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Mathematical Tools
for Neural and Cognitive Science

Fall semester, 2018

Section 3a: Early auditory system (an extended LSI example)

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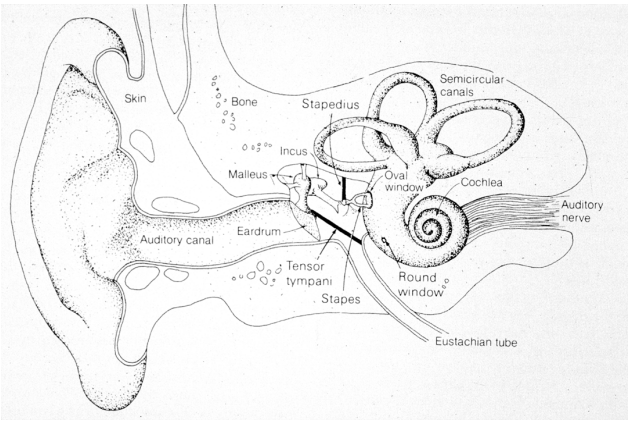

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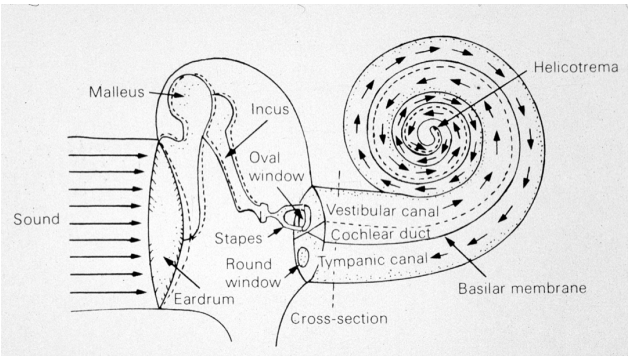

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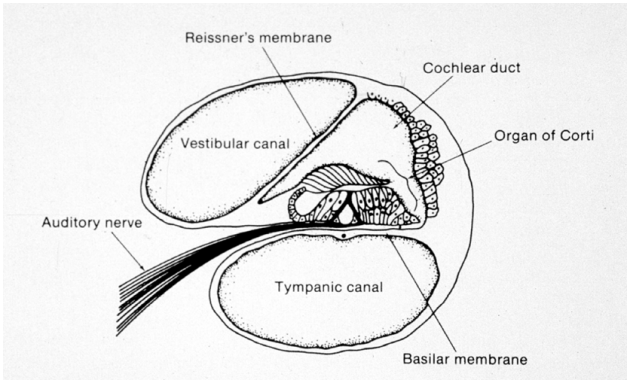
Reissner's membrane
Cochlear duct
Vestibular canal
Organ of Corti
Auditory nerve
Tympanic canal
Basilar membrane

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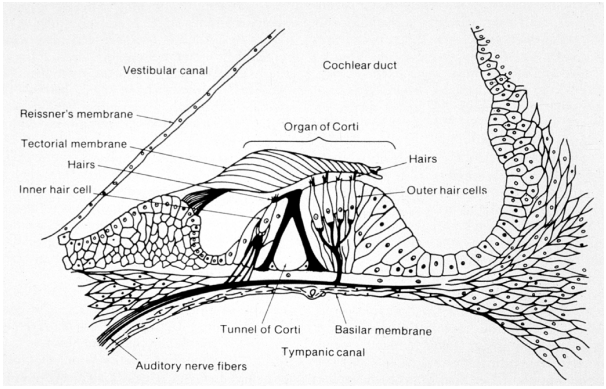
Vestibular canal
Cochlear duct
Reissner's membrane
Organ of Corti
Tectorial membrane
Hairs
Hairs
Inner hair cell
Outer hair cells
Tunnel of Corti
Basilar membrane
Tympanic canal
Auditory nerve fibers

