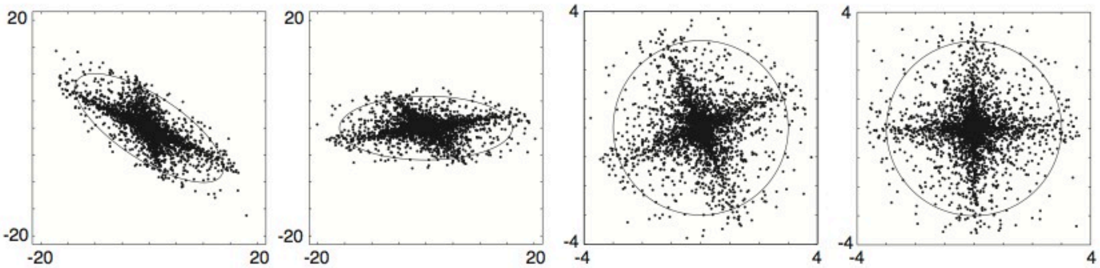
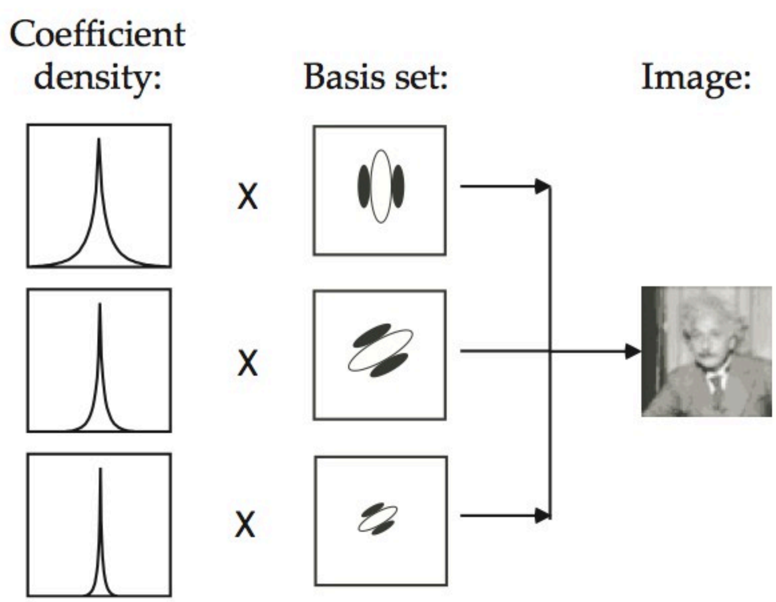
“Independent” Components Analysis (ICA)



For Linearly Transformed Factorial (LTF) sources: guaranteed independence
(with some minor caveats)

[Comon 94; Cardoso 96; Bell/Sejnowski 97; ...]

Model II (LTF)



Trouble in paradise

- Statistics: Images don't obey ICA source model
  - Image subband coefficients are clearly not independent samples (by visual inspection!)
  - The responses of ICA filters are highly dependent [Wegmann & Zetzsche 90, Simoncelli 97]
  - All bandpass filters give sparse marginals [Baddeley 96] => Oriented filters are a shallow optimum [Bethge 06; Lyu & Simoncelli 08]

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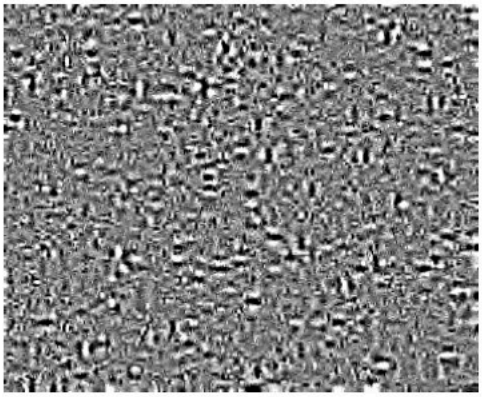
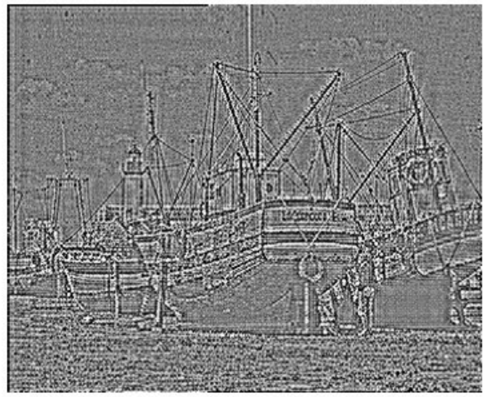
- Biology: Visual system uses a cascade
  - Where's the retina? The LGN?
  - What happens after V1? Why don't responses get sparser? [Baddeley etal 97; Chechik etal 06]

