Math Tools Tutorial

Sep 12, 2019

Some things to know

- Scalars
- Vectors
- Transposes
- Matrices
- Matrix multiplication
- Definition of linearity

Vectors

- Vector addition v1=[a,b]; v2=[c,d]; v1+v2=[a+b,c+d]
- Vector subtraction?

Exercise:

v1=[1;2] (Matlab notation for column vector)

v2=[3;2]

v3=[6;3]

By hand, please compute and draw:

a) v1+v2
b) v1-v3
c) v1+v2+v3
d) v1+v2-v3

Multiplication

Exercise:

Scalar * scalar?

Scalar * vector?

Scalar * matrix?

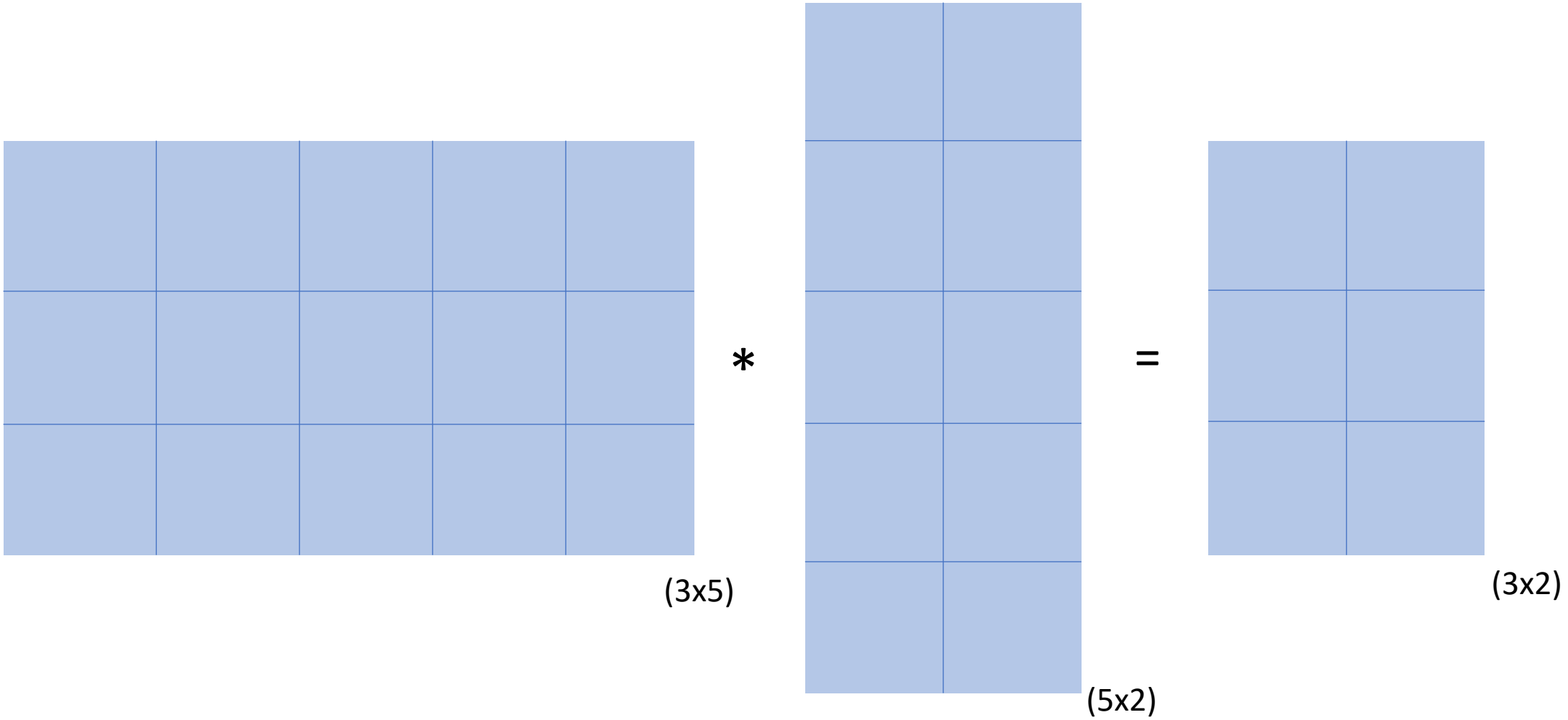
Trickier:

vector * vector?

matrix * vector?

matrix * matrix?

