

Mathematical Tools for Neural and Cognitive Science

Fall semester, 2026

Section 1a: Trichromacy (an extended linear algebra example)

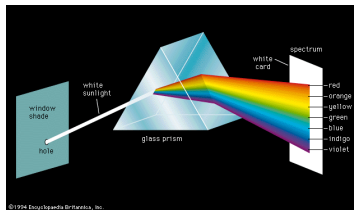
Trichromacy

A spectacular multi-disciplinary story ...

- Physics: spectral nature and additivity of light [Newton, and others, 1600's]
- Perception: Trichromatic matching experiments [Grassman, Young, and others, 1850's]
- Mathematics: Human color matching is explained by a 3-dimensional linear model [late 1800's]
- Engineering: Devices for color reproduction require only three color channels [early 1930's]
- Neurobiology: Trichromacy in humans (and some other primates) arises from 3 cone types [late 1980's]

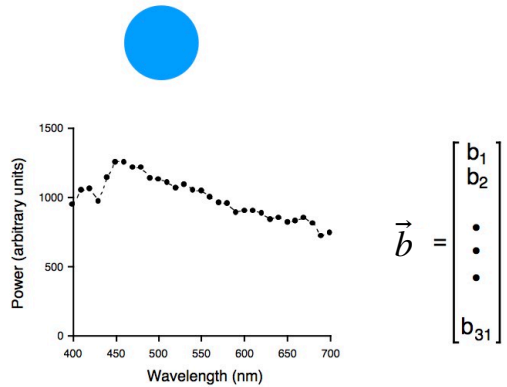
[See Wandell, *Foundations of Vision*, ch2]

Spectral nature of light



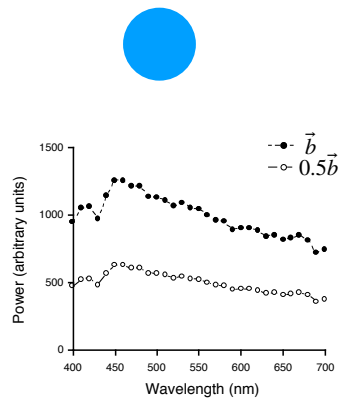
[Newton, 1665]

Representing spectra with vectors



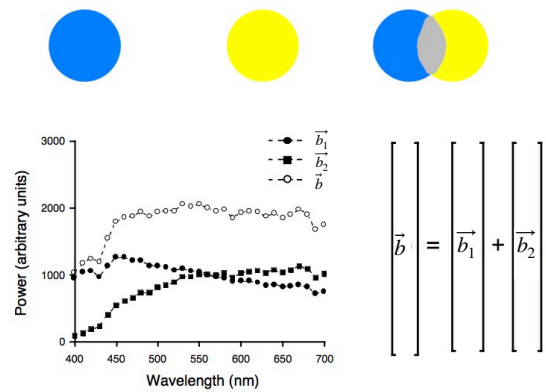
[c/o David Brainard]

Intensity adjustment (scalar multiplication)



[c/o David Brainard]

Additive mixtures



[c/o David Brainard]

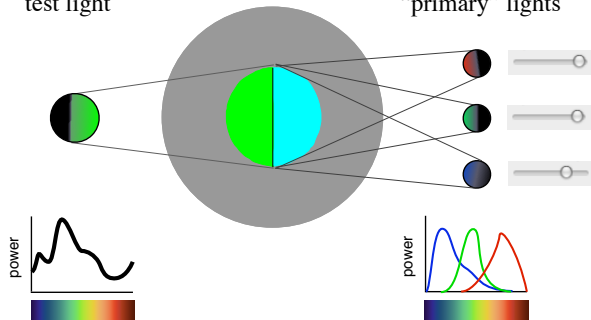
Color perception: what's it for?