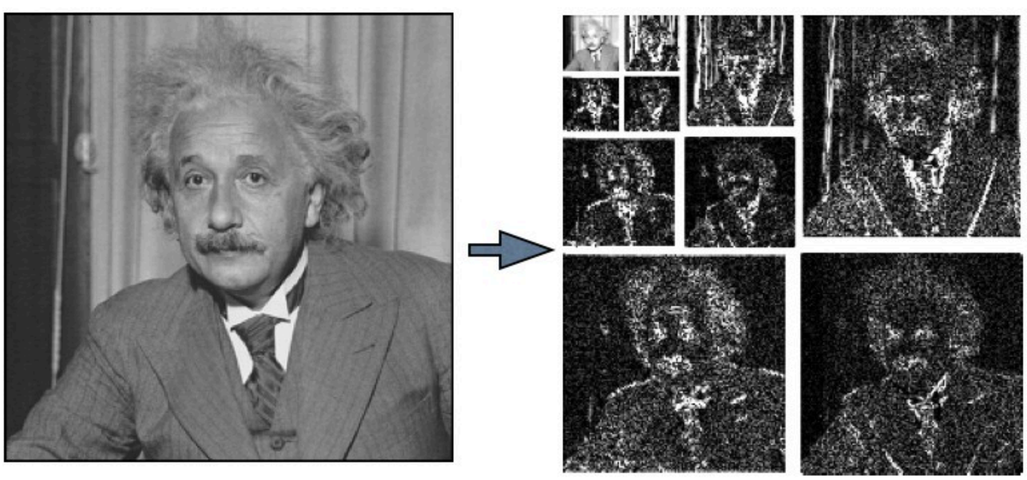
Indications that the model is weak...