Matrix multiplication



$$(m_{rows} \times n_{rows}) * (n_{rows} \times p_{cols}) = (m_{rows} \times p_{cols})$$

Vector multiplication

Column vectors:

xx = [7;8]

yy = [8;9]

Exercise by hand:

- <x,y>
- dot(x,y)
- x^T*y
- $x*y^T$?

Dot products (white board)

What is a dot product, geometrically?

$\langle x,y \rangle = ||x||\ ||y||\ \cos(\Theta)$

||vec|| = vector length = vector magnitude = vector norm

Exercise in MATLAB:
vec1 = [3;4]
vec2 =[-3,4]

What are their lengths?
What is their dot product?
What is the angle between them?

Matrix vector multiplication

x = [2; 4] (column vec)
A = [1 5;
3 7]

Exercise by hand:

A*x?
x*A?
xT*A?
xT*AT?

Applying your knowledge so far…

- You have linear systems (you know for sure they're linear):

You test these systems to get input/output pairs. What are the matrices describing these systems?

System A:

[3, 0] → 7

System B:

[1, 4] → 13

[3, 5] → 21

System C is: [1 2;3 4]

[1,2] → ?

$[3, 4]^T$ → ?

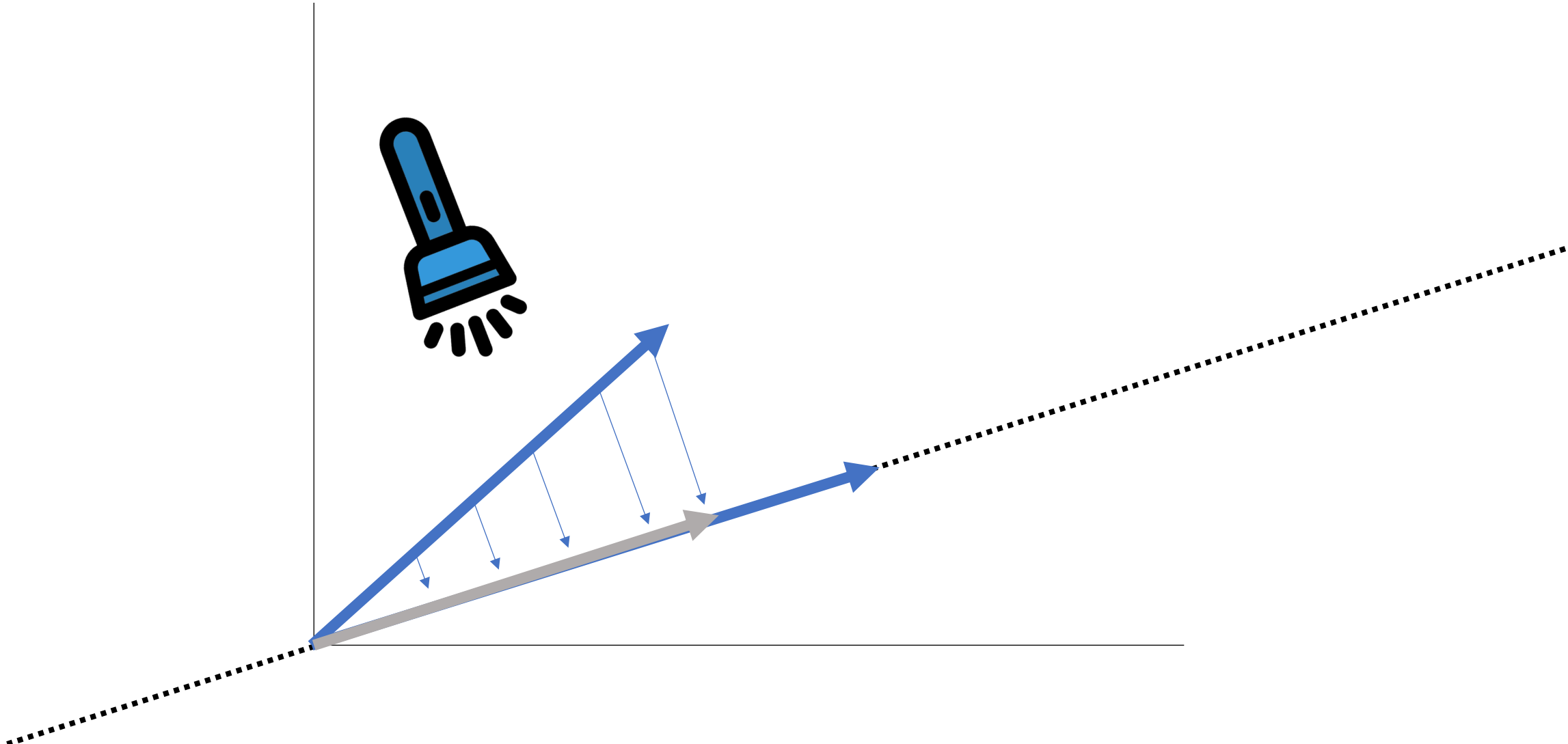
Matrix multiplication

$$A = \begin{matrix} 1 & 4 & 3 & 2 \\ 3 & 2 & 8 & 1 \end{matrix}$$

$$B = \begin{matrix} 1 & 8 & 1 & 0 \\ 6 & 9 & 2 & 3 \\ 4 & 2 & 0 & 1 \end{matrix}$$

What is their matrix product?

Vector projection (white board)