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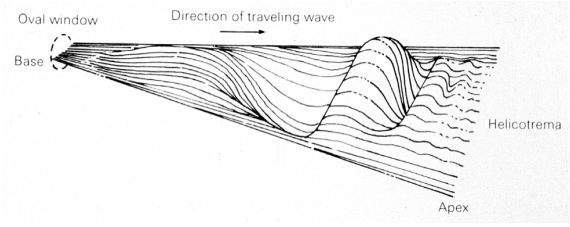
Oval window
Direction of traveling wave
Base
Helicotrema
Apex

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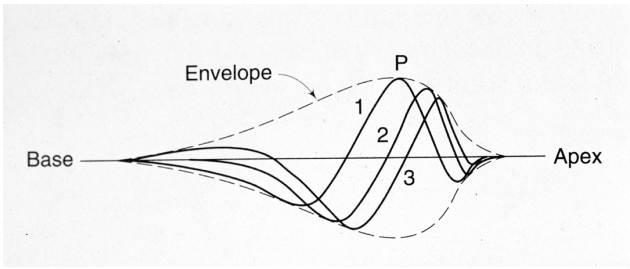
Envelope
P
1
2
3
Base
Apex

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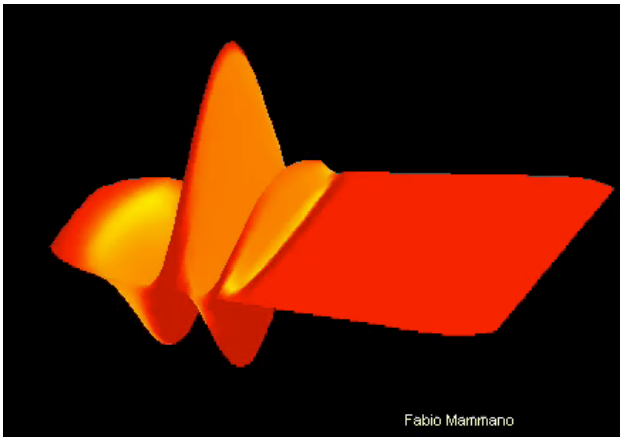
Fabio Mammano

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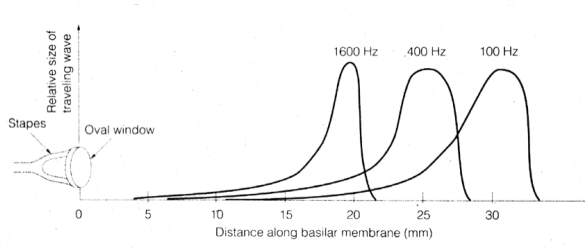
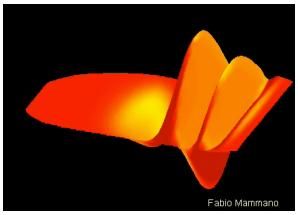
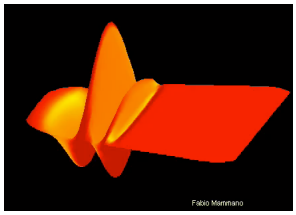

Relative size of traveling wave
Stapes
Oval window
1600 Hz
400 Hz
100 Hz
0
5
10
15
20
25
30
Distance along basilar membrane (mm)

10



400 Hz



4000 Hz

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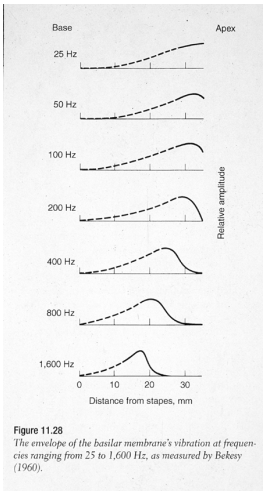

Figure 11.28
The envelope of the basilar membrane's vibration at frequencies ranging from 25 to 1,600 Hz, as measured by Bekesy (1960).