[c/o David Brainard]

Perceptual color matching experiment

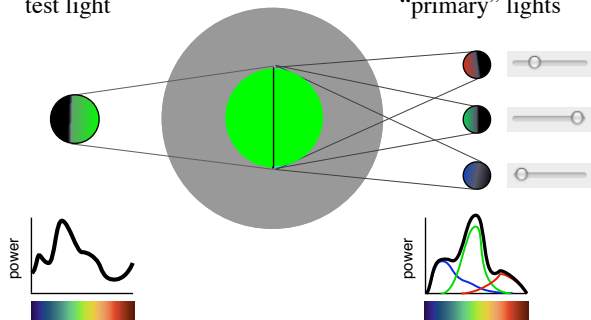
Arbitrary
test light



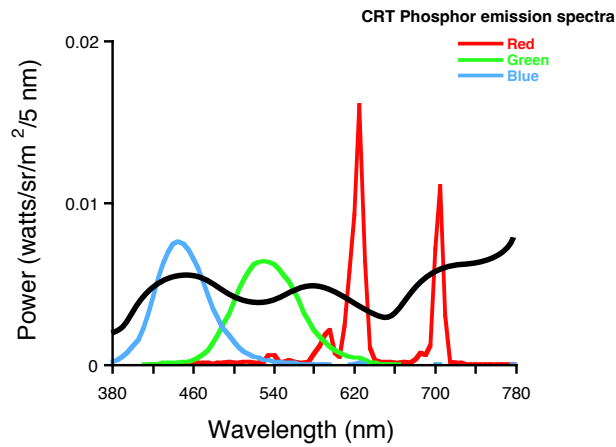
[Young, Helmholtz, Grassman, etc, 1800's]

Perceptual color matching experiment

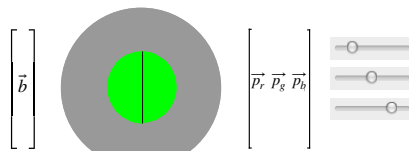
Arbitrary
test light



[Young, Helmholtz, Grassman, etc, 1800's]



Experimental
measurement:



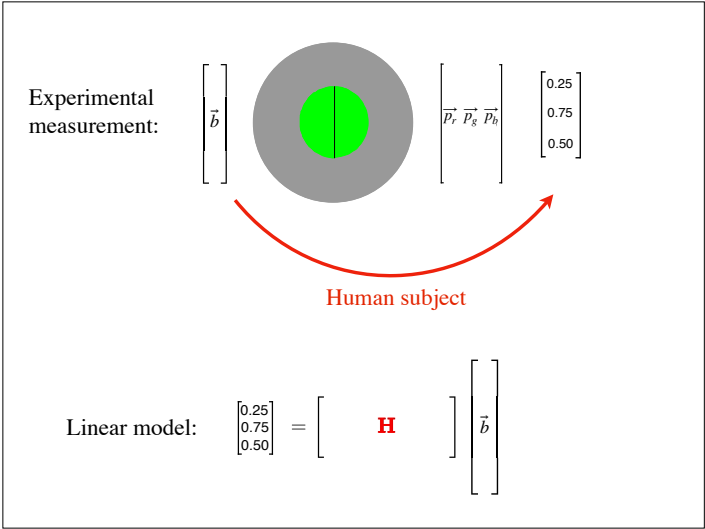
Human subject

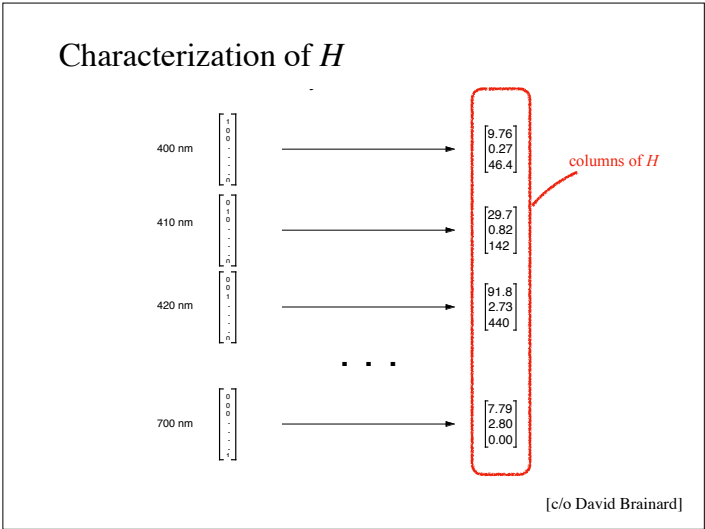
Grassmann's Laws (1853)