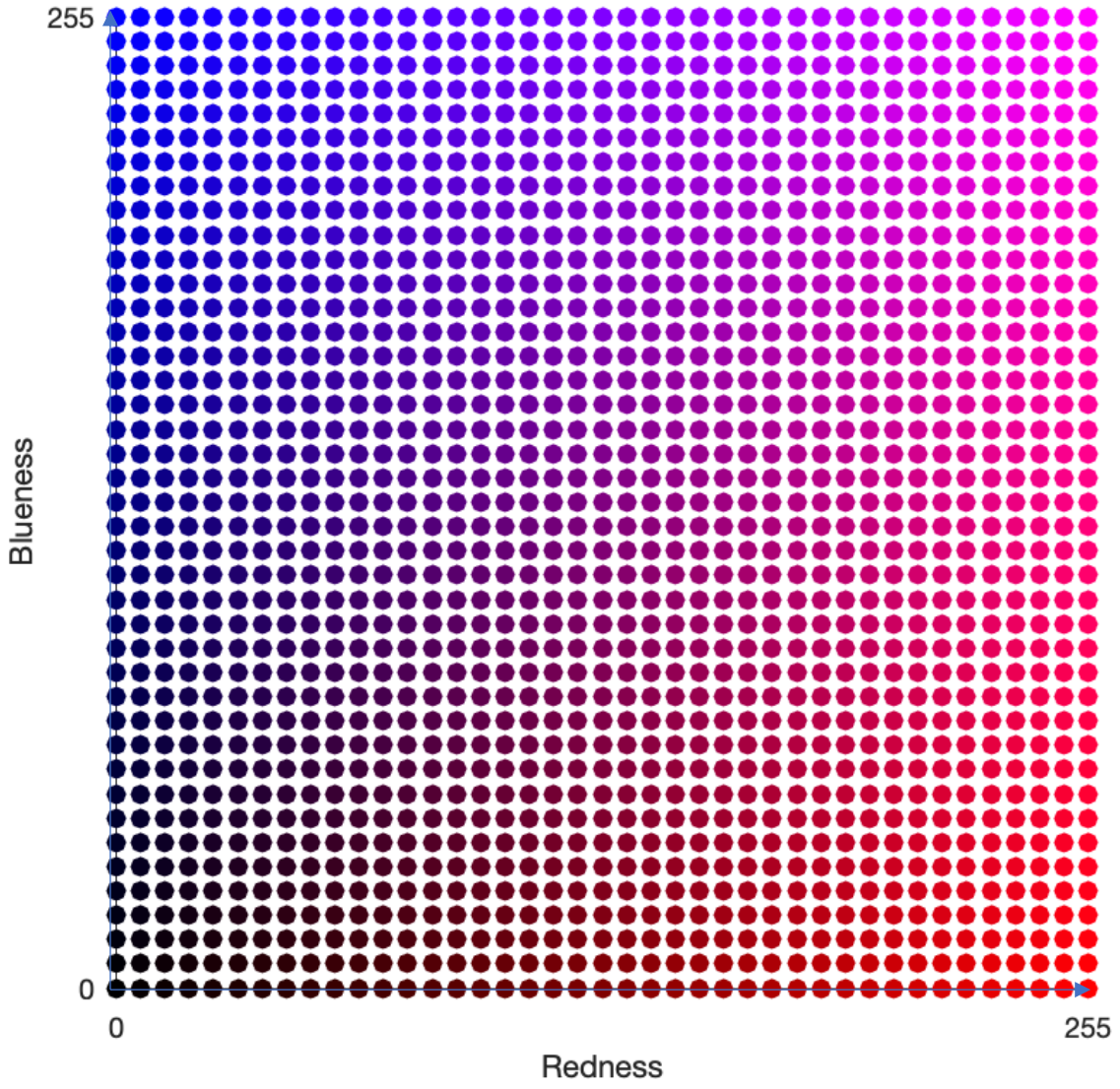
Please open MATLAB script

(R,G,B) = (79, 0, 135)

(R,B) = (79, 135)

You are tasked with creating a new colour for NYU
It can be anything you want!
