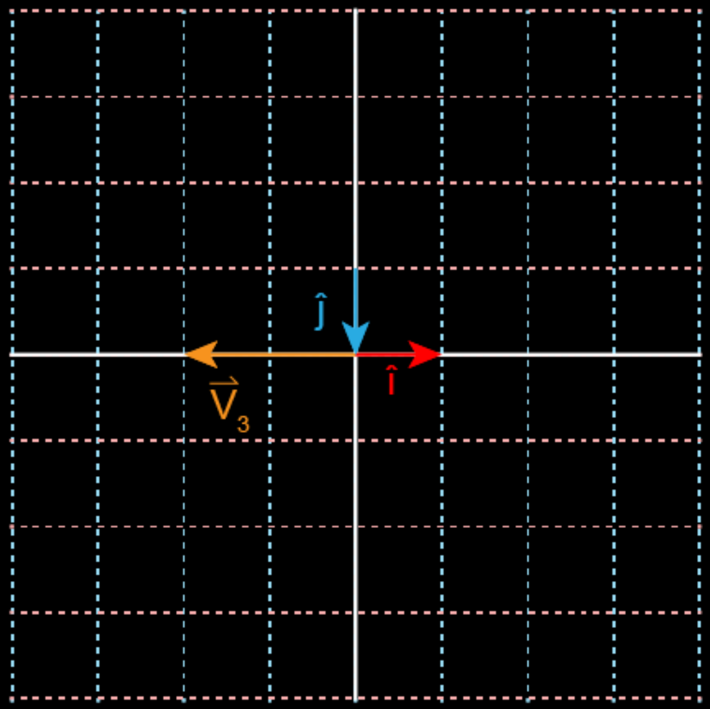
$L(\hat{\imath}) = (2\ ,\ 0)$

$L(\hat{\jmath}) = (0\ ,\ 2)$

$$\vec{V}_3 = \begin{pmatrix} -2 \\ 0 \end{pmatrix} = -2\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$S(a\vec{x} + b\vec{y}) = aS(\vec{x}) + bS(\vec{y})$$

Matrices describe linear transformations

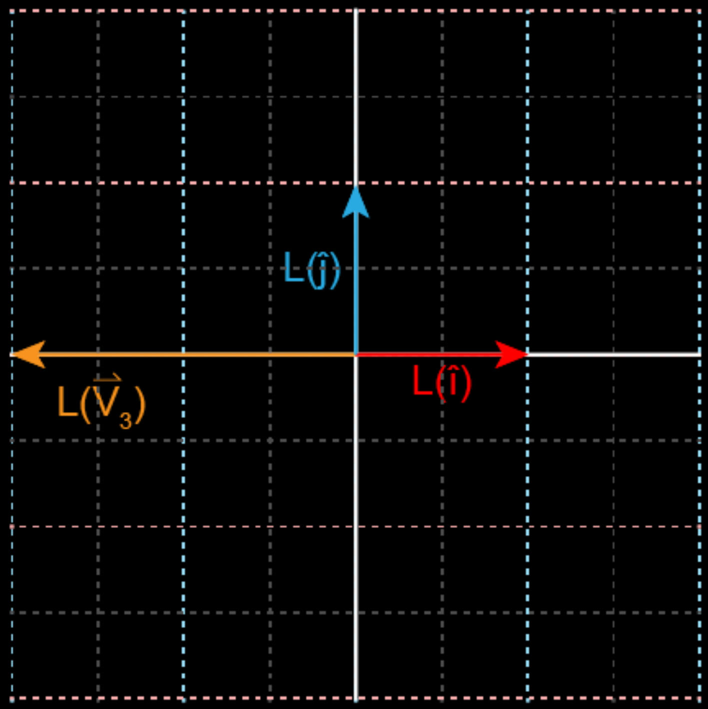


$$\xrightarrow{L(\vec{x})}$$

$\vec{V}_3 = (-2 , 0)$

$\hat{i} = (1 , 0)$

$\hat{j} = (0 , 1)$



$L(\vec{V}_3) = (-4 , 0)$

$L(\hat{i}) = (2 , 0)$

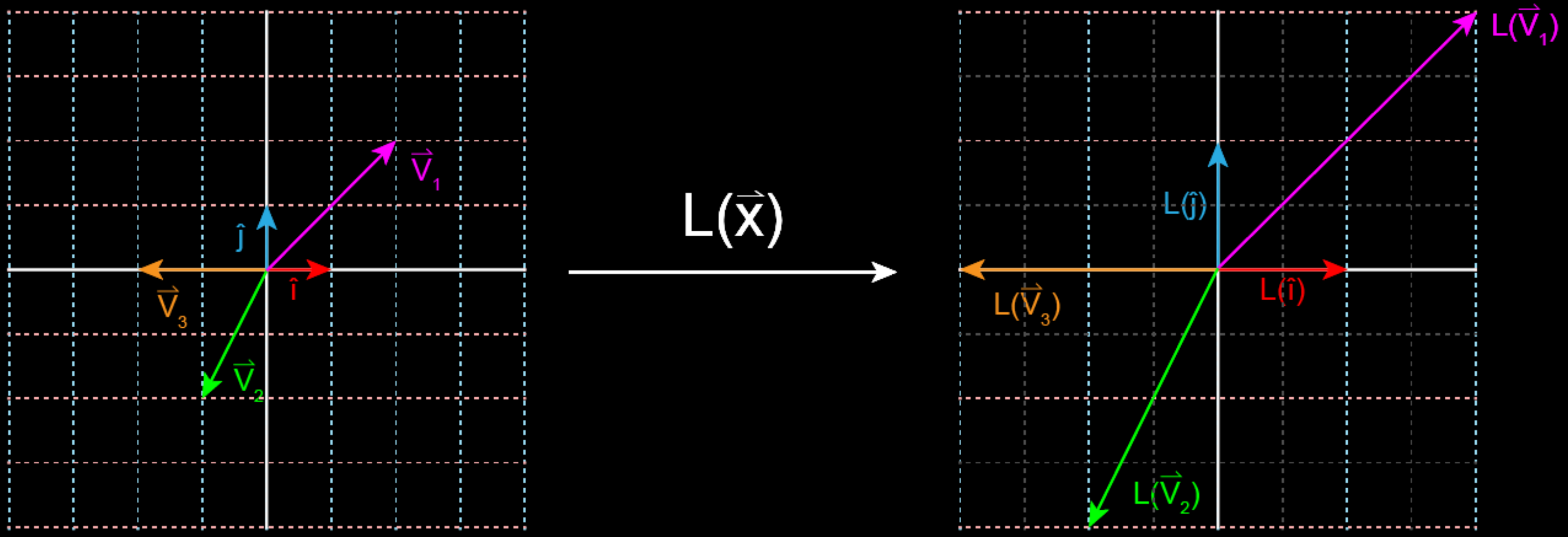
$L(\hat{j}) = (0 , 2)$

$$\vec{V}_3 = \begin{pmatrix} -2 \\ 0 \end{pmatrix} = -2\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$L(\vec{V}_3) = \begin{pmatrix} -4 \\ 0 \end{pmatrix} = -2\begin{pmatrix} 2 \\ 0 \end{pmatrix} + 0 \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$S(a\vec{x} + b\vec{y}) = aS(\vec{x}) + bS(\vec{y})$$

Matrices describe linear transformations



$$\vec{V}_1 = \begin{pmatrix}2\\2\end{pmatrix}$$

$$\vec{V}_2 = \begin{pmatrix}-1\\-2\end{pmatrix}$$

$$\vec{V}_3 = \begin{pmatrix}-2\\0\end{pmatrix}$$

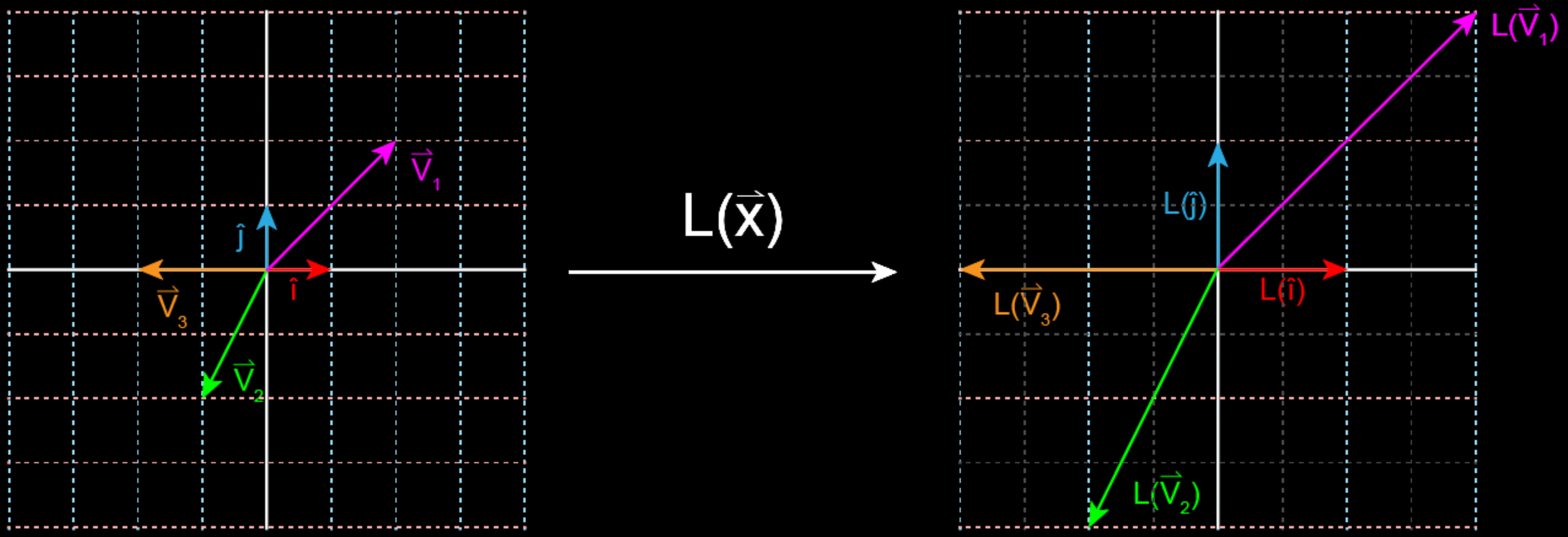
$$L(\vec{V}) = V_x\begin{pmatrix}2\\0\end{pmatrix} + V_y\begin{pmatrix}0\\2\end{pmatrix}$$

$$L(\vec{V}_1) = 2\begin{pmatrix}2\\0\end{pmatrix} + 2\begin{pmatrix}0\\2\end{pmatrix}$$

$$L(\vec{V}_2) = -1\begin{pmatrix}2\\0\end{pmatrix} + -2\begin{pmatrix}0\\2\end{pmatrix}$$

$$L(\vec{V}_3) = -2\begin{pmatrix}2\\0\end{pmatrix} + 0\begin{pmatrix}0\\2\end{pmatrix}$$

Matrices describe linear transformations



$\vec{V}_1 = \begin{pmatrix}2\\2\end{pmatrix}$

$\vec{V}_2 = \begin{pmatrix}-1\\-2\end{pmatrix}$

$\vec{V}_3 = \begin{pmatrix}-2\\0\end{pmatrix}$

$$L(\vec{V}) = V_x\begin{pmatrix}2\\0\end{pmatrix} + V_y\begin{pmatrix}0\\2\end{pmatrix}$$

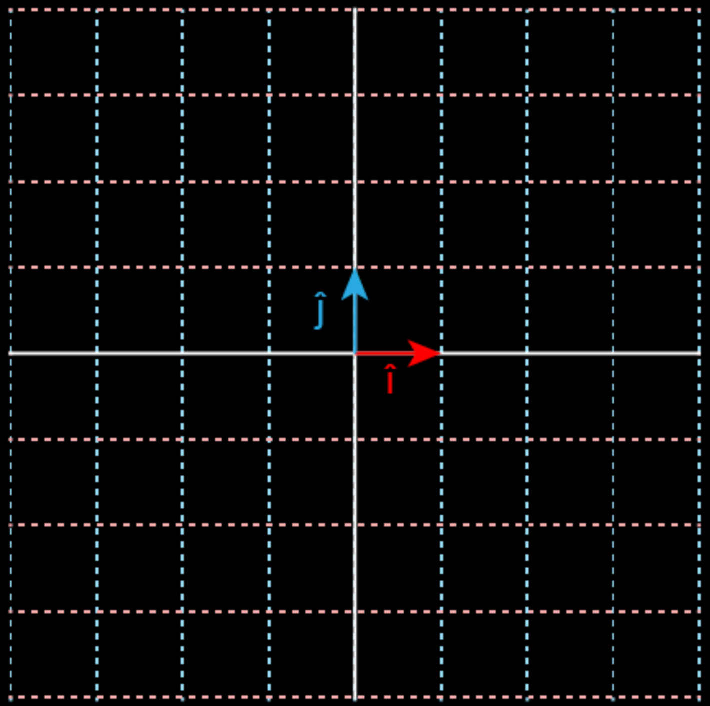
$$L(\vec{V}) = \begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}\begin{pmatrix}V_x\\V_y\end{pmatrix}$$

$$L(\vec{V}_1) = 2\begin{pmatrix}2\\0\end{pmatrix} + 2\begin{pmatrix}0\\2\end{pmatrix}$$