Sample from model

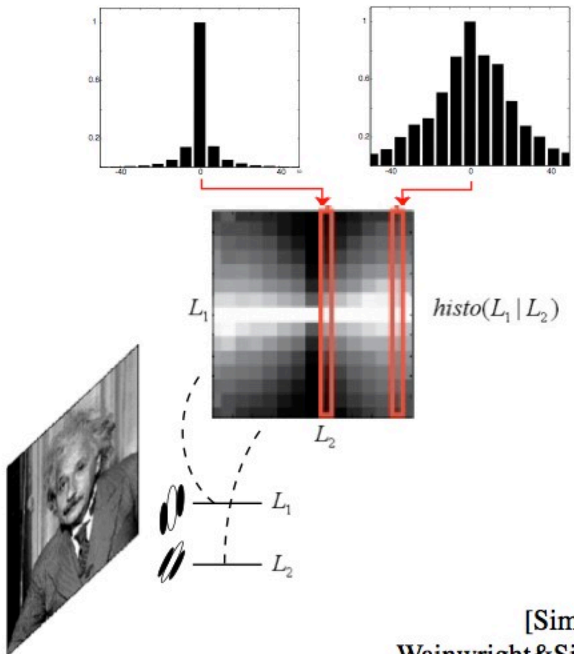


Image, ICA-transformed
and Gaussianized



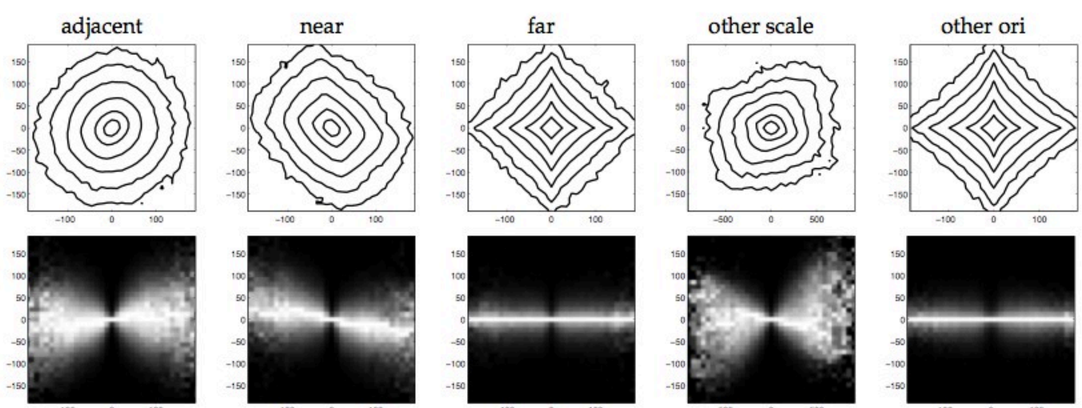
- Large-magnitude subband coefficients are found at neighboring positions, orientations, and scales.

Conditional densities reveal nonlinear dependencies



[Simoncelli 97; Buccigrossi&Simoncelli 99; Wainwright&Simoncelli 99; Schwartz&Simoncelli 01]

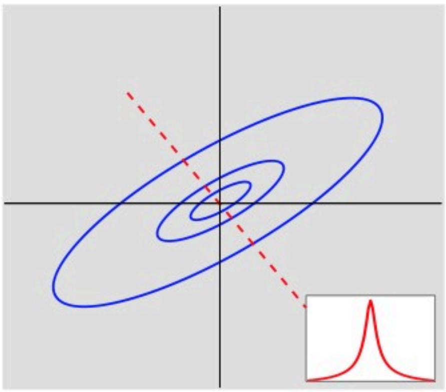
Joint densities



- Nearby: densities are approximately circular/elliptical
- Distant: densities are approximately factorial

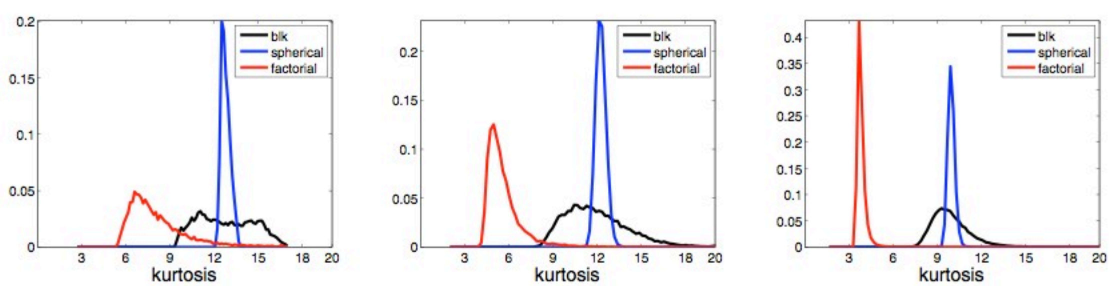
[Simoncelli, '97; Wainwright&Simoncelli, '99]

Non-Gaussian elliptical observations
and models of natural images:



- Zetzsche & Krieger, 1999;
- Huang & Mumford, 1999;
- Wainwright & Simoncelli, 1999;
- Hyvärinen and Hoyer, 2000;
- Parra et al., 2001;
- Srivastava et al., 2002;
- Sendur & Selesnick, 2002;
- Teh et al., 2003;
- Gehler and Welling, 2006
- Lyu & Simoncelli, 2008
- etc.