The only catch: it has to be **more** purple than the current purple

Define purple as: equal parts red and blue



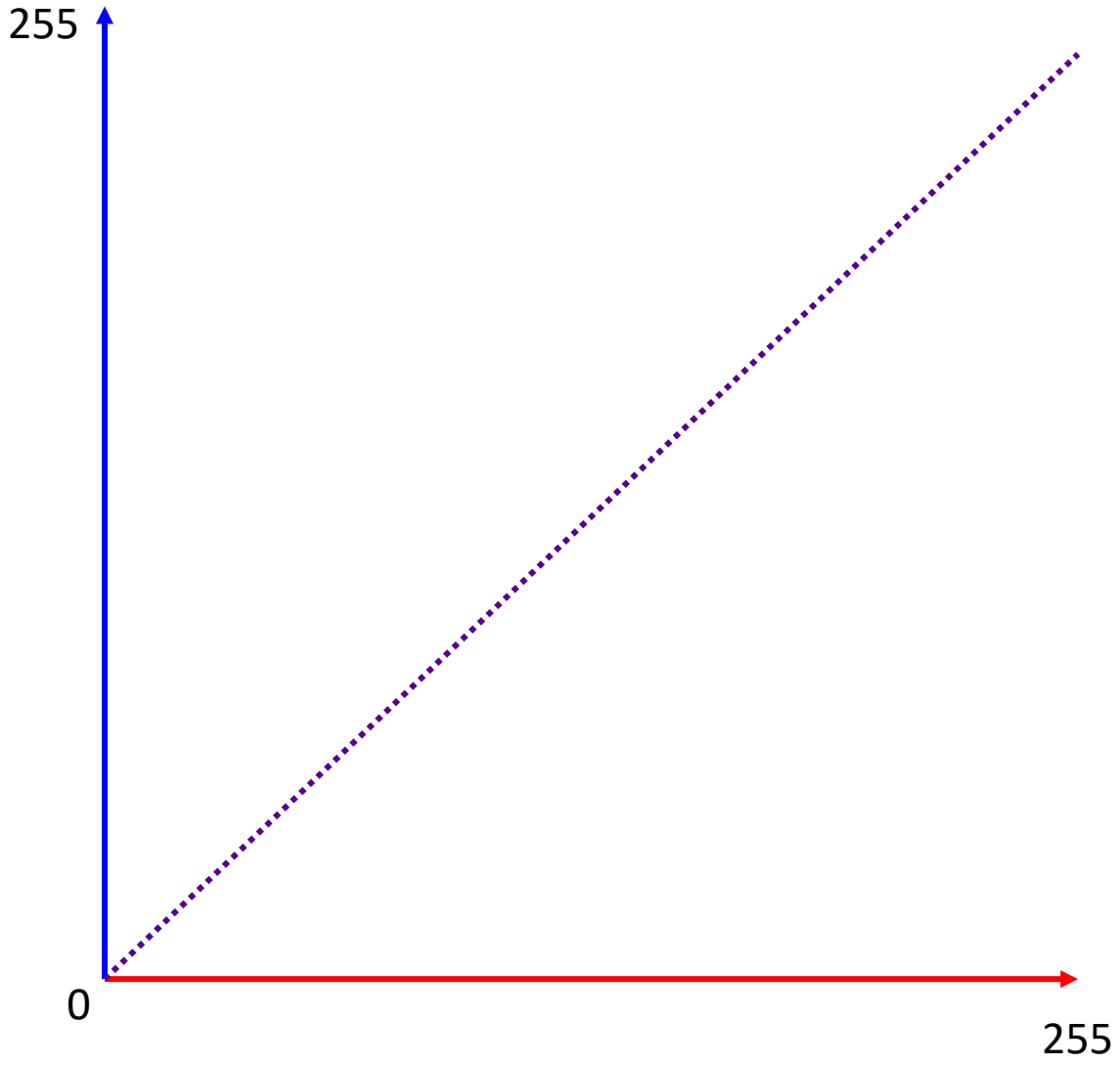
(R,G,B) = (79, 0, 135)