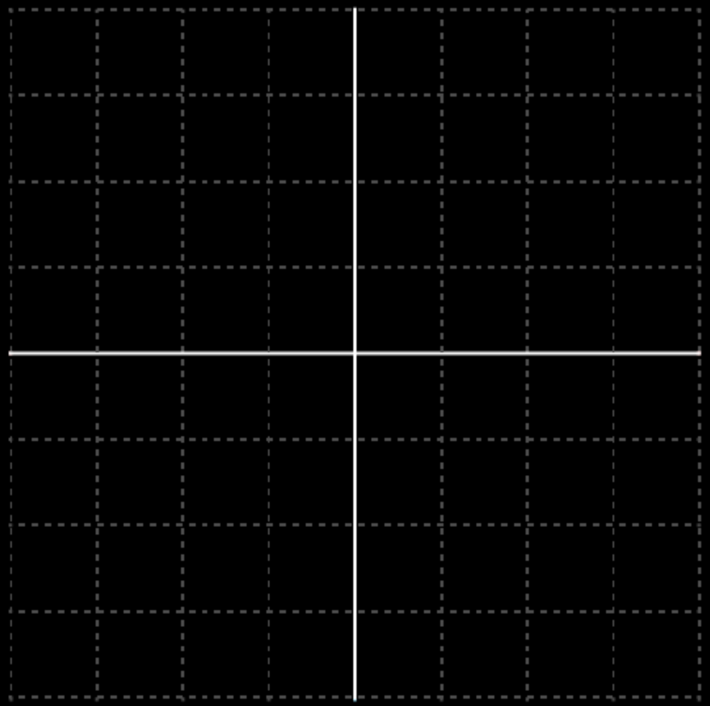
$$L(\vec{V}_2) = -1\begin{pmatrix}2\\0\end{pmatrix} + -2\begin{pmatrix}0\\2\end{pmatrix}$$

$$L(\vec{V}_3) = -2\begin{pmatrix}2\\0\end{pmatrix} + 0\begin{pmatrix}0\\2\end{pmatrix}$$

Matrices describe linear transformations



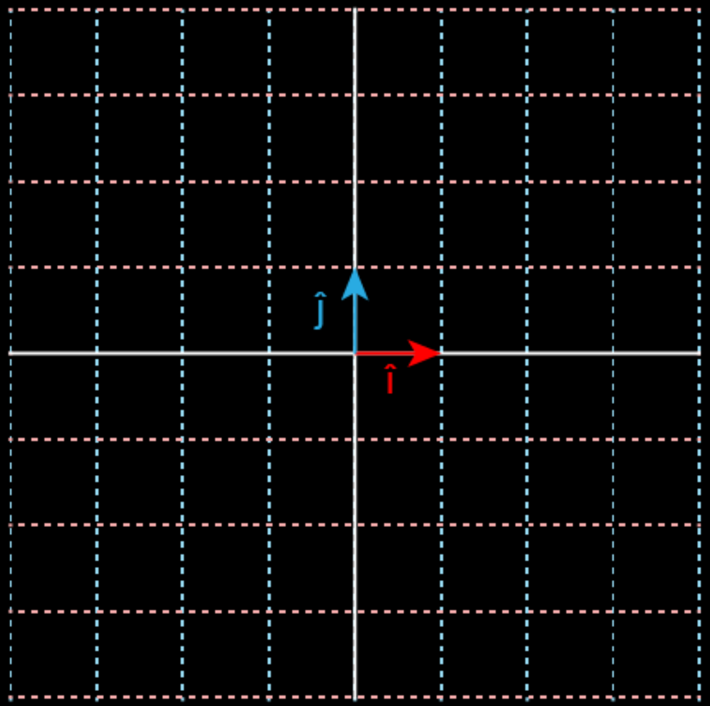
$$L(\vec{V}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$



$\hat{\imath} = (1 , 0)$

$\hat{\jmath} = (0 , 1)$

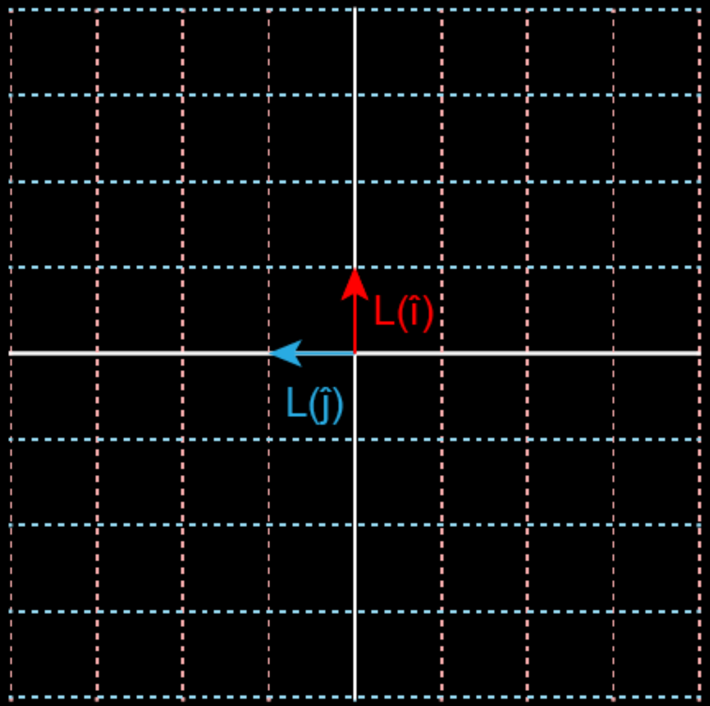
Matrices describe linear transformations



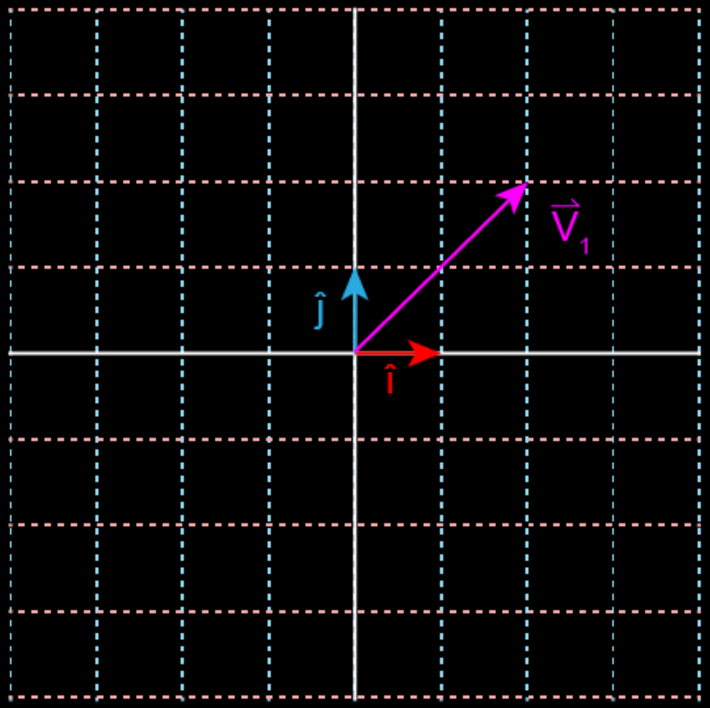
$$L(\vec{V}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

$\hat{\imath} = (1 , 0)$

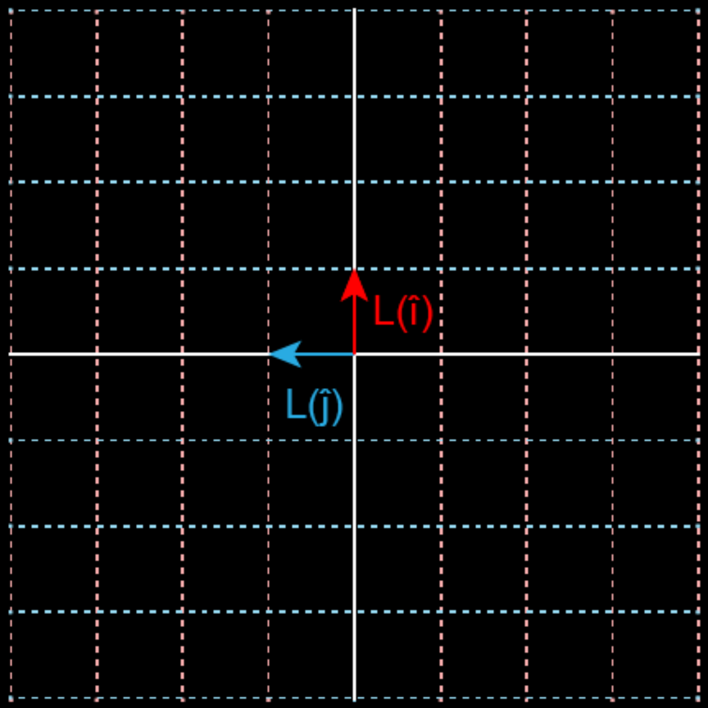
$\hat{\jmath} = (0 , 1)$



Matrices describe linear transformations



$$L(\vec{V}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$



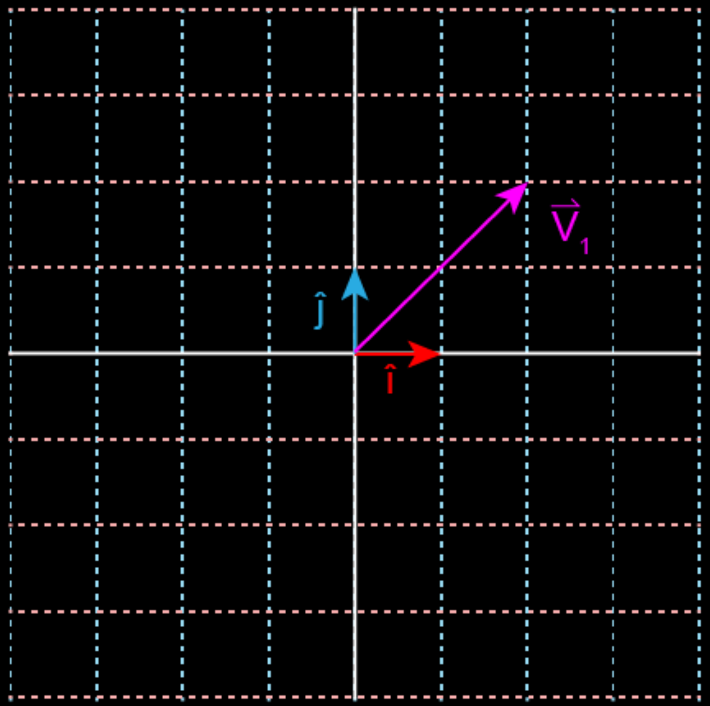
$\vec{V}_1 = (2 , 2)$

$\hat{\imath} = (1 , 0)$

$\hat{\jmath} = (0 , 1)$

$$\vec{V}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Matrices describe linear transformations