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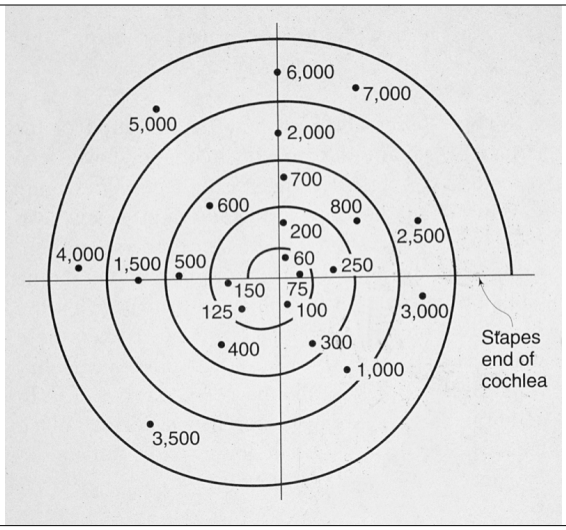

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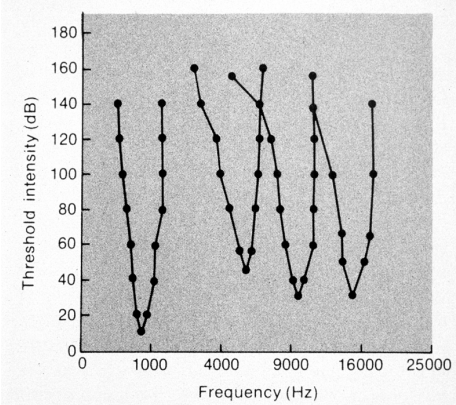

Figure 6–14 *Threshold response curves for auditory nerve fibers in the cat (based on Whitfield, 1968).*

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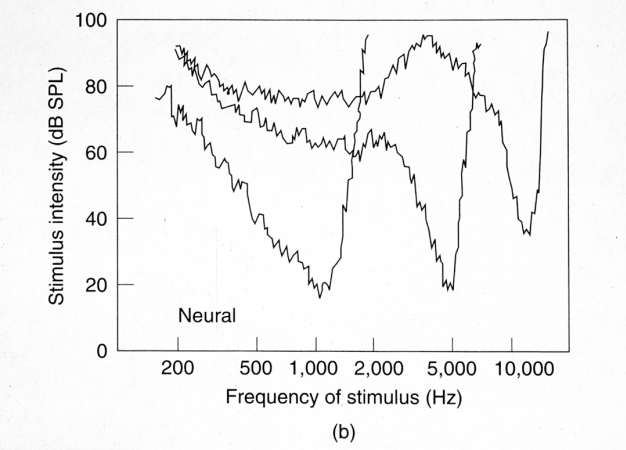

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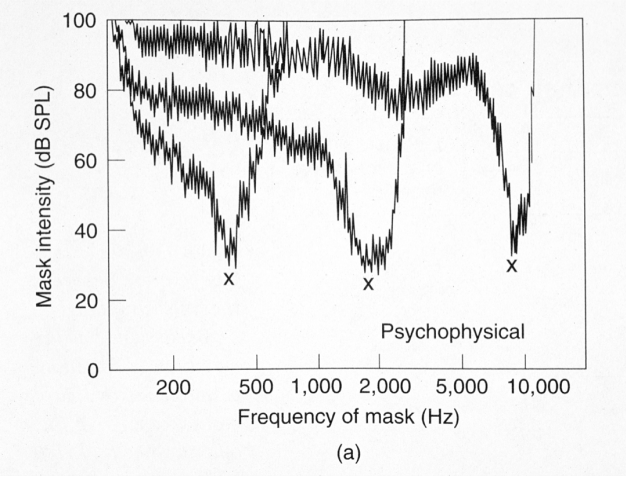