(R,B) = (79, 135)

Proposed colour: [97, 111]

How purple is it?

Is it more/less purple than the current NYU purple?

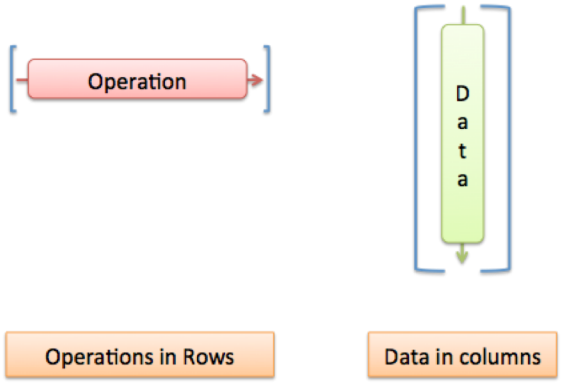


Now you need to test 10 new colours!

Write a for loop to project each of these colours onto the purple axis.
Are any of them **more** purple than NYU's current purple of [79, 135]?

Why are so many of them the same amount of purple, yet have very different (R,B) values?

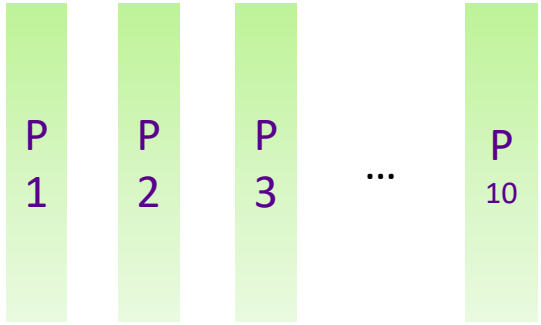
How do we do this without loops?