Spherical vs Sparse

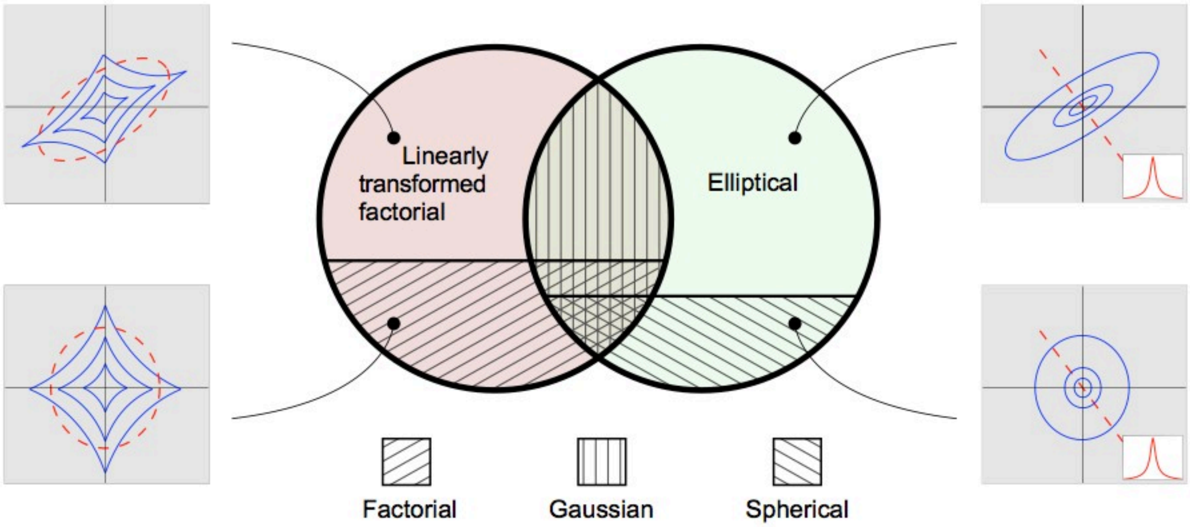


3x3 7x7 15x15

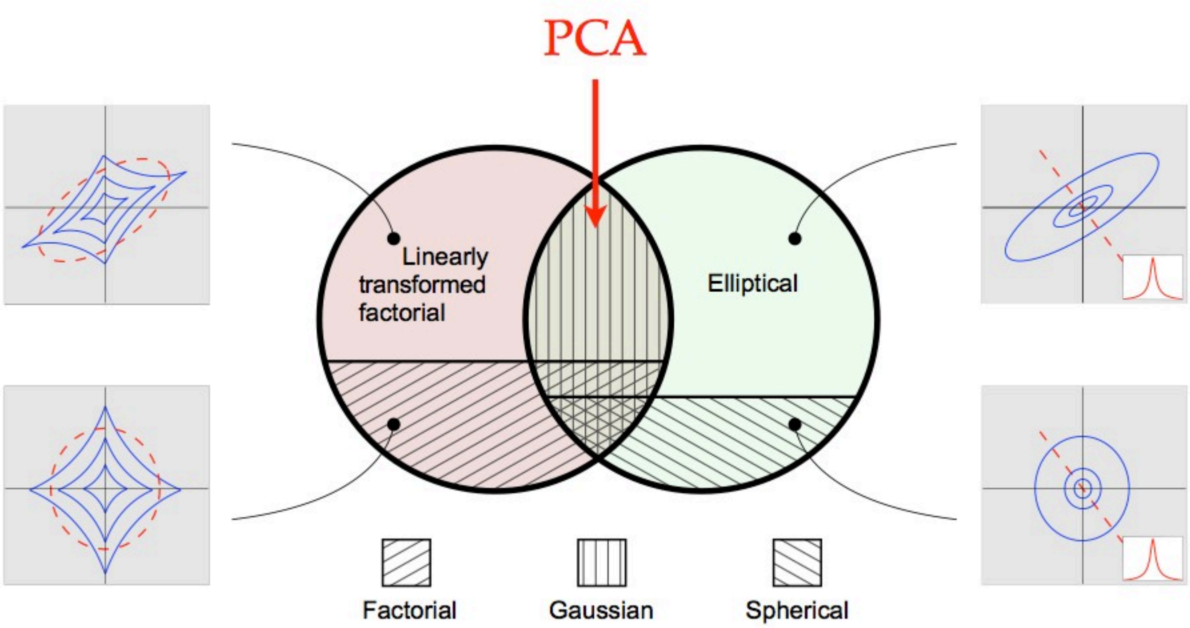
data (ICA'd): —— sphericalized: —— factorialized: ——

• Histograms, kurtosis of projections of image blocks onto random unit-norm basis functions.
• These imply data are closer to spherical than factorial