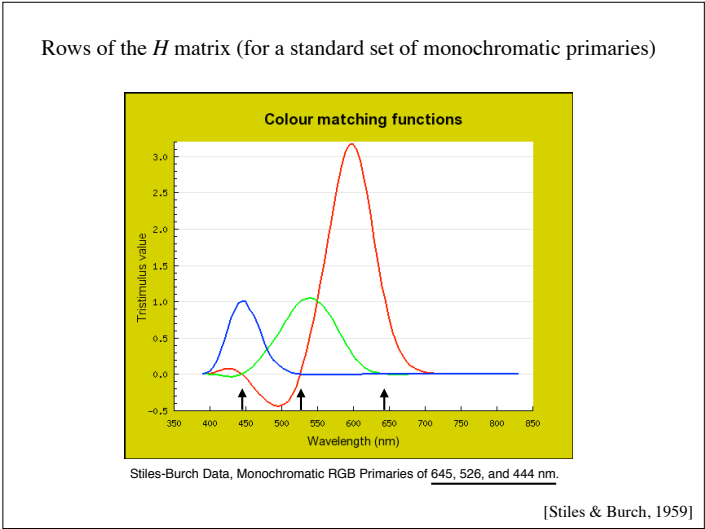


- 1) Any test light can be perceptually matched with a mixture of 3 primary lights
 - 2) Rescaling the test light results in a rescaled mixture
 - 3) Adding two lights results in a sum of their mixtures
- => Color matching can be described by an $N \times 3$ linear system*

* Fine print: i) Normal human observer; ii) photopic intensity levels, not too bright, not too dim; iii) independent primaries; iv) "negative" weights => add to test







Implications

- If P is an $N \times 3$ matrix containing the primary spectra, then the color matching experiment is captured by a $3 \times N$ “matching matrix” H that maps a light $\vec{l}$ to 3 “knob settings”:

$$\vec{l} \sim PH\vec{l}$$

where $\sim$ means “looks the same as”

- Two lights look the same if (and *only* if) they produce the same match settings:

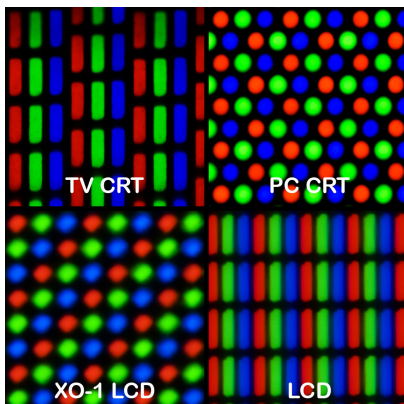
$$\vec{l}_1 \sim \vec{l}_2 \quad \Leftrightarrow \quad H\vec{l}_1 = H\vec{l}_2$$

Useful calculations

- Scientific: given results of one matching experiment, predict the results of another one (with different primaries).
- Practical: calibrate a display device, so as to generate mixtures of three colors that match the appearance of any desired real-world spectrum.