Av

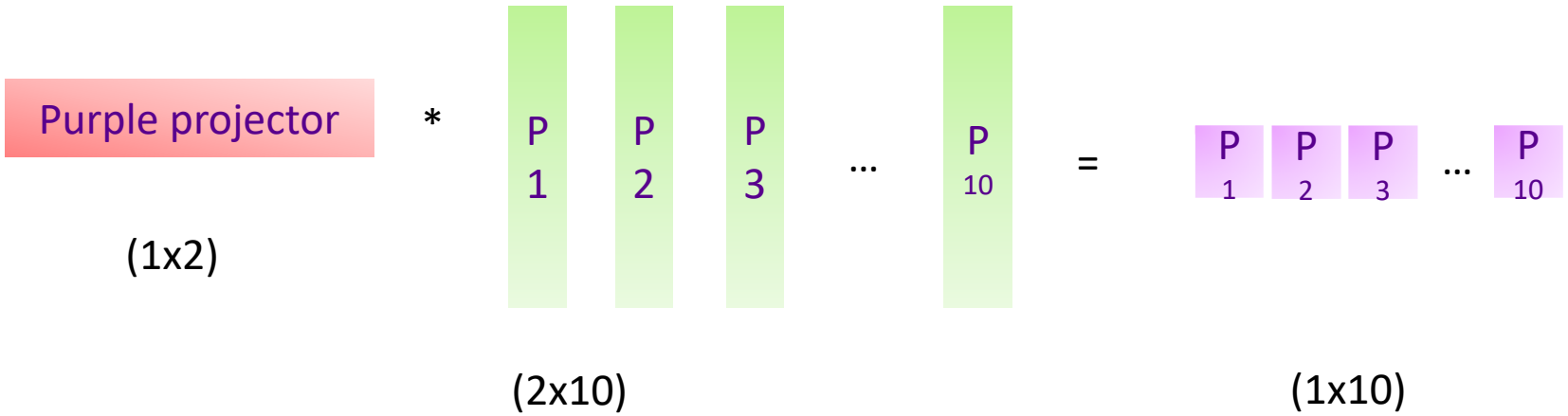
Purple projector

(1x2)