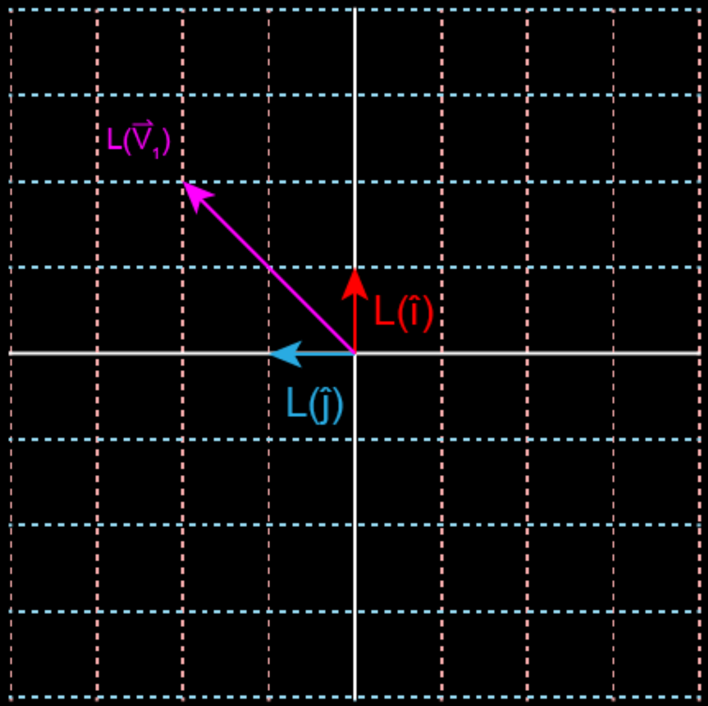


$$L(\vec{V}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

$\vec{V}_1 = (2 , 2)$

$\hat{i} = (1 , 0)$

$\hat{j} = (0 , 1)$



$L(\vec{V}_1) = (-2 , 2)$

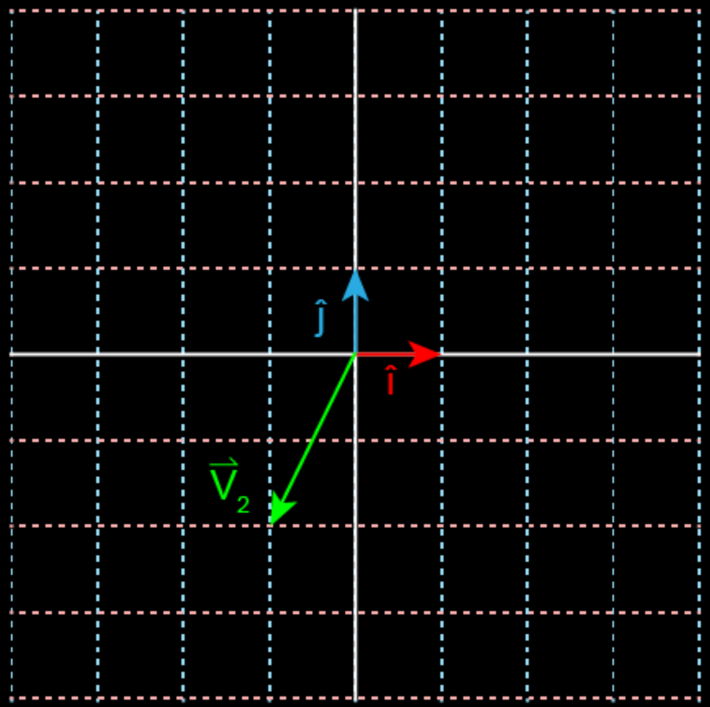
$L(\hat{i}) = (0 , 1)$

$L(\hat{j}) = (-1 , 0)$

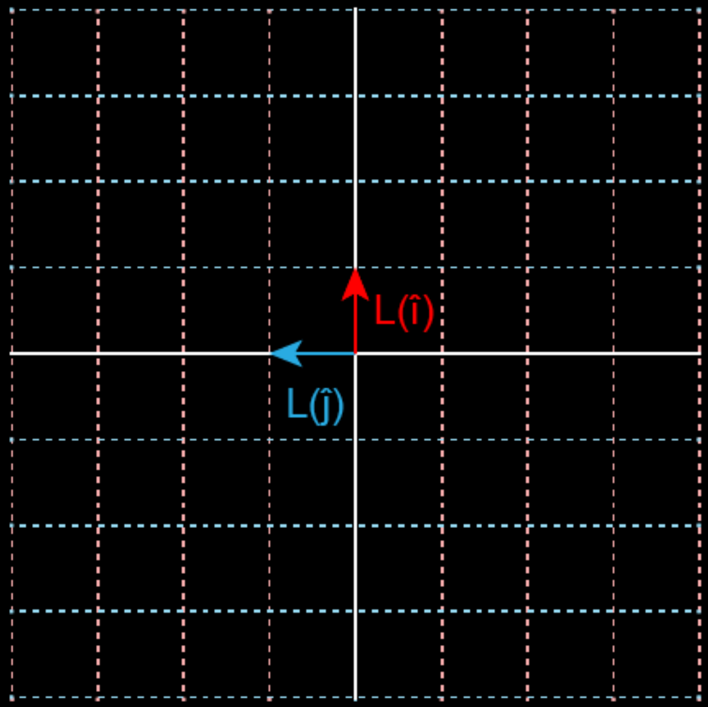
$$\vec{V}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$L(\vec{V}_1) = \begin{pmatrix} -2 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

Matrices describe linear transformations



$$L(\vec{V}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$



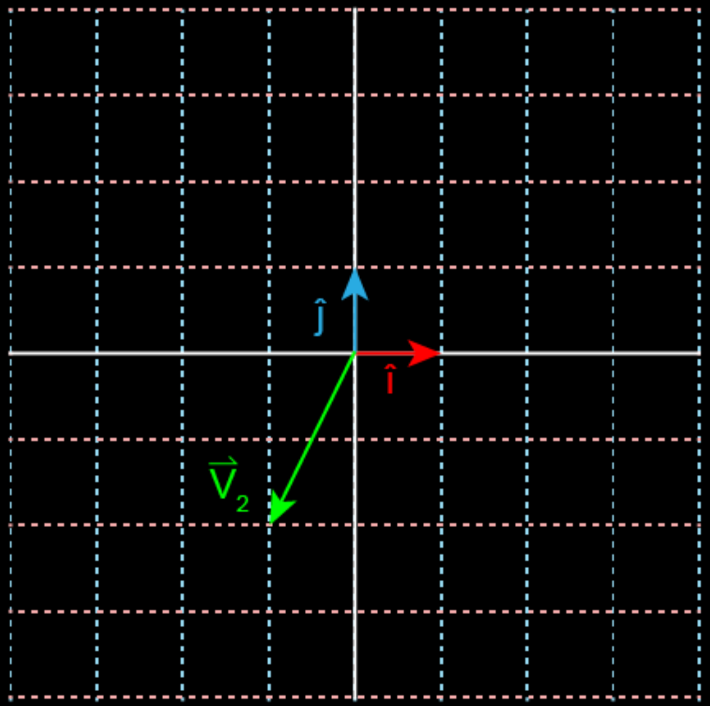
$\vec{V}_2 = (-1 , -2)$

$\hat{i} = (1 , 0)$

$\hat{j} = (0 , 1)$

$$\vec{V}_2 = \begin{pmatrix} -1 \\ -2 \end{pmatrix} = -1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + -2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Matrices describe linear transformations

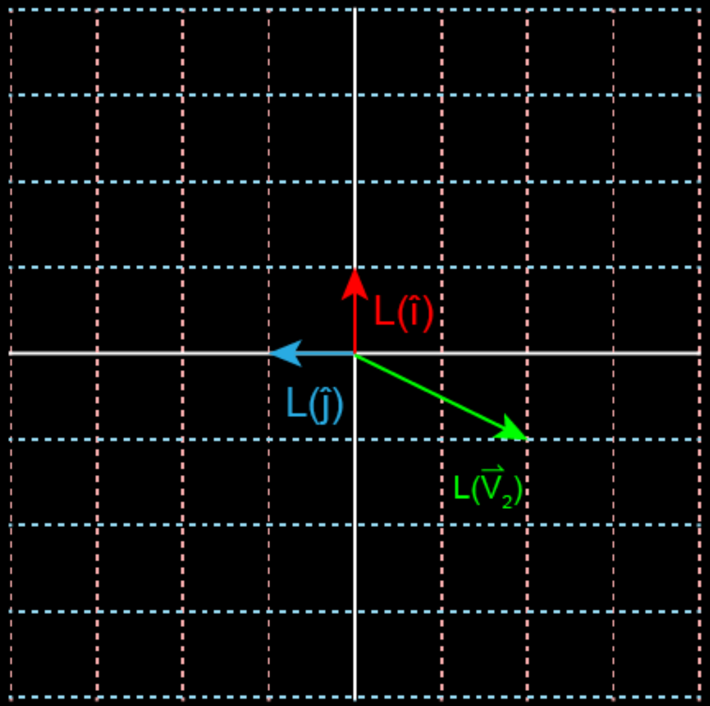


$$L(\vec{V}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

$\vec{V}_2 = (-1\ ,\ -2)$

$\hat{\imath} = (1\ ,\ 0)$

$\hat{\jmath} = (0\ ,\ 1)$



$L(\vec{V}_2) = (1\ ,\ -2)$

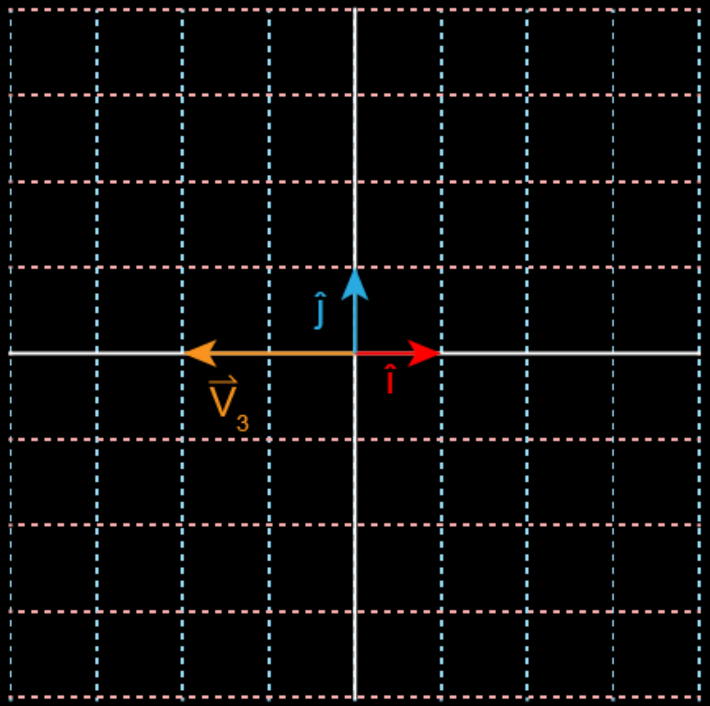
$L(\hat{\imath}) = (0\ ,\ 1)$

$L(\hat{\jmath}) = (-1,\ 0)$

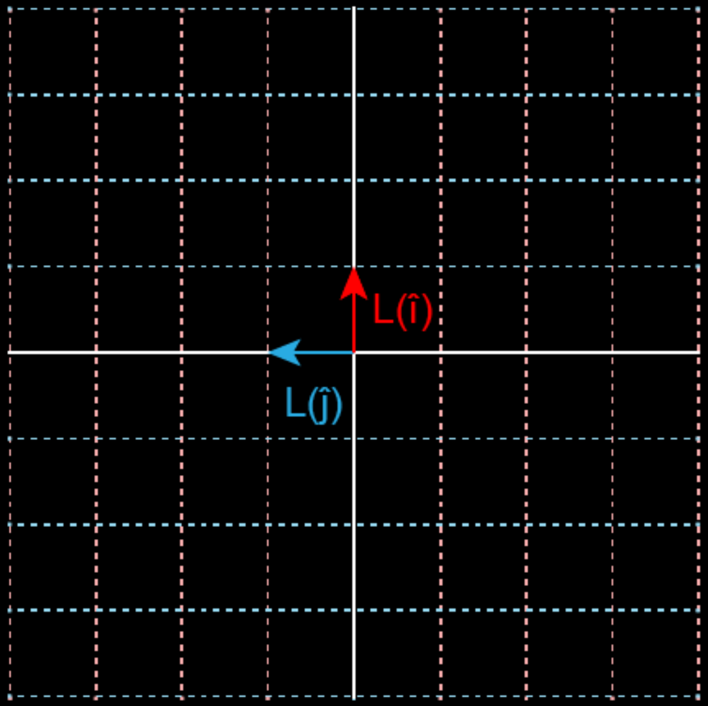
$$\vec{V}_2 = \begin{pmatrix} -1 \\ -2 \end{pmatrix} = -1\begin{pmatrix} 1 \\ 0 \end{pmatrix} + -2\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$L(\vec{V}_2) = \begin{pmatrix} 2 \\ -1 \end{pmatrix} = -1\begin{pmatrix} 0 \\ 1 \end{pmatrix} + -2\begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