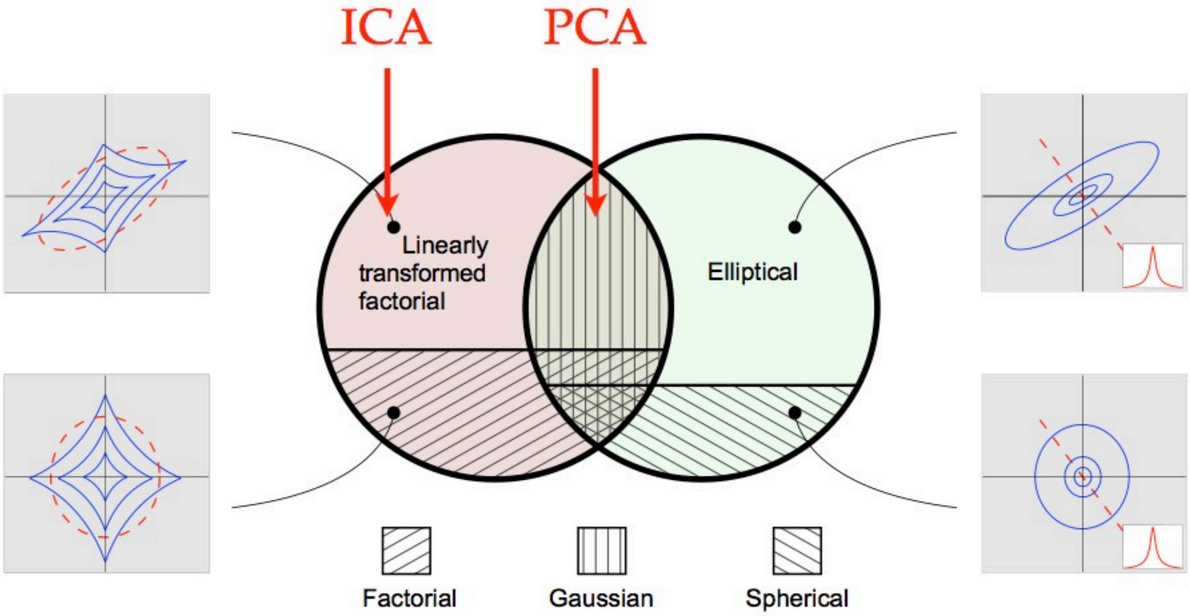
[Lyu & Simoncelli 08]



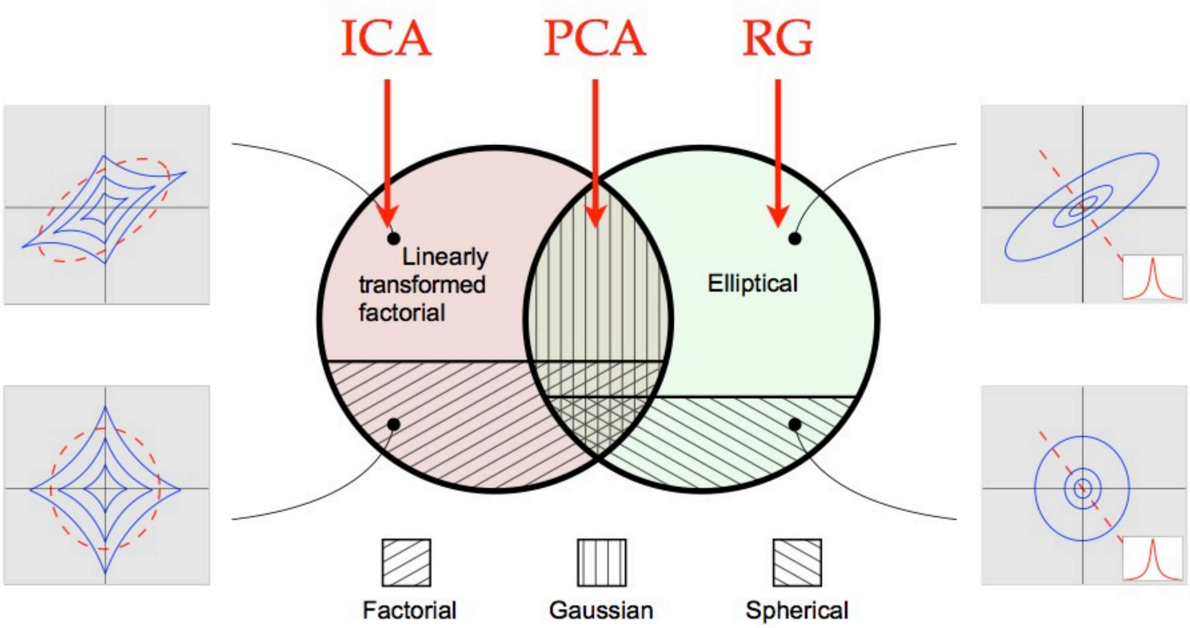
[Lyu & Simoncelli, 2008]