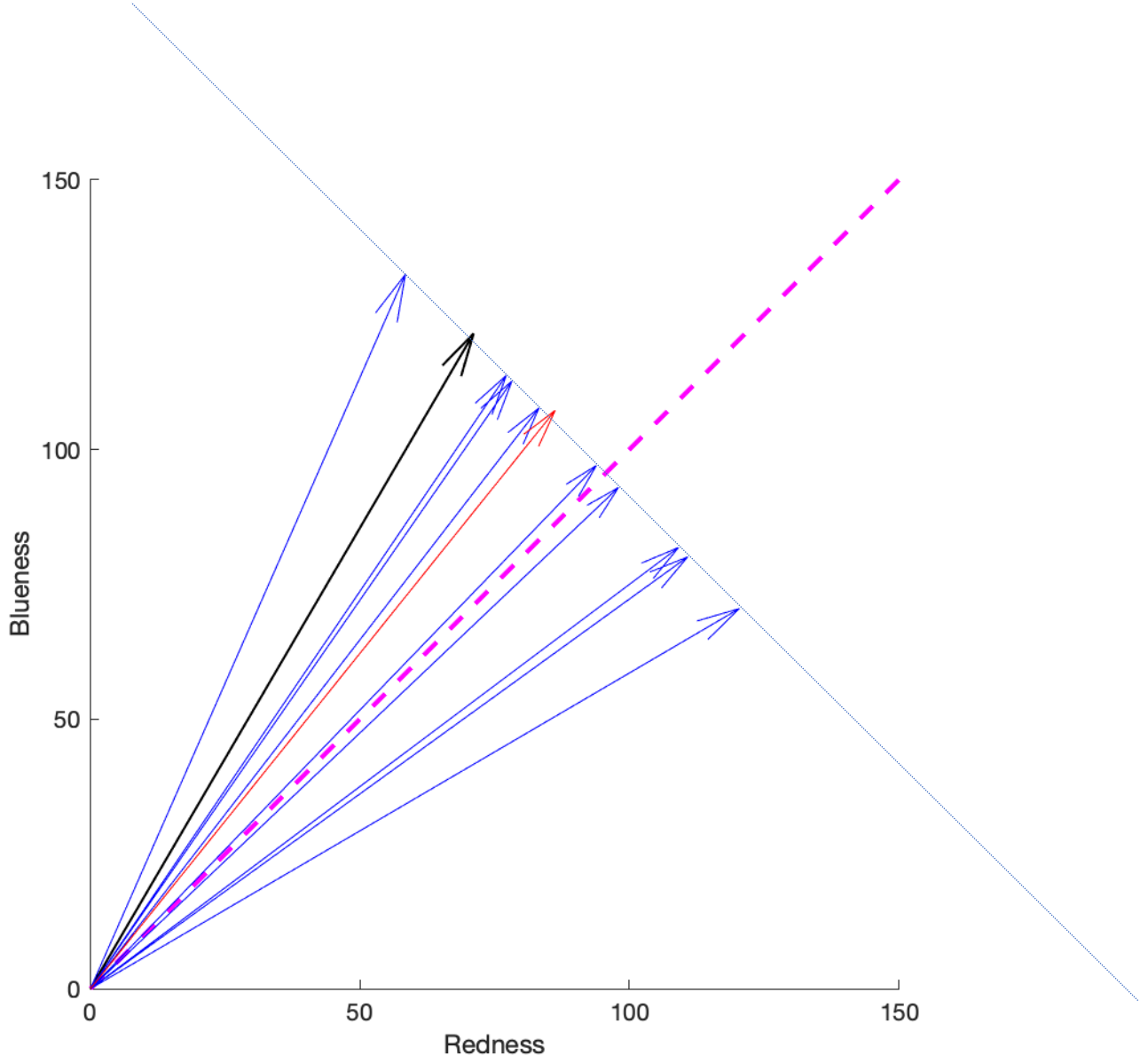


(2x10)

“Vectorization”