Matrices describe linear transformations



$$L(\vec{V}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$



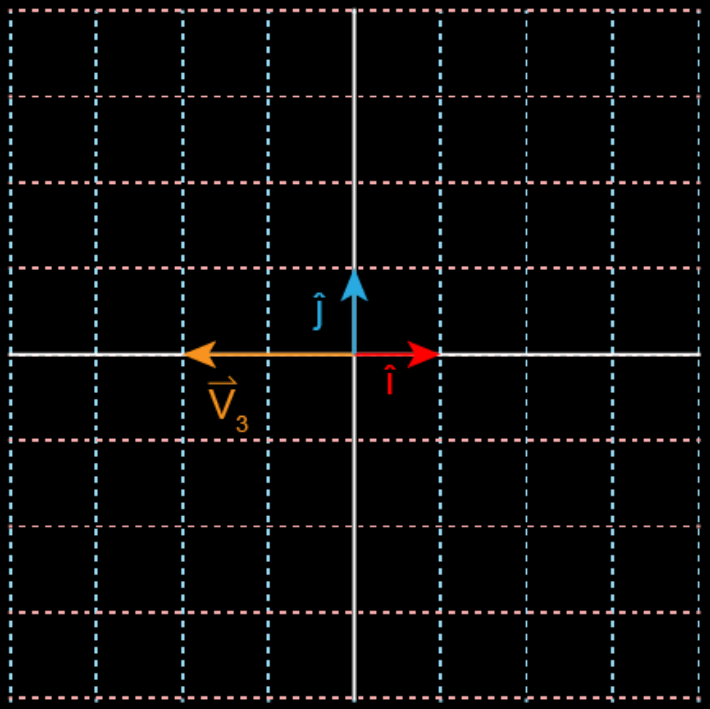
$\vec{V}_3 = (-2\ ,\ 0)$

$\hat{\imath} = (1\ ,\ 0)$

$\hat{\jmath} = (0\ ,\ 1)$

$$\vec{V}_3 = \begin{pmatrix} -2 \\ 0 \end{pmatrix} = -2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Matrices describe linear transformations

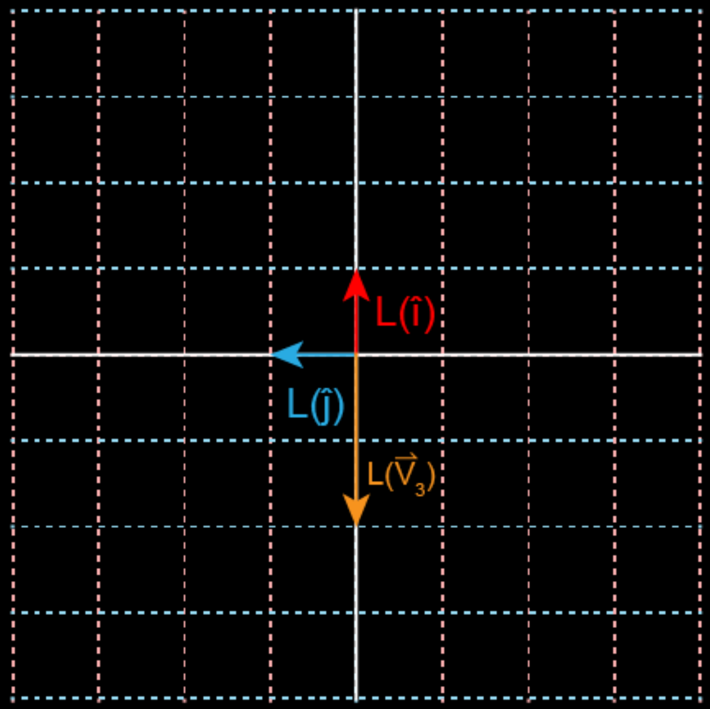


$$L(\vec{V}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

$\vec{V}_3 = (-2 , 0)$

$\hat{\imath} = (1 , 0)$

$\hat{j} = (0 , 1)$



$L(\vec{V}_3) = (0 , -2)$

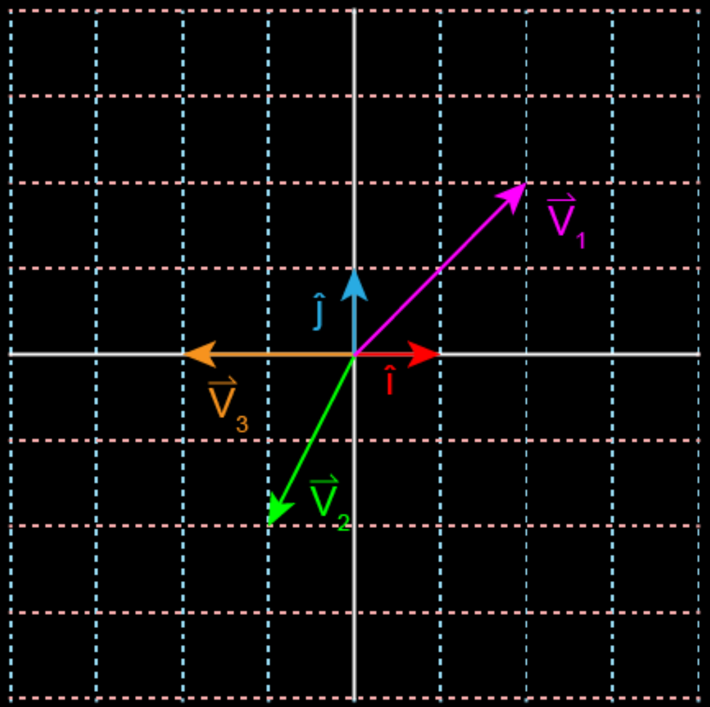
$L(\hat{\imath}) = (0 , 1)$

$L(\hat{j}) = (-1 , 0)$

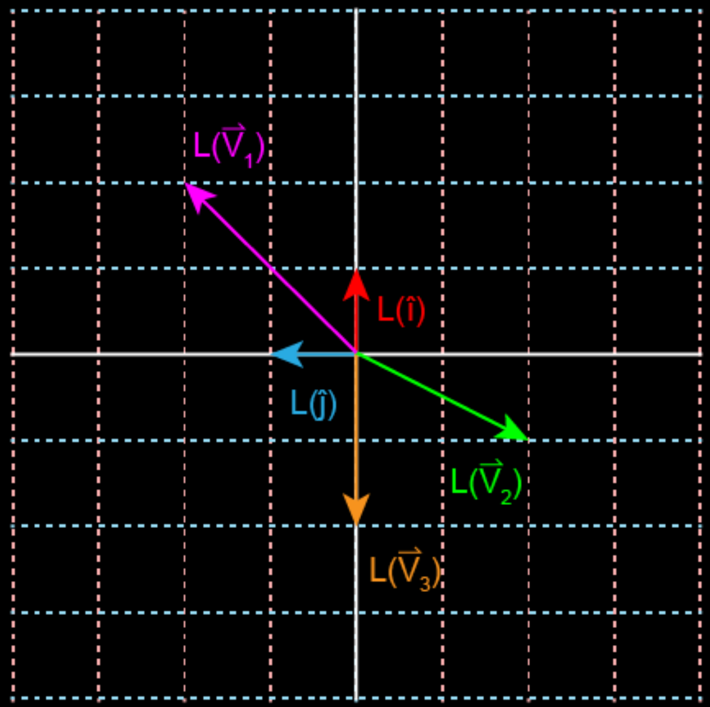
$$\vec{V}_3 = \begin{pmatrix} -2 \\ 0 \end{pmatrix} = -2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$L(\vec{V}_3) = \begin{pmatrix} 0 \\ -2 \end{pmatrix} = -2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 0 \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

Matrices describe linear transformations



$$L(\vec{V}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$



$$\vec{V}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\vec{V}_2 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\vec{V}_3 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

$$L(\vec{V}) = V_x \begin{pmatrix} 0 \\ 1 \end{pmatrix} + V_y \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$L(\vec{V}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

$$L(\vec{V}_1) = 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

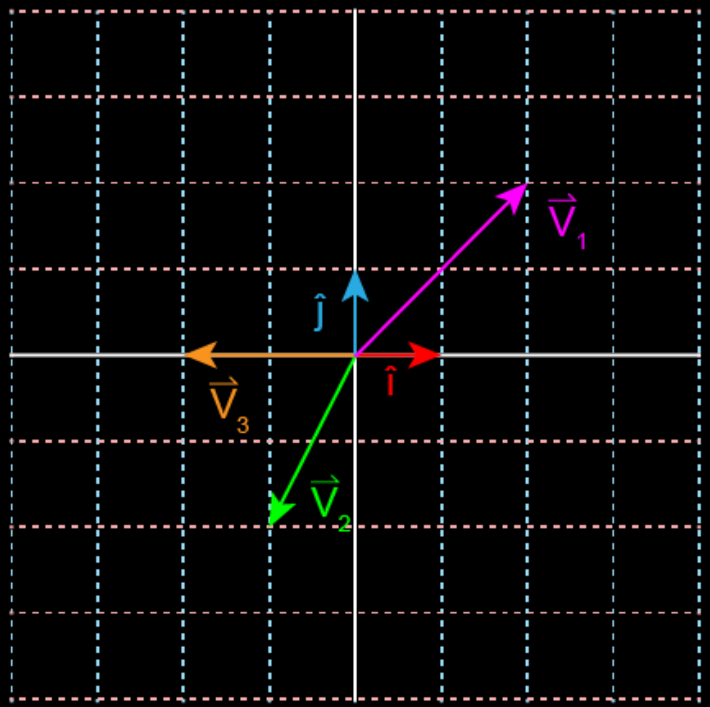
$$L(\vec{V}_2) = -1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + -2 \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$L(\vec{V}_3) = -2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 0 \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

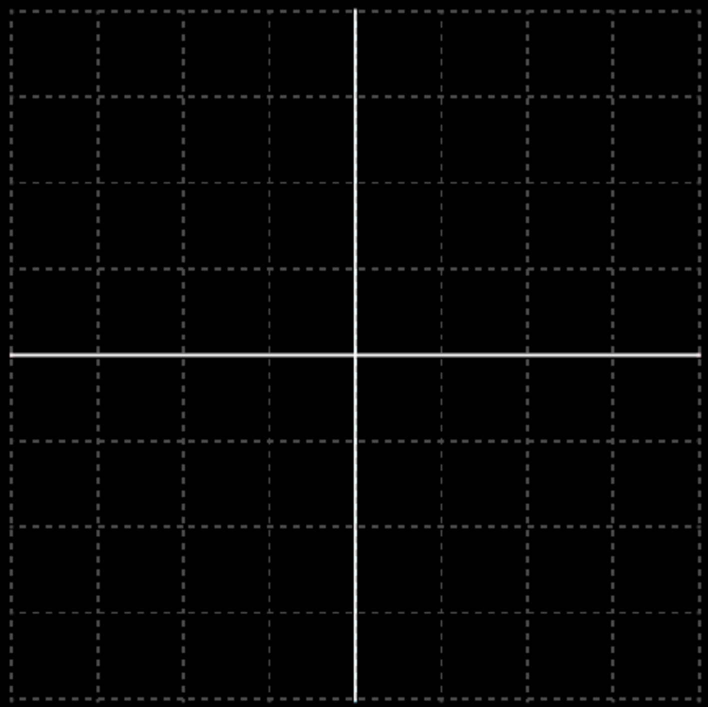
Exercises

Predict the linear transformation (without math!)

