Mathematical Tools for Neural and Cognitive Science

Fall semester, 2024

Section 3: Linear Shift-Invariant Systems

## Linear shift-invariant (LSI) systems

- Linearity (previously discussed): "linear combination in, linear combination out"
- Shift-invariance (new property): "shifted vector in, shifted vector out"
- These two properties are independent (think of some examples that have both, one, or neither)

















- Structured matrix
- Boundaries? zero-padding, reflection, circular
- Examples: impulse, delay, average, difference





































#### The Discrete Fourier transform (DFT)

- Construct an orthogonal matrix of sin/cos pairs, covering different numbers of cycles
- Frequency multiples of 2π/N radians/sample, (specifically, 2πk/N, for k = 0, 1, 2, ... N/2)
- For k = 0 and k = N/2, only need the cosine part (thus, N/2 + 1 cosines, and N/2 1 sines)
- When we apply this matrix to an input vector, think of output as *paired* coordinates
- Common to plot these pairs as amplitude/phase

[details on board...]



























# Recap...

- Linear system
  - defined by superposition
  - characterized by a matrix
- Linear Shift-Invariant (LSI) system
  - defined by superposition and shift-invariance
  - characterized by a vector, which can be either:
    - » the impulse response
    - » the frequency response (amplitude and phase). Specifically, the Fourier Transform of the impulse response specifies an amplitude multiplier and a phase shift for each frequency.





### Visualizing the (Discrete) Fourier Transform

- Two conventional choices for frequency axis:
  - Plot frequencies from k = 0 to k = N/2(in matlab: 1 to N/2+1)
  - Plot frequencies from k = -N/2 to k = N/2 1 (in matlab: recenter using ffshift)
- Typically, we plot amplitude (and optionally, phase), instead of the real/imaginary (cosine/sine) components

### Some examples

- constant
- sinusoid (see next slide)
- impulse
- Gaussian "lowpass"
- Derivative of Gaussian "bandpass"
- DoG (difference of 2 Gaussians) "bandpass"
- Gabor (Gaussian windowed sinusoid) "bandpass"

[on board]



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• Represent/analyze periodic signals

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• Analyze/design LSI *systems*. In particular, how do you identify the nullspace?



