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Critical Bands

- Loudness summation
- Critical band masking

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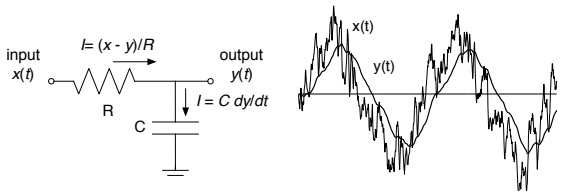
"Designing" Filters



$$y(t) = V_C = \frac{1}{C}\int I\,dt \quad \text{or} \quad I = C\frac{dV_C}{dt}$$

$$x(t) - y(t) = V_R = IR$$

$$x - y = RC\frac{dy}{dt} = \tau\frac{dy}{dt}$$

For $x = 0$, solutions have the form $y = Ae^{-t/\tau}$

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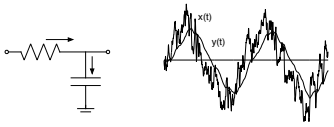
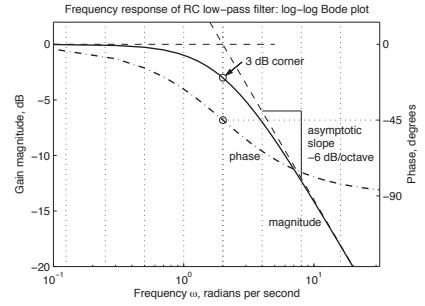
"Designing" Filters





$$H(\omega) = \frac{1}{i\tau\omega + 1}$$

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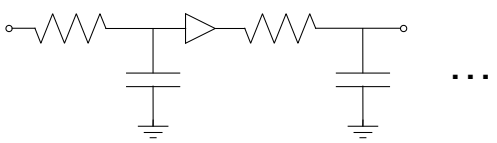
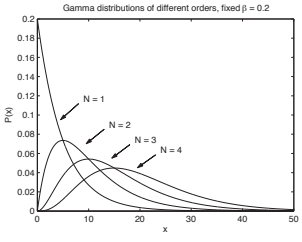
Cascaded or Gamma Filters



...

$$y \propto t^{n-1} e^{-t/\tau}$$

$$H(\omega) = \left(\frac{1}{i\omega\tau + 1} \right)^n$$



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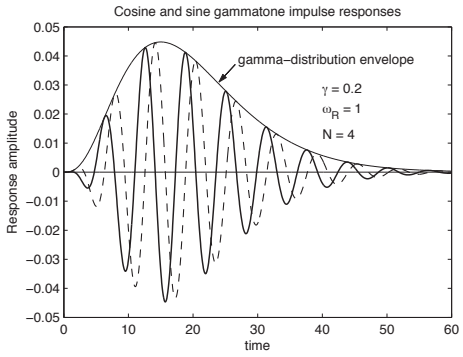
Gammatone Filters

$$y \propto t^{n-1} e^{-t/\tau} \cos(\omega t - \phi)$$



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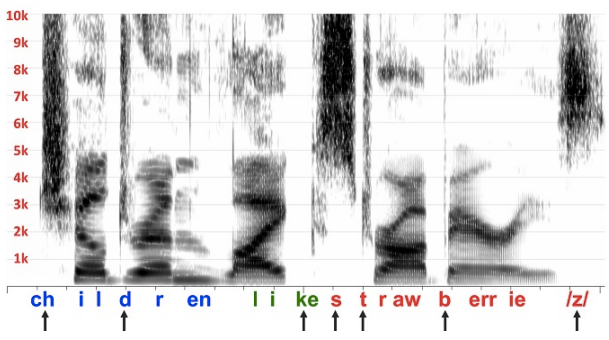
Speech Spectrogram



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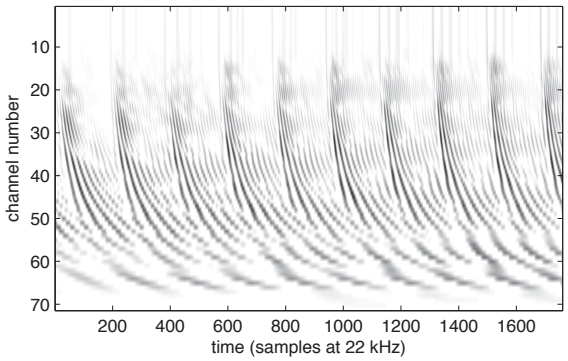
Cochleogram