Exercise 1



$$L(\vec{V}) = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$



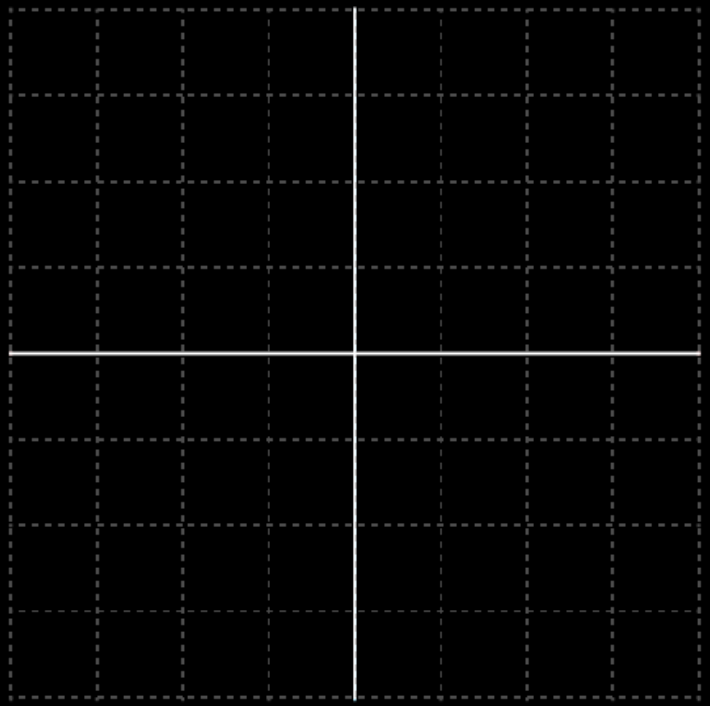
$$\vec{V}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\vec{V}_2 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\vec{V}_3 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

Exercise 2

$$L(\vec{V}) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$



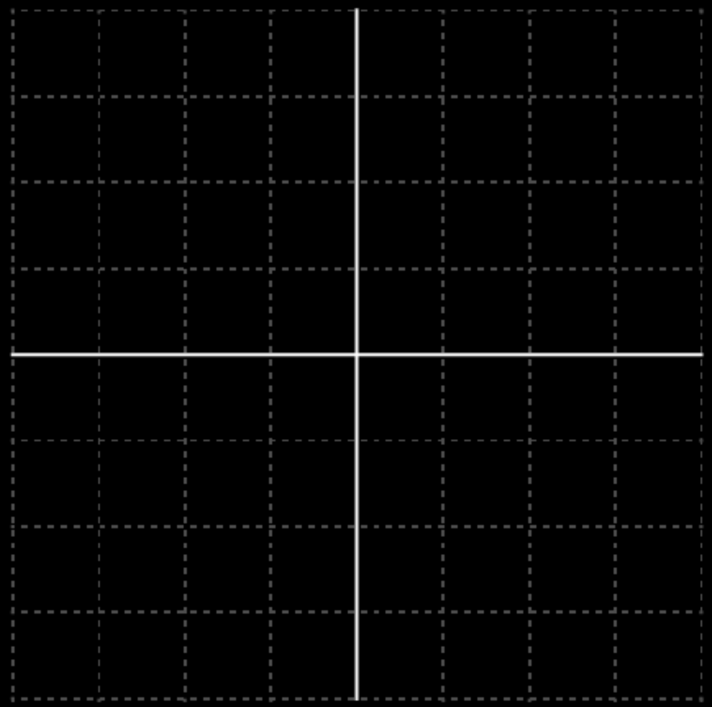
$$\vec{V}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\vec{V}_2 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\vec{V}_3 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

Exercise 3

$$L(\vec{V}) = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

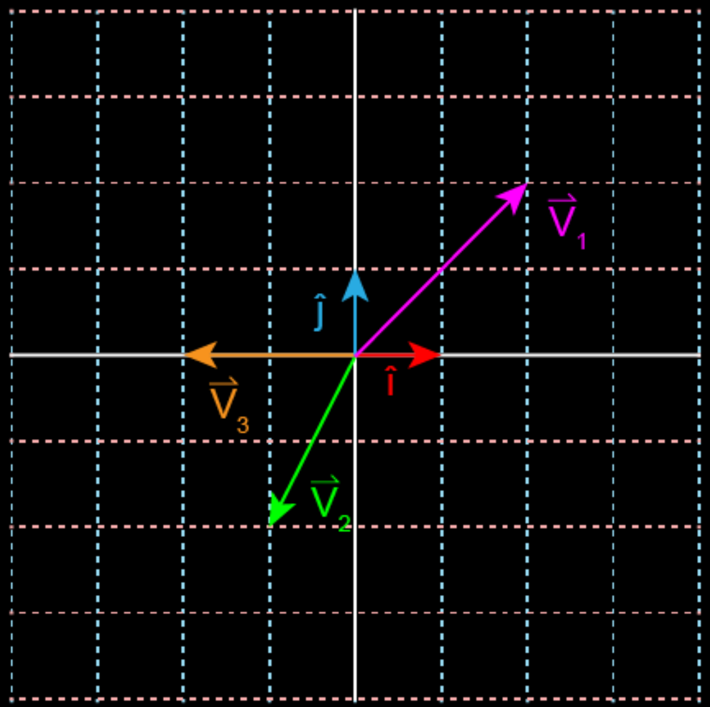


$$\vec{V}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\vec{V}_2 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\vec{V}_3 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

Exercise 4



$$L(\vec{V}) = \begin{pmatrix} 2 & 0 \\ 0 & 0.5 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

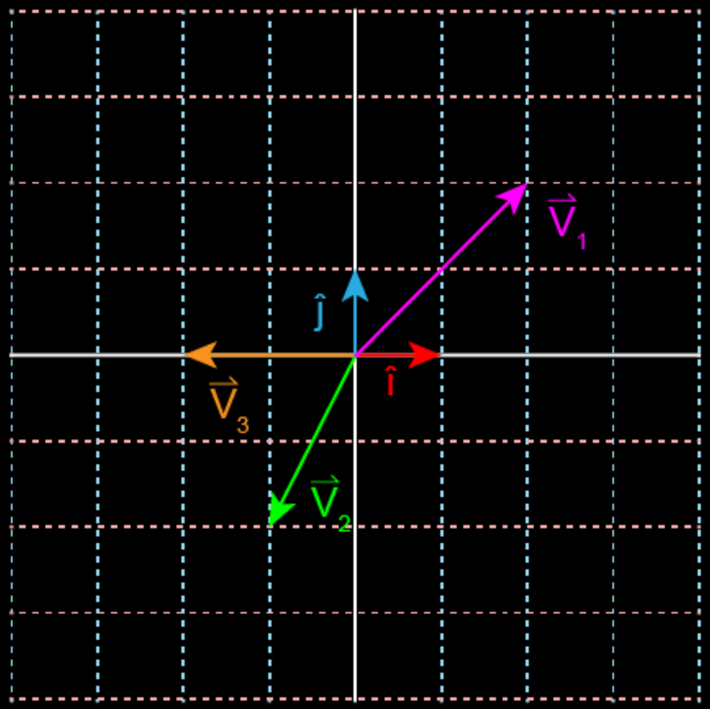
$\longrightarrow$

$$\vec{V}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

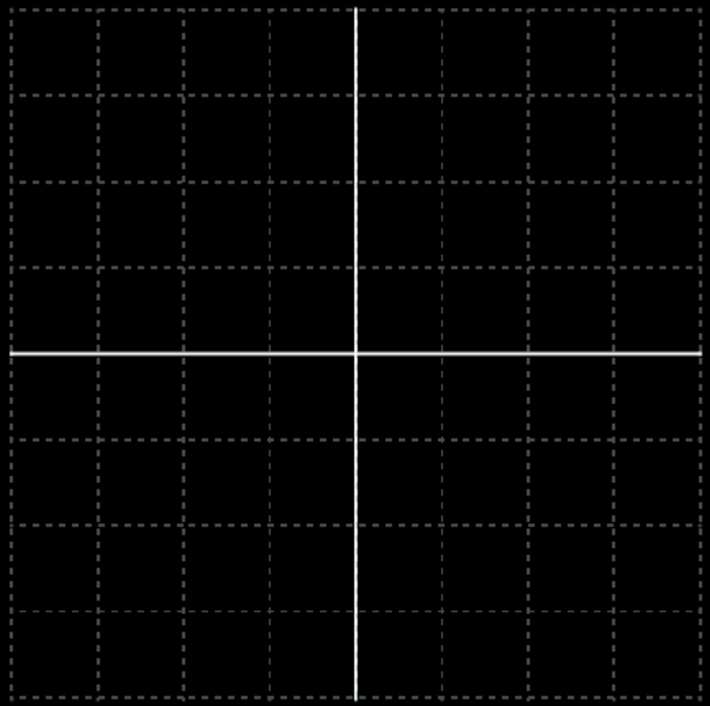
$$\vec{V}_2 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\vec{V}_3 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

Exercise 5



$$L(\vec{V}) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} V_x \\ V_y \end{pmatrix}$$



$$\vec{V}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\vec{V}_2 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\vec{V}_3 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

Exercise 6

$$L(\vec{V}) = \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

$$\vec{V}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\vec{V}_2 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\vec{V}_3 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

Exercise 7

$$L(\vec{V}) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

$$\longrightarrow$$

$$\vec{V}_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\vec{V}_2 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\vec{V}_3 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

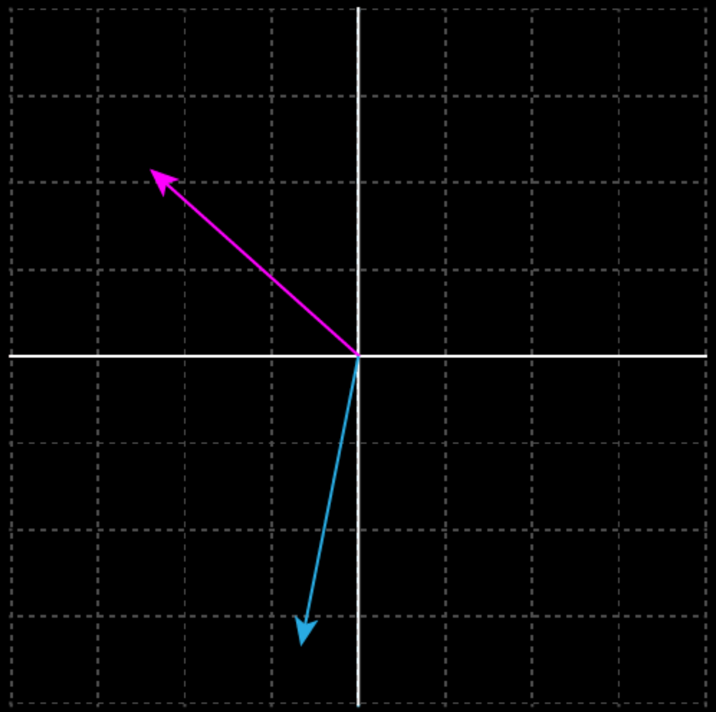
Dot product

How are two vectors related in space?

Are they similar (aligned in space)?

Are they different (orthogonal in space)?

Can we easily quantify this?



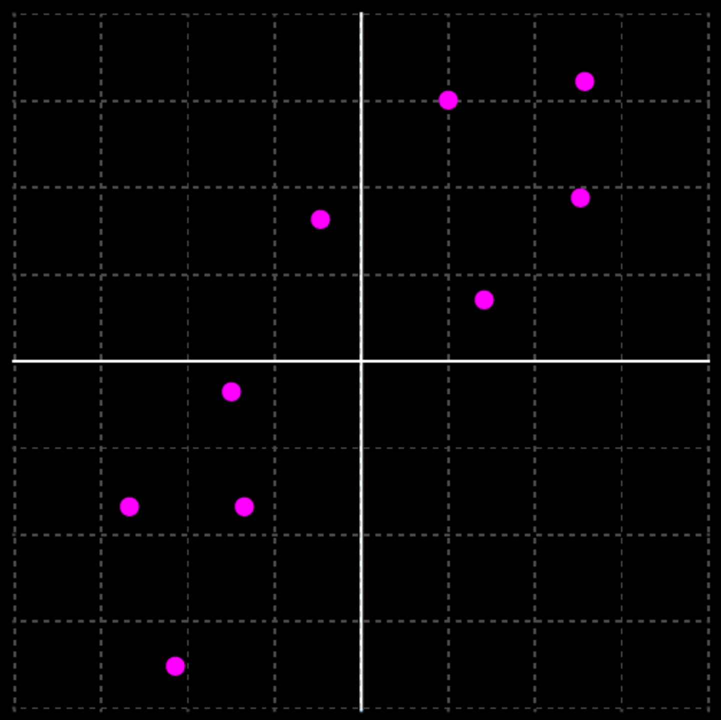
$$\begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} \quad ? \quad \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$$

How are two vectors related in space?

Are they similar (aligned in space)?

Are they different (orthogonal in space)?

Can we easily quantify this?



How are two vectors related in space?