[Lyu & Simoncelli, 2008]



[Lyu & Simoncelli, 2008]

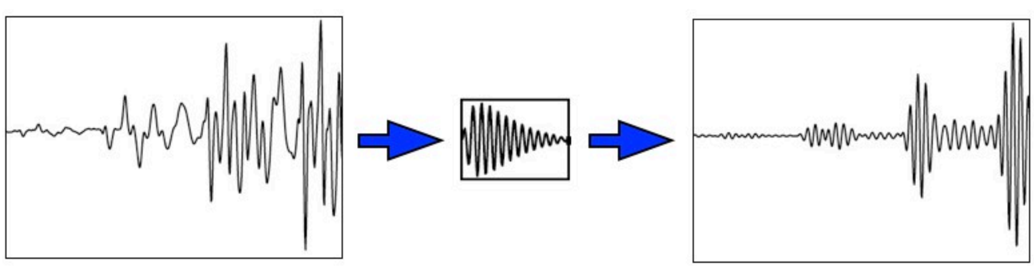


[Lyu & Simoncelli, 2008]

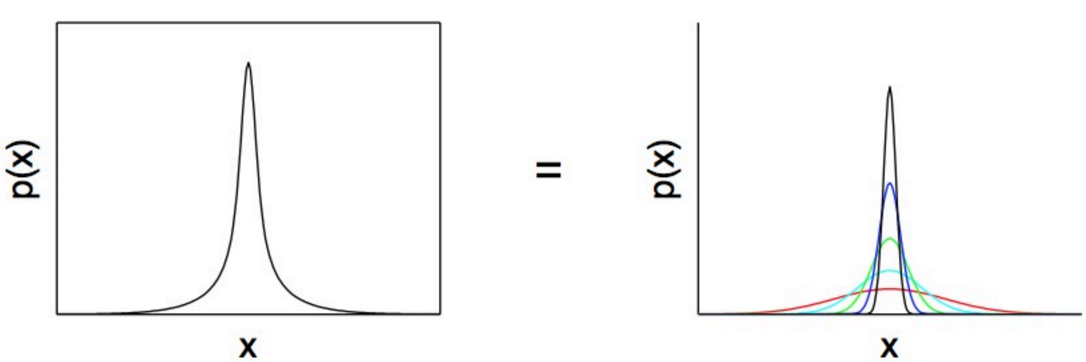
Putting it all together...

- Subband coefficients are marginally non-Gaussian
- Coeffient pairs, or local clusters, are approximately elliptical
- Image subbands contain a small number of very large coefficients (that's what lets us separate them from noise), and these tend to occur near each other
- So suppose coefficients are locally Gaussian, but the variance is *fluctuating* over the image (known as heteroscedasticity)

Marginal statistics - sound



Signal is *heteroskedastic* (has varying variance):

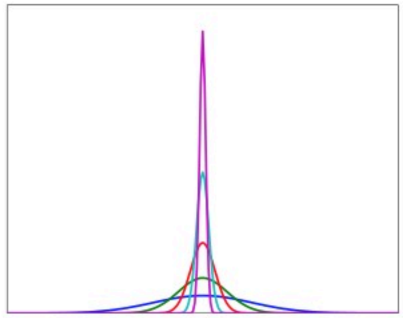


Modeling heteroscedasticity

Assume a hidden scaling variable for each patch

Gaussian scale mixture (GSM)
[Andrews & Mallows 74]:

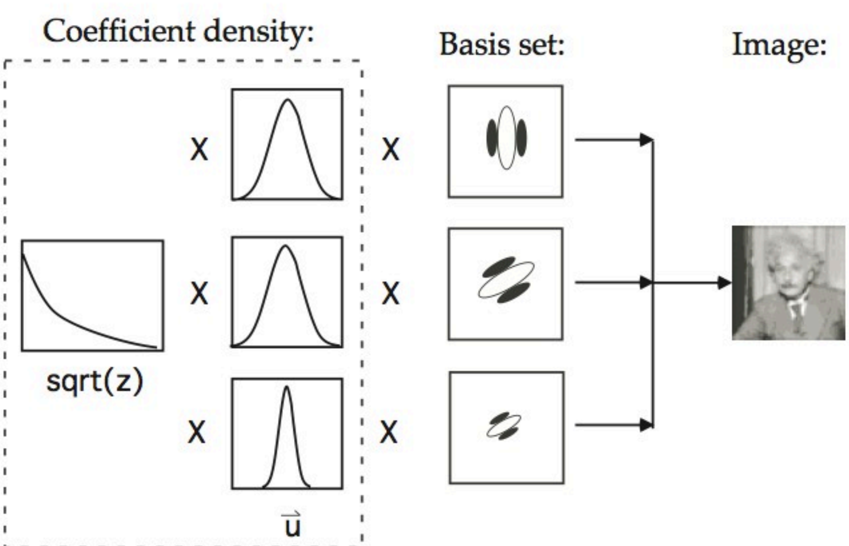
$$\vec{x} = \sqrt{z}\vec{u}$$



- $\vec{u}$ is Gaussian, $z > 0$
- z and $\vec{u}$ are independent
- $\vec{x}$ is elliptically symmetric, with covariance $\propto C_u$
- marginals of $\vec{x}$ are leptokurtotic

[Wainwright&Simoncelli 99]

Model III (GSM)



Denoising: Joint

$$\mathbb{E}(x|\vec{y}) = \int dz\, \mathcal{P}(z|\vec{y})\, \mathbb{E}(x|\vec{y}, z)$$
$$= \int dz\, \mathcal{P}(z|\vec{y}) \left[zC_u(zC_u + C_w)^{-1}\vec{y}\right]_{\text{ctr}}$$

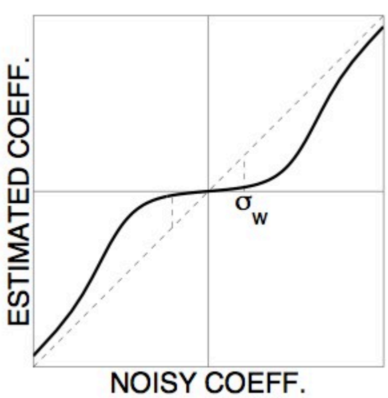
where

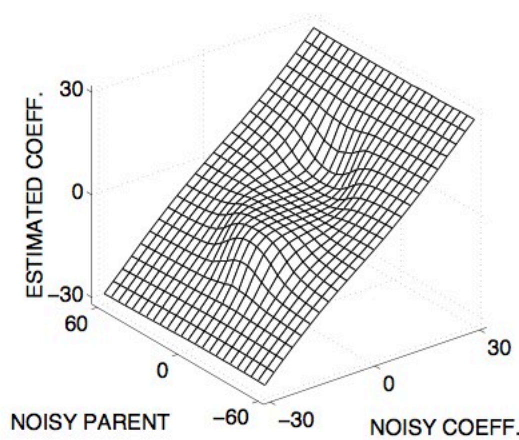
$$\mathcal{P}(z|\vec{y}) = \frac{\mathcal{P}(\vec{y}|z)\, \mathcal{P}(z)}{\mathcal{P}\vec{y}}, \quad \mathcal{P}(\vec{y}|z) = \frac{\exp(-\vec{y}^T(zC_u + C_w)^{-1}\vec{y}/2)}{\sqrt{(2\pi)^N |zC_u + C_w|}}$$

Numerical computation of solution is reasonably efficient if one jointly diagonalizes C_u and C_w ...