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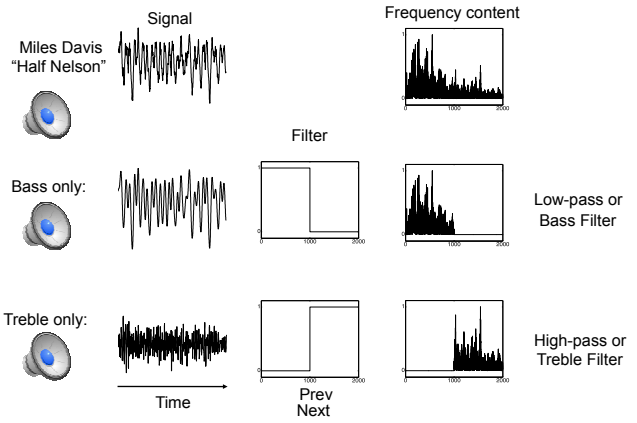
Example – Bass/Treble filters



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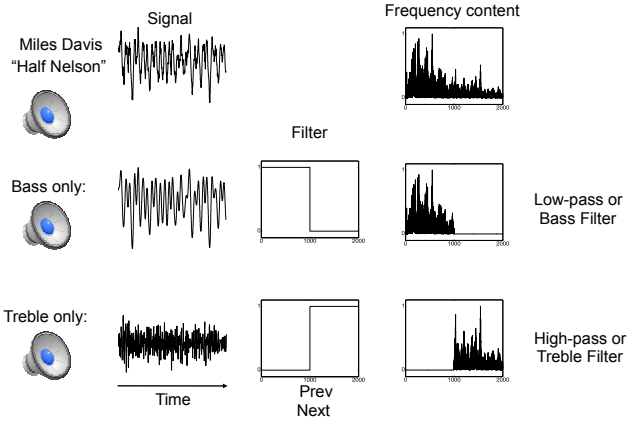
Example – Bass/Treble filters



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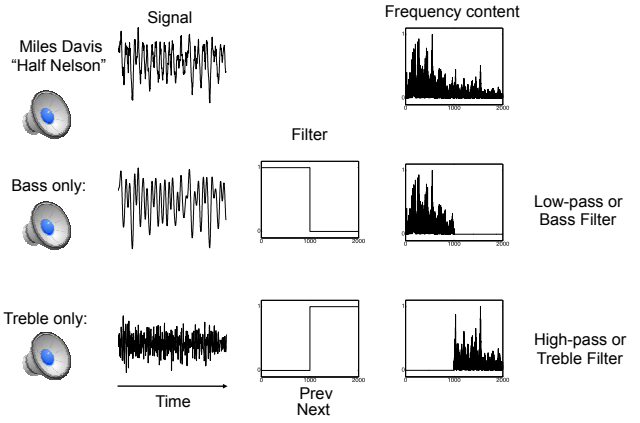
Example – Bass/Treble filters



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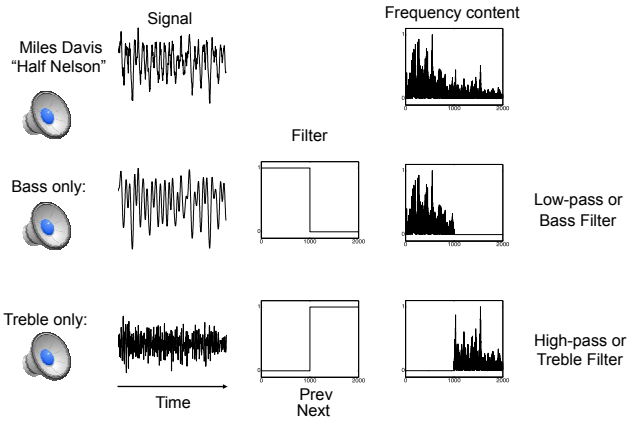
Example – Bass/Treble filters