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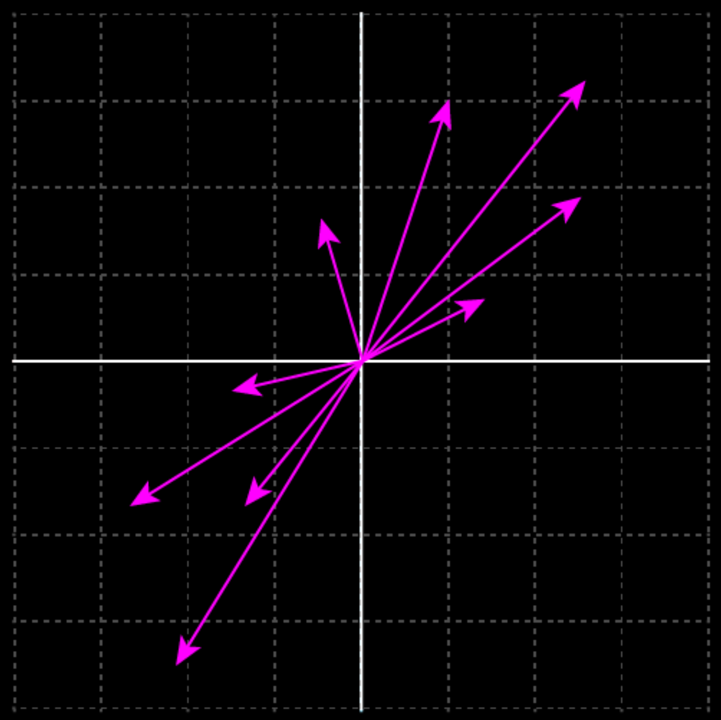

How are two vectors related in space?

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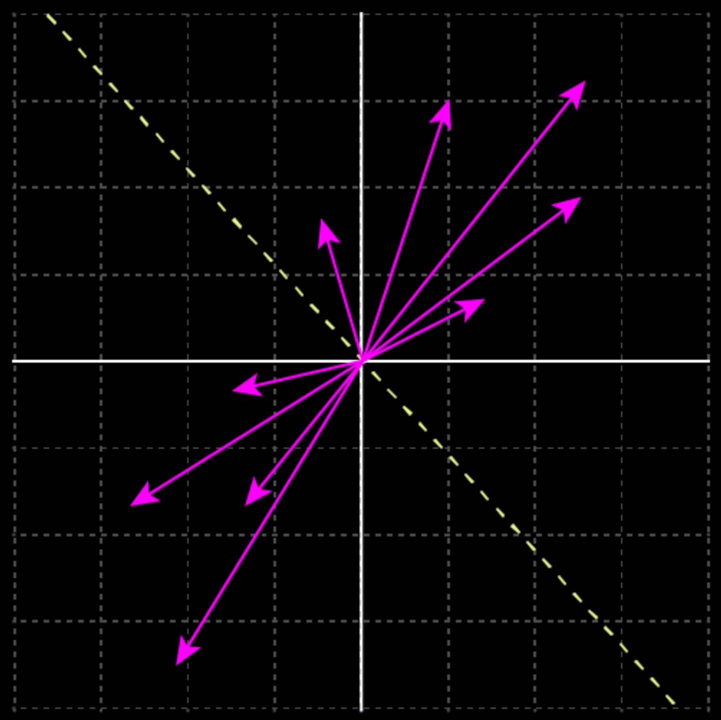

How are two vectors related in space?

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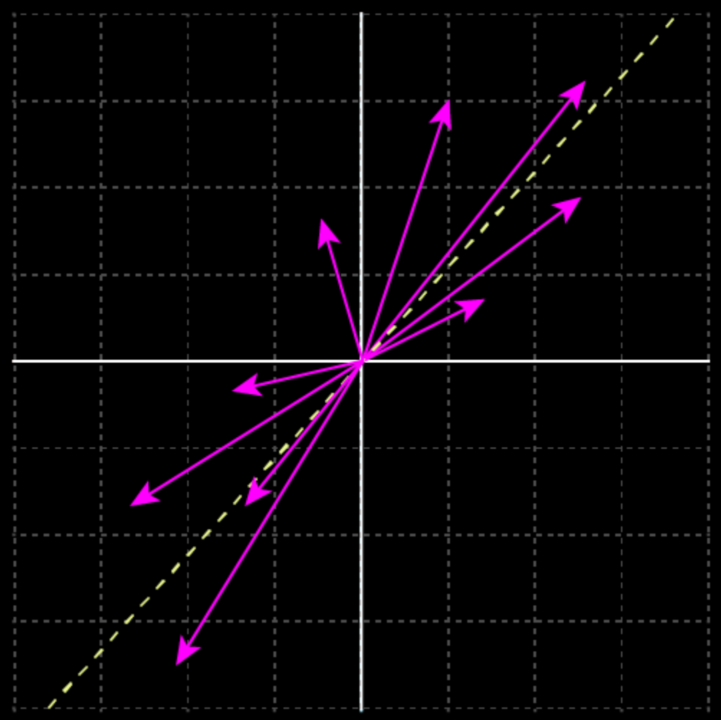
Are they different (orthogonal in space)?

Can we easily quantify this?



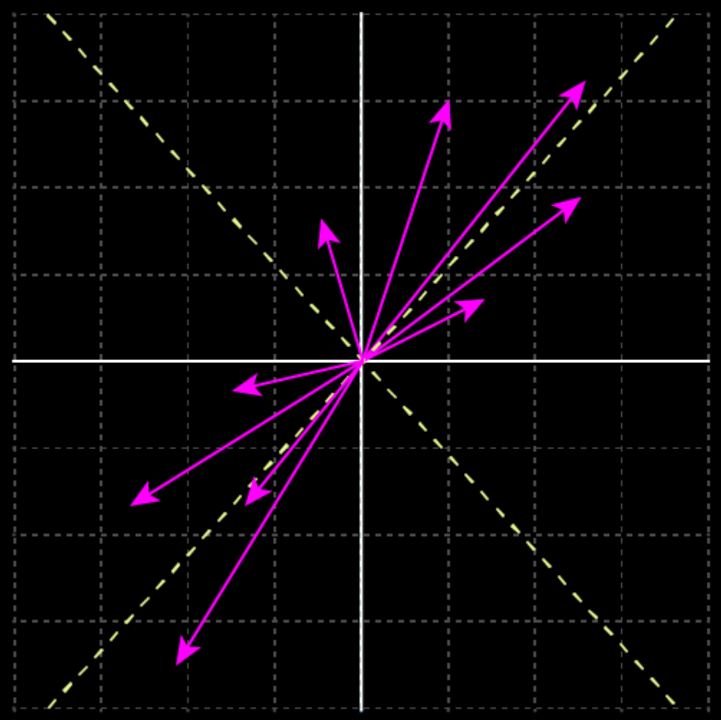
How are two vectors related in space?

Are they similar (aligned in space)?

Are they different (orthogonal in space)?

Can we easily quantify this?

Rotate line until you find a line that has maximum alignement with data points



Mathematical

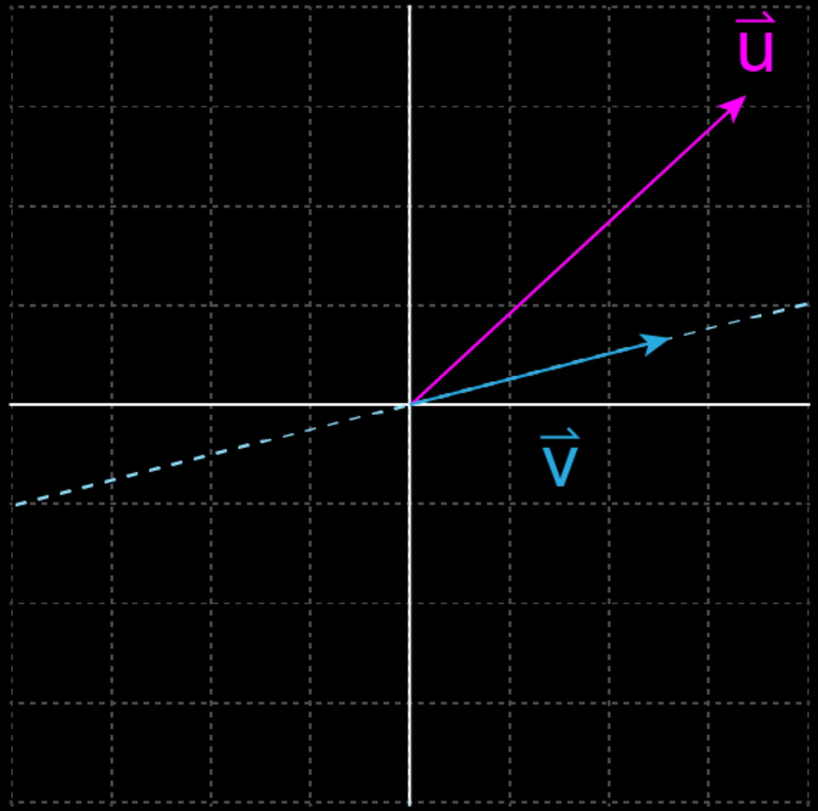
$$\vec{V} \cdot \vec{U}$$

$$= V_1 * U_1 + V_2 * U_2 + \ldots + V_n * U_n$$

$$= ||\vec{v}|| * ||\vec{u}|| * \cos(\theta_{vu})$$

Geometry

$$\vec{V} \cdot \vec{U}$$



Component of $\vec{u}$ that lies along the line defined by $\vec{v}$, scaled by the length of $\vec{v}$

If $||\vec{v}|| = 1$ (unit vector), then dot product is the projection of $\vec{u}$ onto line defined by $\vec{v}$

Linear Transformation

$$\vec{V} \cdot \vec{U}$$

$$= v_1 * u_1 + v_2 * u_2 + \ldots + v_n * u_n$$

$$= \begin{pmatrix} v_1 & v_2 & \ldots & v_n \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}$$

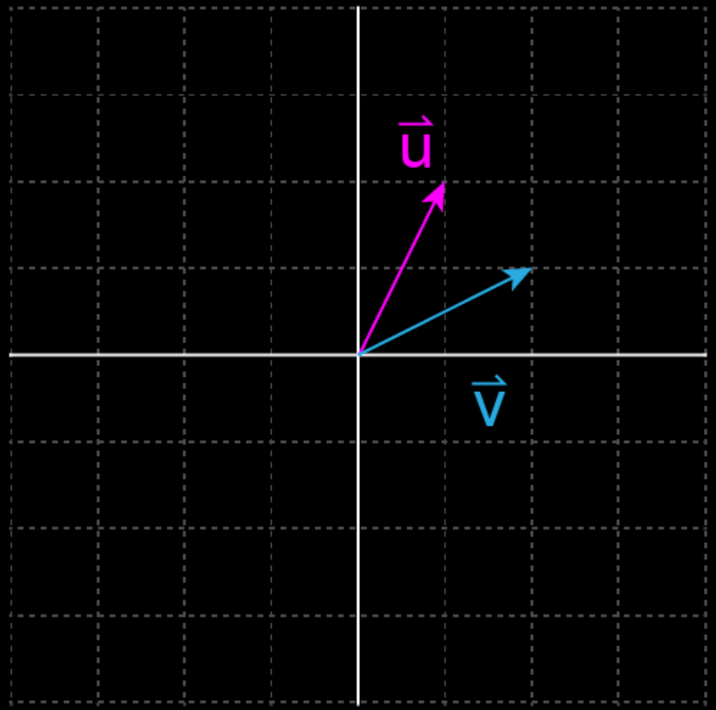
Linear Transformation

$$\vec{V} \cdot \vec{U}$$

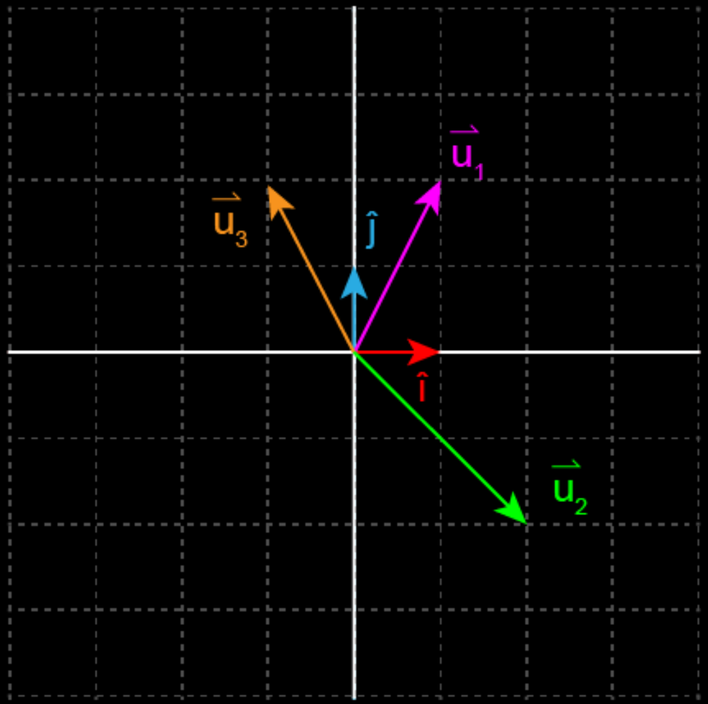
$$= v_1 * u_1 + v_2 * u_2 + \dots + v_n * u_n$$

$$= \begin{pmatrix} v_1 & v_2 & \dots & v_n \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$