[Portilla et. al. 2003]

Example estimators



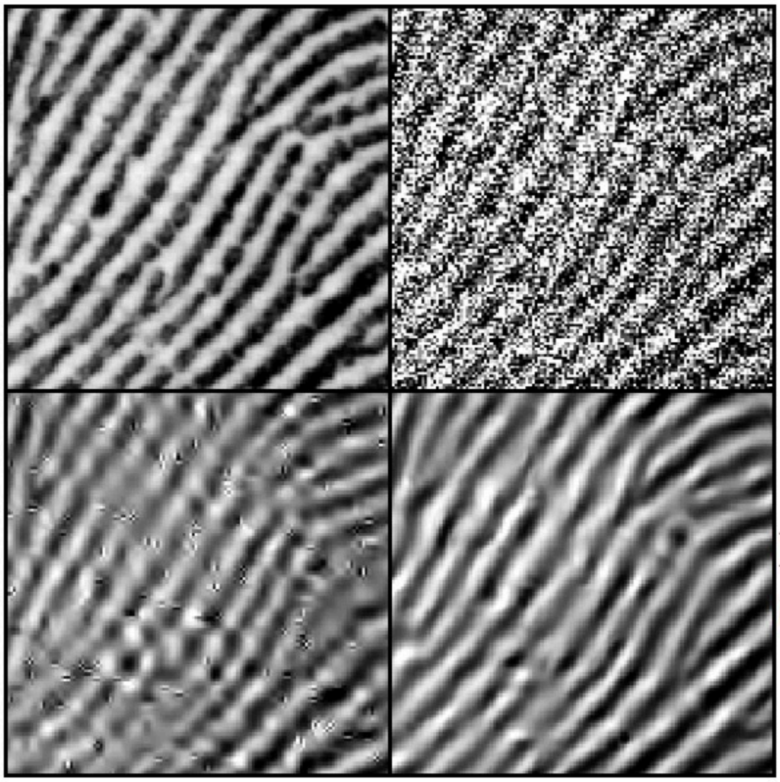


Estimators for the scalar and single-neighbor cases

[Portilla et. al. 2003]

Original

Noisy
(8.1 dB)



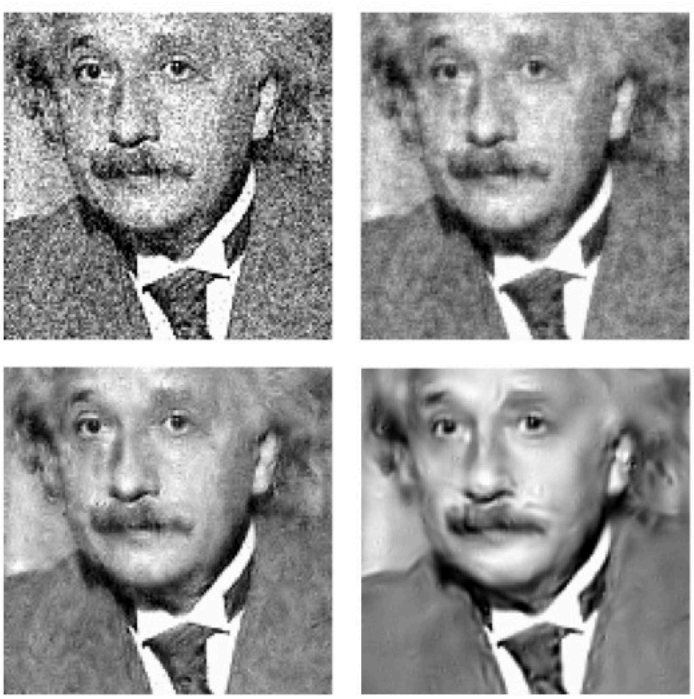
UndWvlt
Thresh
(19.0 dB)

BLS-GSM
(21.2 dB)

[Portilla et. al. 2003]

noisy
(4.8)

I-linear
(10.61)



II-marginal
(11.98)

III-GSM
nbd: $5 \times 5 + p$
(13.60)

[Portilla et. al. 2003]

Real sensor noise

400 ISO

GSM denoised

[Portilla et. al. 2003]