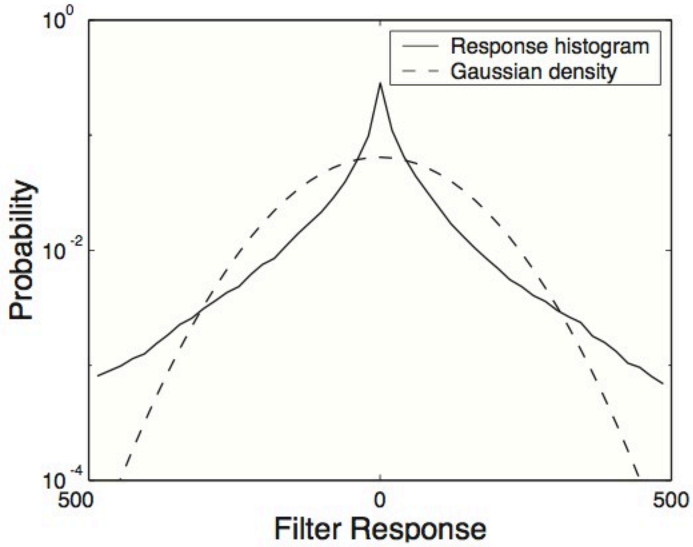
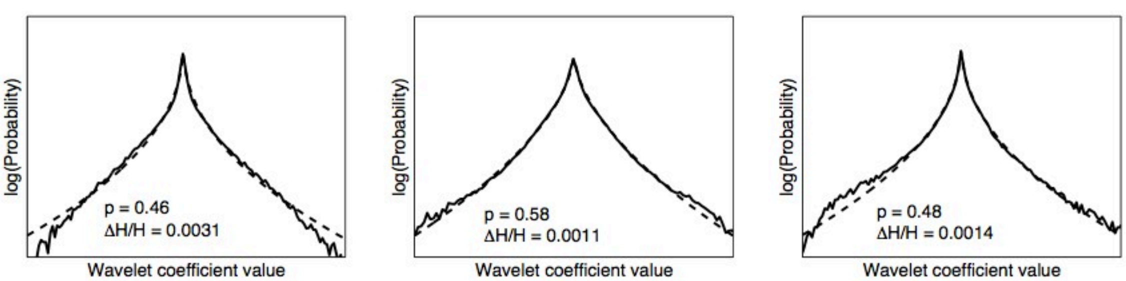
Bandpass Filter Responses



[Burt&Adelson 82; Field 87; Mallat 89; Daugman 89, ...]

Marginal densities



Well-fit by a generalized Gaussian:

$$P(x) \propto \exp -|x/s|^p$$

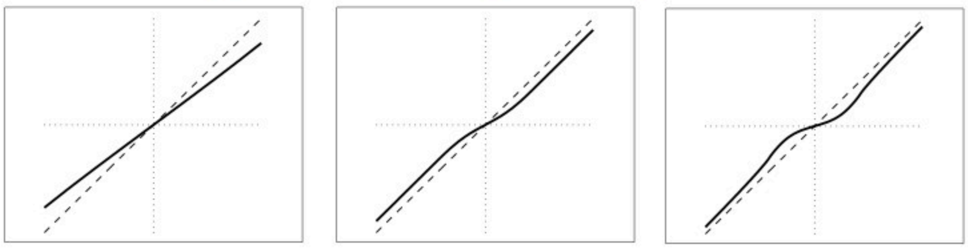
[Mallat 89; Simoncelli&Adelson 96; Moulin&Liu 99; ...]

II. BLS for non-Gaussian prior

- Assume marginal distribution [Mallat '89]:

$$P(x) \propto \exp -|x/s|^p$$

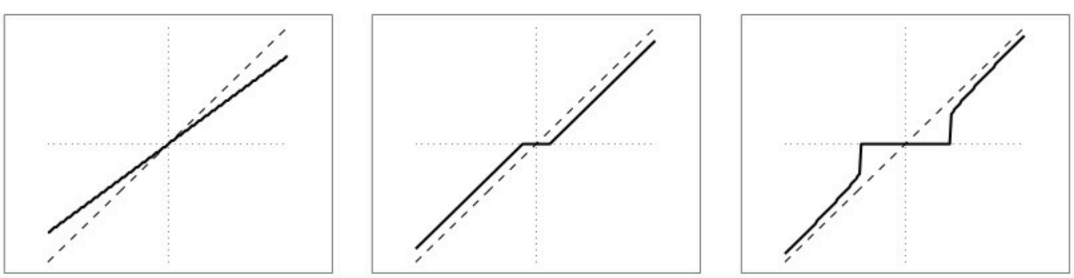
- Then Bayes estimator is generally nonlinear:



p = 2.0 *p = 1.0* *p = 0.5*

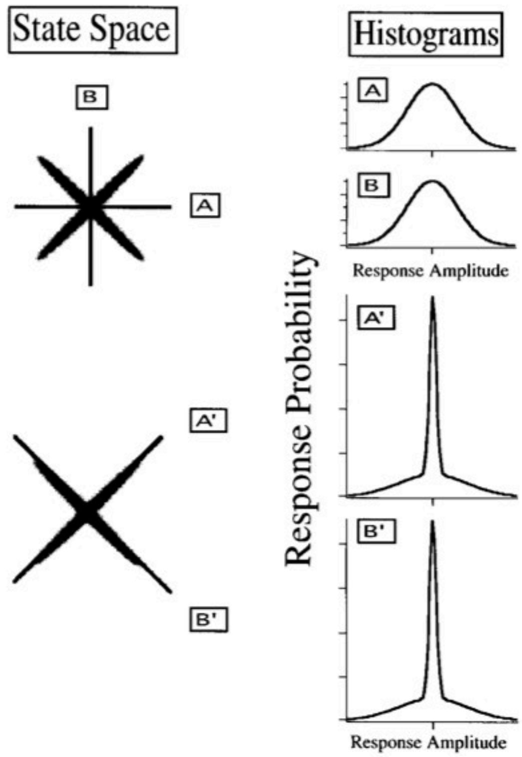
[Simoncelli & Adelson, '96]