Exercise 8



$$L(\vec{u}) = \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

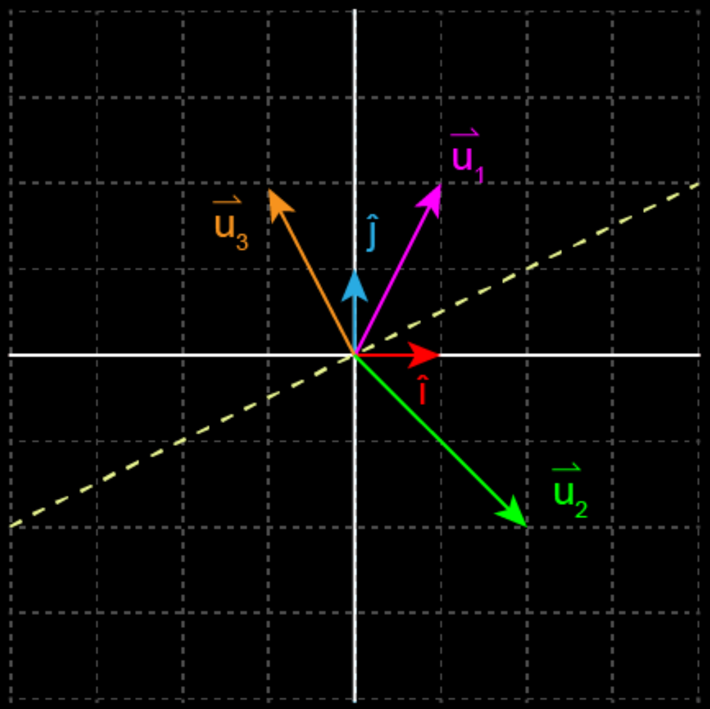
$$\longrightarrow$$

$$\vec{u}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

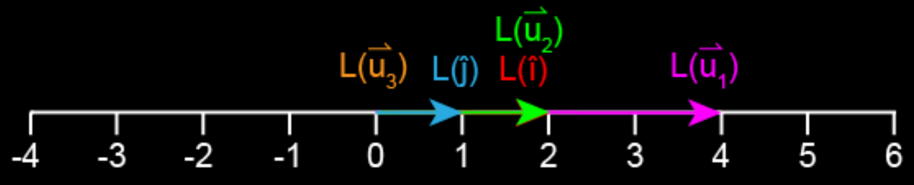
$$\vec{u}_2 = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$$\vec{u}_3 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Exercise 8



$$L(\vec{u}) = \begin{pmatrix} 2 & 1 \end{pmatrix}\begin{pmatrix} u_x \\ u_y \end{pmatrix}$$



$\vec{u}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\qquad L(\vec{u}_1) = 4$

$\vec{u}_2 = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ $\qquad L(\vec{u}_2) = 2$

$\vec{u}_3 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\qquad L(\vec{u}_3) = 0$

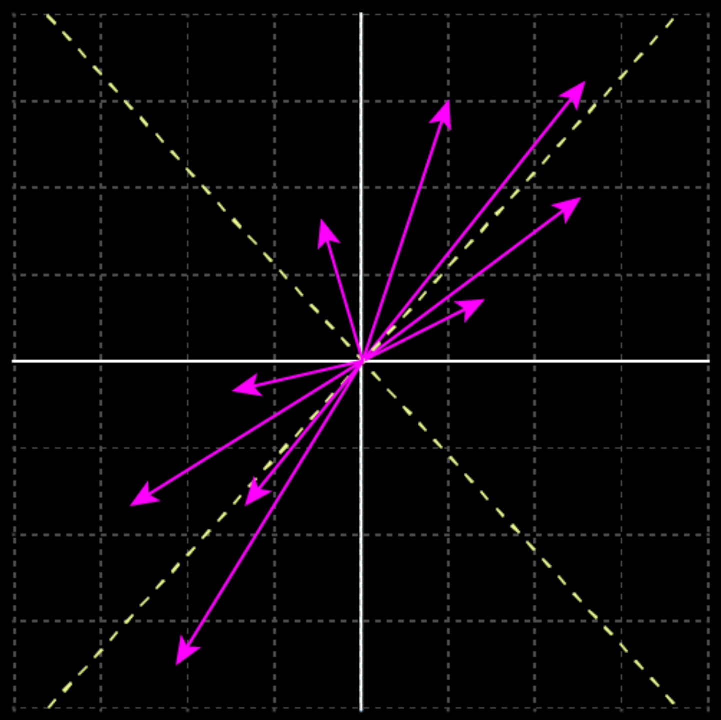
How are two vectors related in space?

Are they similar (aligned in space)?

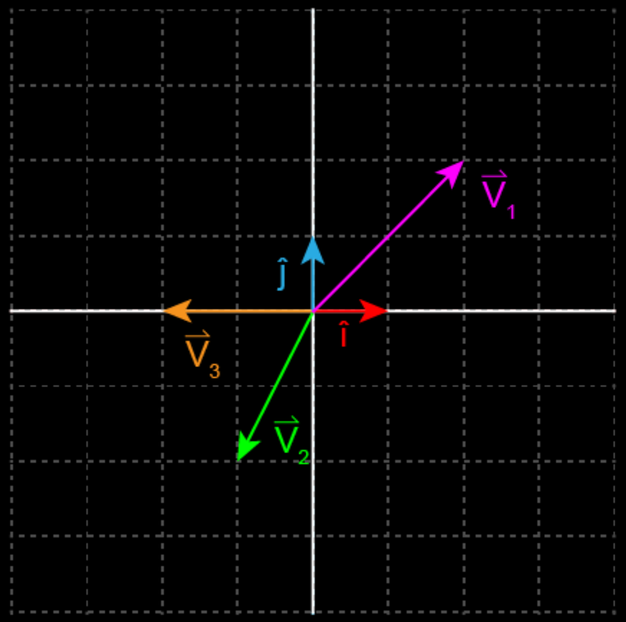
Are they different (orthogonal in space)?

Can we easily quantify this?

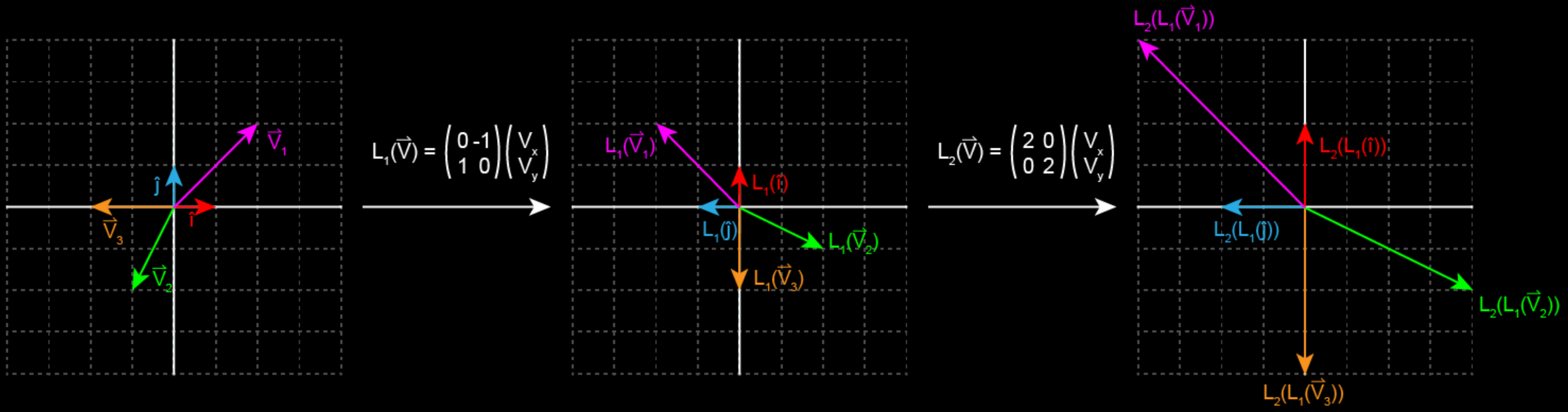
Rotate line until you find a line that has maximum alignement with data points



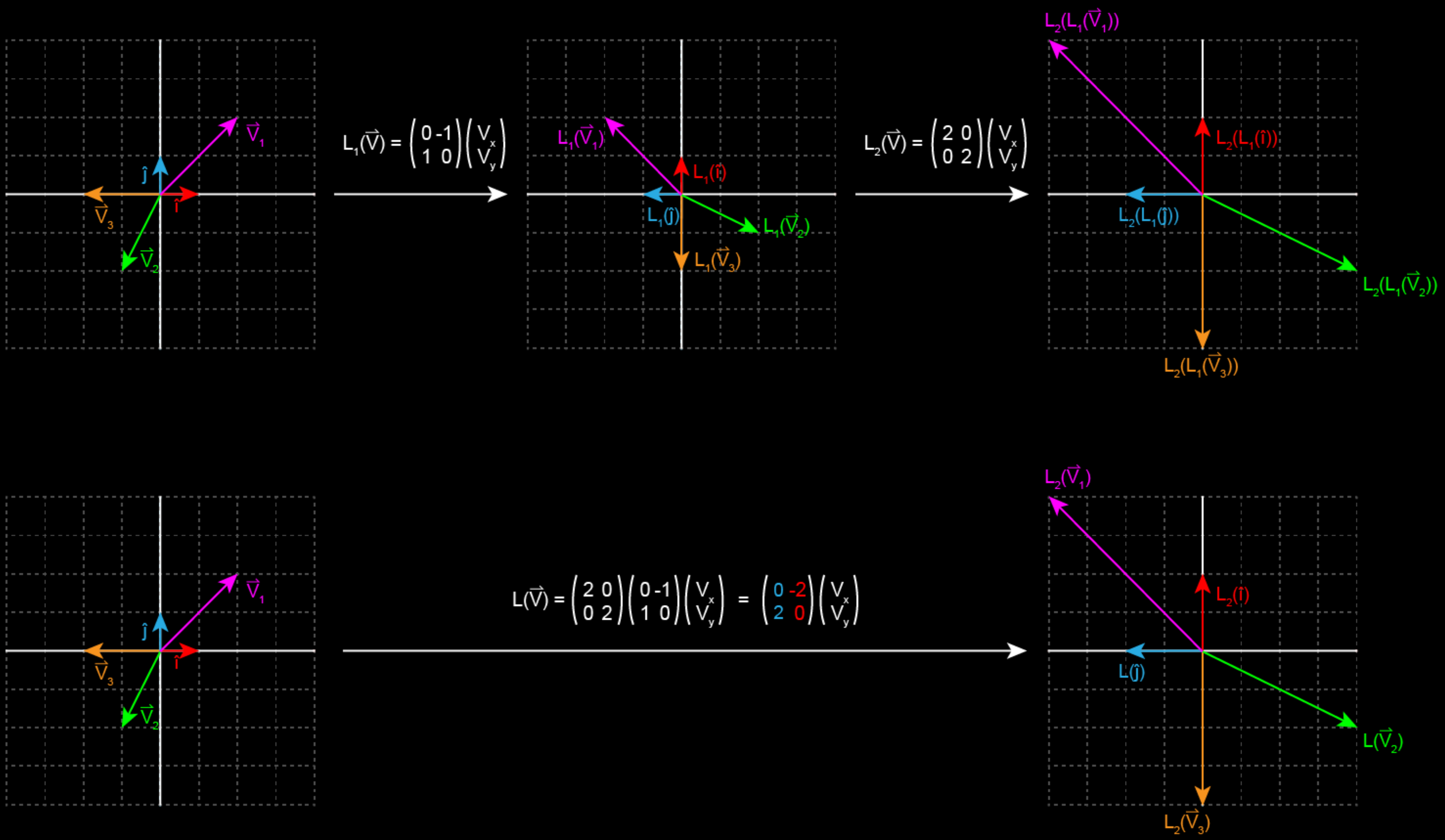
Matrix Multiplication



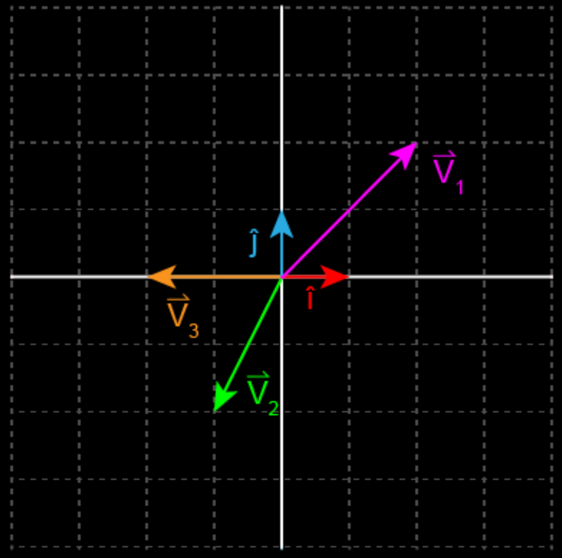
$$L(\vec{V}) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$