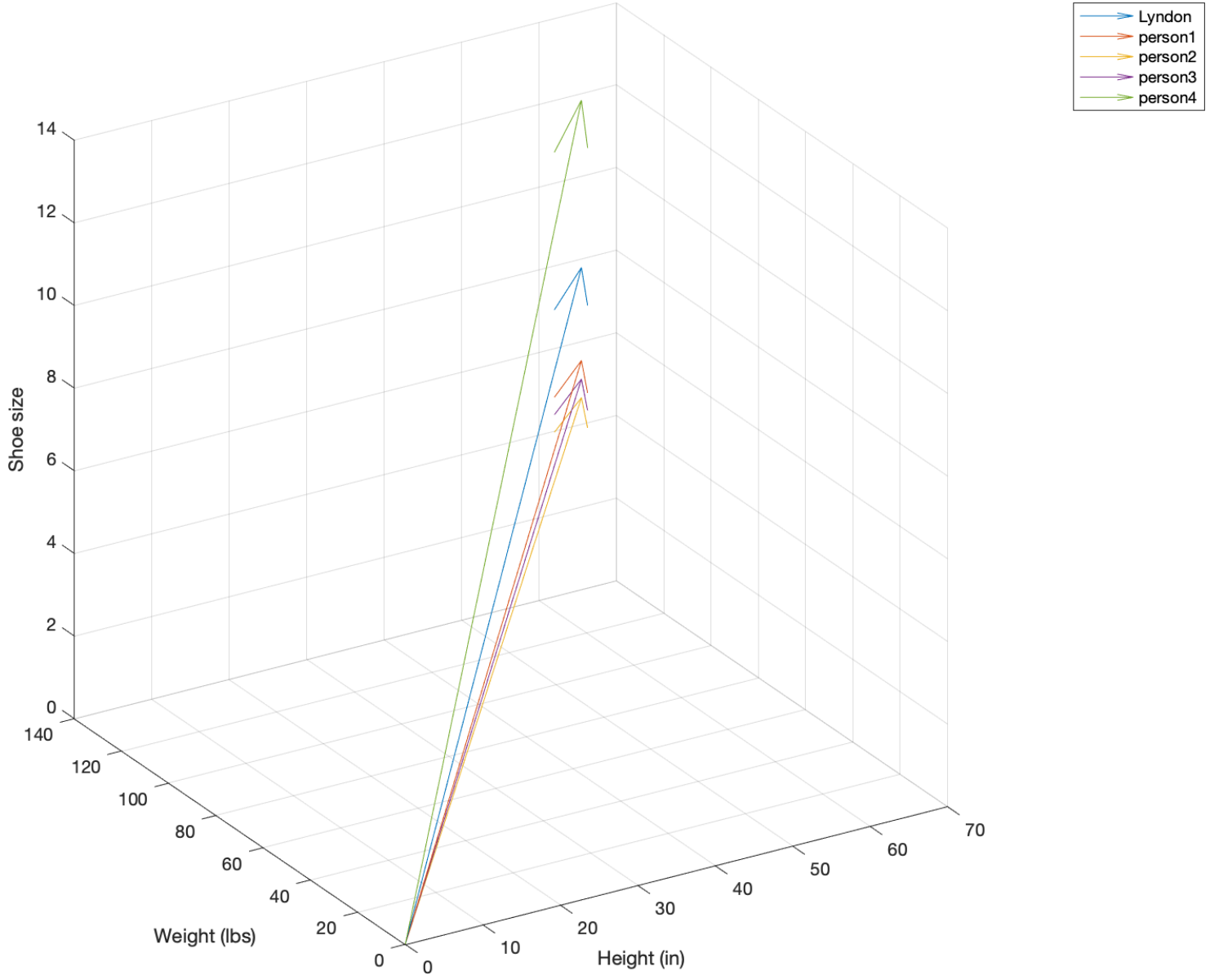


Why are most of them the same amount of purple?



Project 3D → 2D

- MATLAB script



Linear systems theory

- How do you determine if a system is linear?

[1, 2] → [8, 5]

[-1, 3] → [2, 0]

[1, 7] → [18, 10]

Linearity: superposition → additivity, homogeneity

Special matrices

- Identity matrix
- Orthogonal/orthonormal
- Diagonal matrix
- Matrix inverses/pseudoinverses

Matrix factorization/decomposition

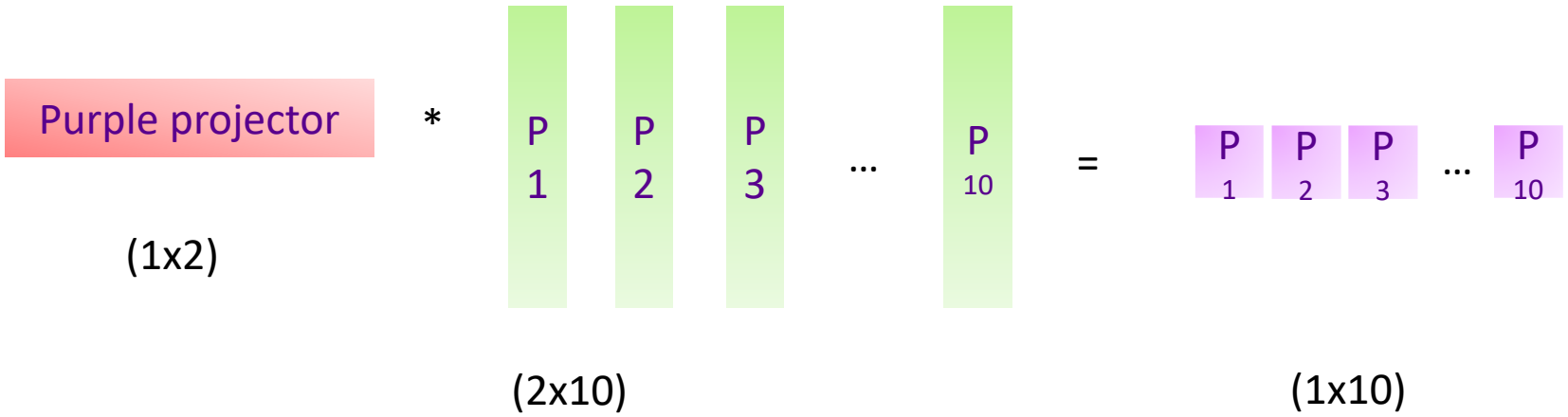
- Any sequence of linear transformations (matrices) can be concisely written as ONE matrix

$$(A*B*C)v = Dv$$

OR

$$Dv = (A*B*C)v$$

Back to purple space



u_{purple}=[.7071; .7071]
$u_{ortho\text{-}purple}$=[-7071; .7071]