[derive on board]

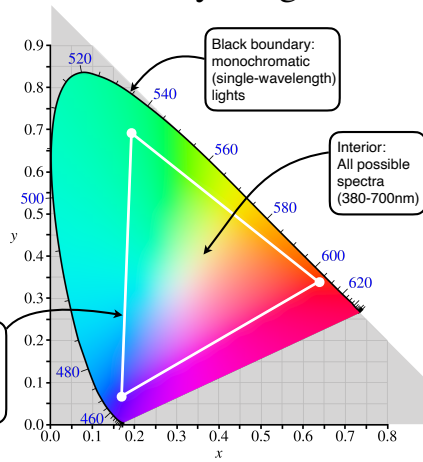
Trichromatic display technologies



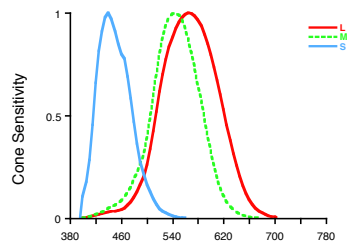
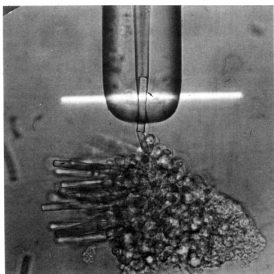
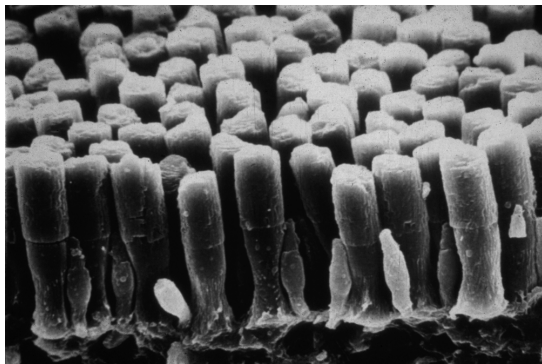
CIE “Chromaticity Diagram”

For tristimulus values {X,Y,Z}

Define
 $x = X/(X+Y+Z)$
 $y = Y/(X+Y+Z)$
 $z = Z/(X+Y+Z)$
 $= 1 - x - y$

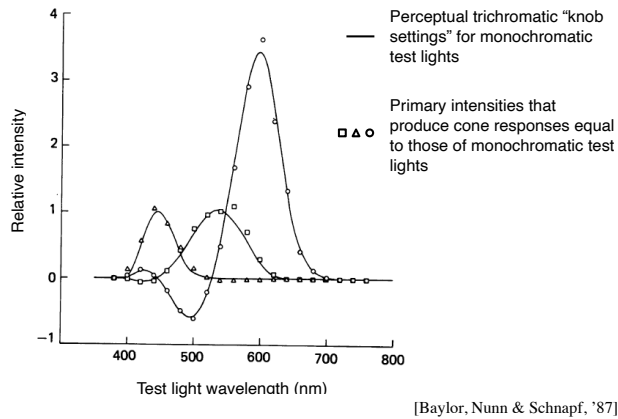


The underlying mechanism...



[Baylor, Nunn & Schnapf, 1987]

Confirmation of prediction (100+ years later!)

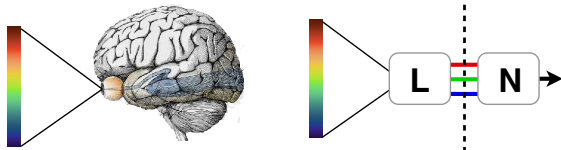


Some dimensionality caveats

- Normal human vision, at high (“photopic”) light levels is tri-chromatic
- At low (“scotopic”) light levels, we are all monochromats
- At intermediate (“mesopic”) light levels, we are quadrachromats!
- Common genetic forms of color blindness are due to lack of one or two cone types

Summary:

- Perception: any light with can be matched with a mixture of 3 primaries
- Theory/model: the visual system projects the wavelength spectra of light onto a 3-dimensional subspace



- Prediction/generalization: perceptual “metamers” - lights that appear identical, but have physically distinct wavelength spectra
- Engineering: CIE standards for color representation (1931)
- Physiological mechanism: cone photoreceptors (1987)