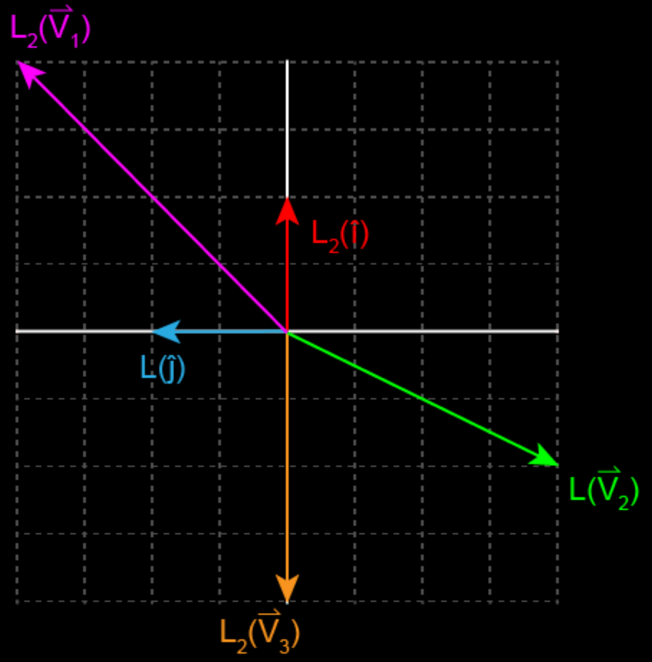
$\hat{j}$
$\hat{i}$
$\vec{V}_1$
$\vec{V}_2$
$\vec{V}_3$
$L_1(\vec{V}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} V_x \\ V_y \end{pmatrix}$
$L_1(\vec{V}_1)$
$L_1(\hat{i})$
$L_1(\hat{j})$
$L_1(\vec{V}_2)$
$L_1(\vec{V}_3)$
$L_2(\vec{V}) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} V_x \\ V_y \end{pmatrix}$
$L_2(L_1(\vec{V}_1))$
$L_2(L_1(\hat{i}))$
$L_2(L_1(\hat{j}))$
$L_2(L_1(\vec{V}_2))$
$L_2(L_1(\vec{V}_3))$



$\vec{V}_1$
$\hat{j}$
$\vec{V}_3$
$\hat{i}$
$\vec{V}_2$
$L_1(\vec{V}) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$
$L_1(\vec{V}_1)$
$L_1(\hat{i})$
$L_1(\hat{j})$
$L_1(\vec{V}_2)$
$L_1(\vec{V}_3)$
$L_2(\vec{V}) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$
$L_2(L_1(\vec{V}_1))$
$L_2(L_1(\hat{i}))$
$L_2(L_1(\hat{j}))$
$L_2(L_1(\vec{V}_2))$
$L_2(L_1(\vec{V}_3))$
$\vec{V}_1$
$\hat{j}$
$\vec{V}_3$
$\hat{i}$
$\vec{V}_2$
$L(\vec{V}) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$
$L_2(\vec{V}_1)$
$L_2(\hat{i})$
$L(\hat{j})$
$L(\vec{V}_2)$
$L_2(\vec{V}_3)$



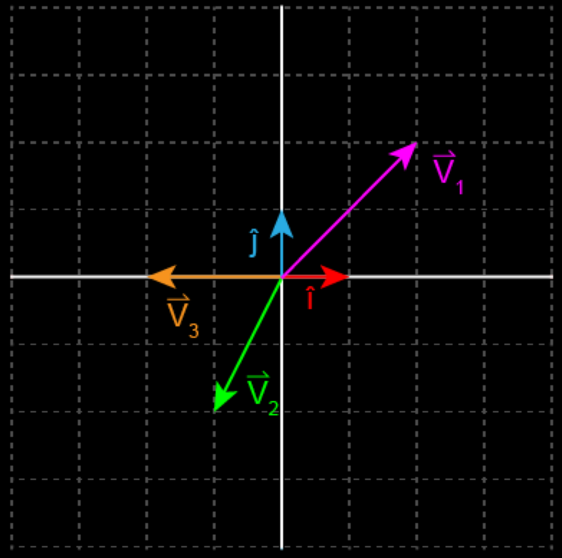
$$L(\vec{V}) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} V_x \\ V_y \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} V_x \\ V_y \end{pmatrix}$$



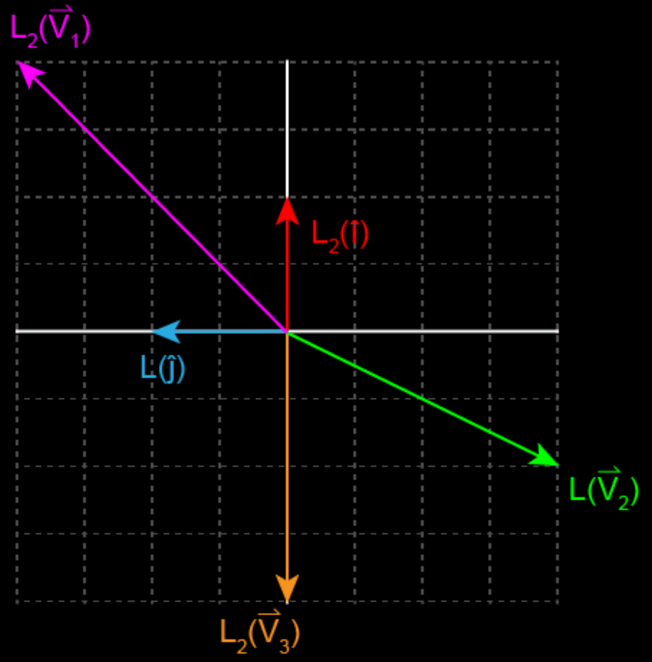
Where do $\hat{\imath}$ and $\hat{\jmath}$ end up after $L_1(\vec{V})$ and $L_2(\vec{V})$?

$$\begin{array}{cc} L_2(\vec{V}) & L_1(\vec{V}) \\ \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} & \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \end{array}$$

$\hat{\imath}$ after $L_1(\vec{V})$ $\quad$ $\hat{\jmath}$ after $L_1(\vec{V})$



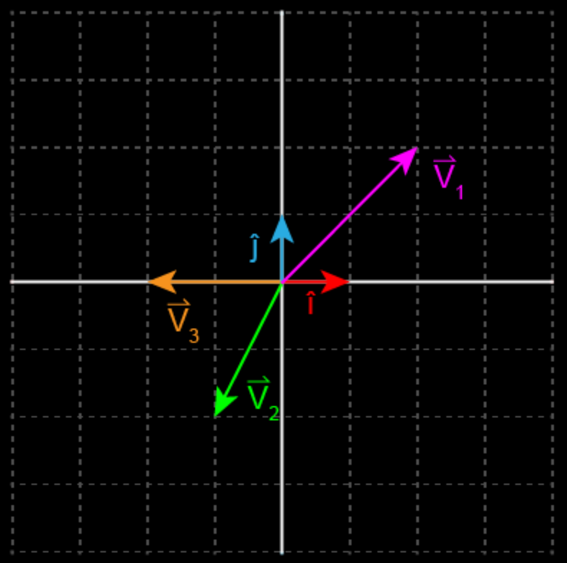
$$L(\vec{V}) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} V_x \\ V_y \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} V_x \\ V_y \end{pmatrix}$$



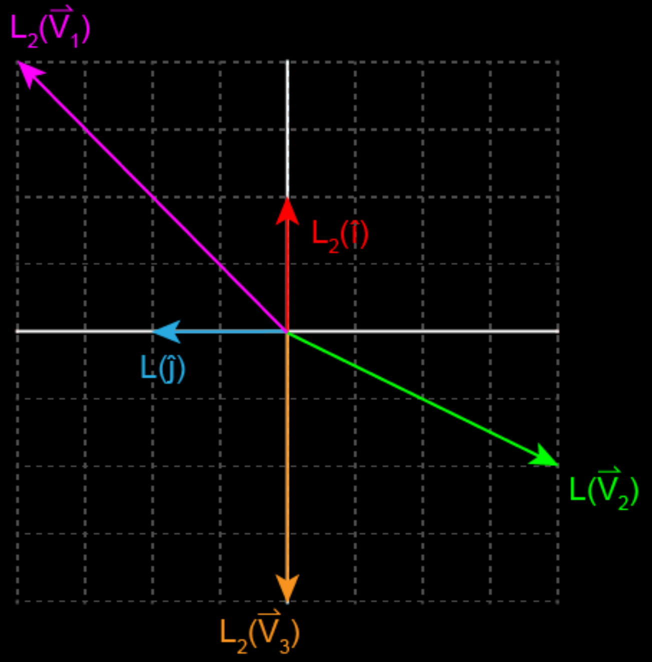
Where do $\hat{i}$ and $\hat{j}$ end up after $L_1(\vec{V})$ and $L_2(\vec{V})$?

$$\begin{array}{cc} L_2(\vec{V}) & L_1(\vec{V}) \\ \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} & \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \end{array} = \left(\underbrace{0\begin{pmatrix} 2 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 0 \\ 2 \end{pmatrix}}_{\text{Transformation of } L_1(\hat{i}) \text{ by } L_2(\vec{V})} , \underbrace{-1\begin{pmatrix} 2 \\ 0 \end{pmatrix} + 0\begin{pmatrix} 0 \\ 2 \end{pmatrix}}_{\text{Transformation of } L_1(\hat{i}) \text{ by } L_2(\vec{V})} \right)$$

$\hat{i}$ after $L_1(\vec{V})$ $\hat{j}$ after $L_1(\vec{V})$



$$L(\vec{V}) = \begin{pmatrix}2&0\\0&2\end{pmatrix}\begin{pmatrix}0&-1\\1&0\end{pmatrix}\begin{pmatrix}V_x\\V_y\end{pmatrix} = \begin{pmatrix}0&-2\\2&0\end{pmatrix}\begin{pmatrix}V_x\\V_y\end{pmatrix}$$



Where do $\hat{\imath}$ and $\hat{j}$ end up after $L_1(\vec{V})$ and $L_2(\vec{V})$?

$$\begin{array}{cc} L_2(\vec{V}) & L_1(\vec{V}) \\ \begin{pmatrix}2&0\\0&2\end{pmatrix} & \begin{pmatrix}0&-1\\1&0\end{pmatrix}\end{array} = \left(0\begin{pmatrix}2\\0\end{pmatrix}+1\begin{pmatrix}0\\2\end{pmatrix},\ -1\begin{pmatrix}2\\0\end{pmatrix}+0\begin{pmatrix}0\\2\end{pmatrix}\right) = \begin{pmatrix}0&-2\\2&0\end{pmatrix}$$

$\hat{\imath}$ after $L_1(\vec{V})$

$\hat{j}$ after $L_1(\vec{V})$

Transformation of $L_1(\hat{\imath})$ by $L_2(\vec{V})$

Transformation of $L_1(\hat{\imath})$ by $L_2(\vec{V})$

$\hat{\imath}$ after $L_1(\vec{V})$ and $L_2(\vec{V})$

$\hat{\imath}$ after $L_1(\vec{V})$ and $L_2(\vec{V})$

Exercises

Practice matrix multiplication on the following matrices

Exercises

$$L(\vec{V}) = \begin{pmatrix} 4 & 1 \\ -5 & 1 \end{pmatrix} \begin{pmatrix} -3 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

$$L(\vec{V}) = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 7 \end{pmatrix} \begin{pmatrix} 1 & -1 & 3 & 2 \\ 0 & -1 & 3 & -1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix}$$

$$L(\vec{V}) = \begin{pmatrix} 2 & 0 & -2 & 1 \\ 3 & 1 & -2 & 6 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & 3 \\ 3 & -1 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

$$L(\vec{V}) = \begin{pmatrix} 5 & 2 & 0 \\ 0 & 1 & 0 \\ -2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & -2 & 4 \\ 1 & 0 & -1 \\ 2 & -2 & -2 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

Questions?