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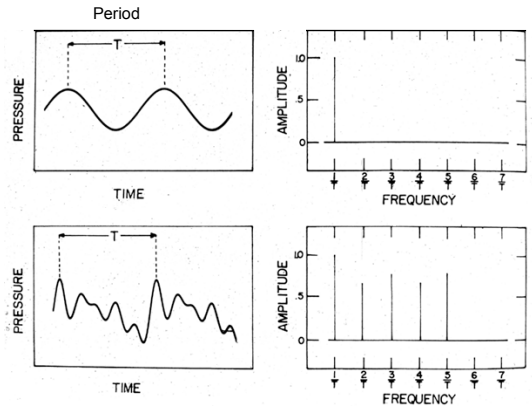
Fourier spectrum representation of sound



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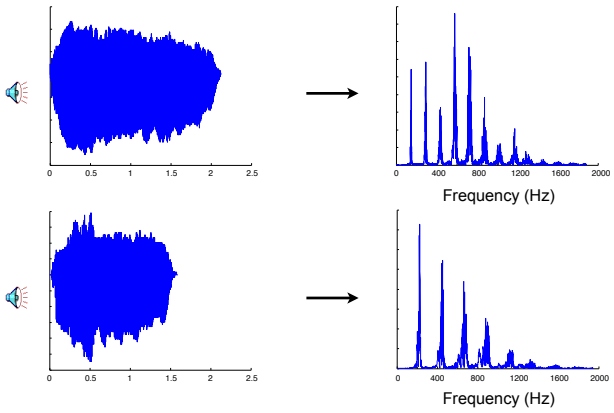
This works for all “musical sounds”



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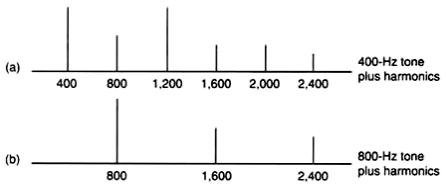
Fundamental frequency and harmonics



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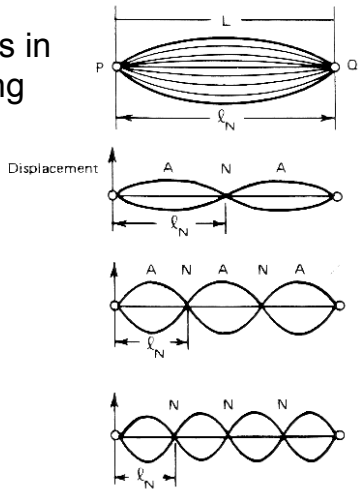
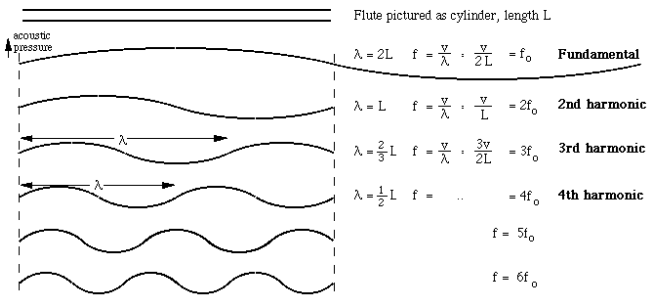
Standing waves in a vibrating string



Flute (open pipe) harmonics



Flute pictured as cylinder, length L

$\lambda = 2L \quad f = \frac{v}{\lambda} = \frac{v}{2L} = f_o$ **Fundamental**

$\lambda = L \quad f = \frac{v}{\lambda} = \frac{v}{L} = 2f_o$ **2nd harmonic**

$\lambda = \frac{2}{3}L \quad f = \frac{v}{\lambda} = \frac{3v}{2L} = 3f_o$ **3rd harmonic**

$\lambda = \frac{1}{2}L \quad f = \quad = 4f_o$ **4th harmonic**

$f = 5f_o$

$f = 6f_o$

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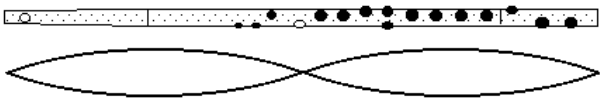
Flute (open pipe) harmonics

Other notes (shorten the pipe)

Reinforcing a harmonic (forcing a “node”)



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