MAP shrinkage



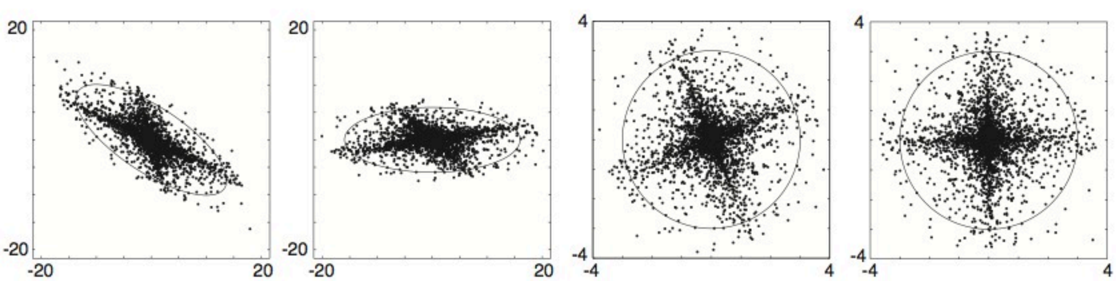
p=2.0 p=1.0 p=0.5

[Simoncelli 99]



[Field, "what is the goal of sensory coding", 1994]

"Independent" Components Analysis (ICA)



For Linearly Transformed Factorial (LTF) sources: guaranteed independence
(with some minor caveats)

[Comon 94; Cardoso 96; Bell/Sejnowski 97; ...]

Independent Component Analysis

Solve for a set of axes (not necessarily orthogonal) along which the data are least Gaussian.

Examples:

- FOBI - simplest algorithm (Cardoso, 1989)
- Fast ICA - fixed-point algorithm with fast convergence (Hyvarinen, 1997)

Closely related: **Projection pursuit**. Seek projections of data that are non-Gaussian (Friedman & Tukey, 1974).

Icassp'89. pp. 2109-2112

SOURCE SEPARATION USING HIGHER ORDER MOMENTS

Jean-Francois Cardoso

Ecole Nat. Sup. des Telecommunications - Dept SIGNAL
46 rue Barrault, 75634 PARIS CEDEX 13, FRANCE.
and
CNRS-URA 820, GRECO-TDSI.

ABSTRACT

This communication presents a simple algebraic method for the extraction of independent components in multidimensional data. Since statistical independence is a much stronger property than uncorrelation, it is possible, using higher-order moments, to identify source signatures in array data without any a-priori model for propagation or reception, that is, without directional vector parametrization, provided that the emitting sources be independent with different probability distributions. We propose such a "blind" identification procedure. Source signatures are directly identified as covariance eigenvectors after data have been orthonormalized and non linearily weighted. Potential applications to Array Processing are illustrated by a simulation consisting in a simultaneous range-bearing estimation with a passive array.

INTRODUCTION

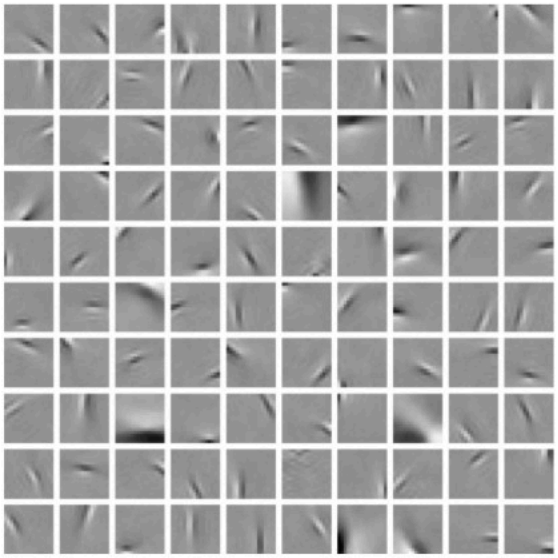
For a lot of reasons (of various kinds), the most common Signal Processing methods deal with second-order statistics, expressed in terms of covariance matrices. It is well known that Gaussian stochastic processes are exhaustively described by their second-order statistics. Nonetheless, when the Gaussian assumption is not valid, some information is lost by retaining only second-order statistics.

in Array Processing has been done within this framework [6,7,8]. However, actual physical settings are often such that source signatures (directional vectors) depart from the assumed model. As expected, model-based methods are very sensitive to such discrepancies. Multipath, unknown antenna deformation are among the common causes of severe performance degradation.

It is the purpose of this communication to present a simple algebraic method allowing source identification when NO a priori information about the propagation and the reception is available. The key requirement is that the observed data consist in a linear superimposition of statistically independent components. It may seem strange that such a blind identification procedure be possible, but it should be recalled that statistical independence between sources is a much stronger requirement than mere uncorrelation. The question of blind separation of multidimensional components by taking advantage of statistical independence has already been adressed in recent litterature. A non-linear adaptive procedure has been proposed in [9,10] while a direct solution using explicitely cumulants was given for the case of two sources and two sensors in [11]. In contrast, we propose here a simple algebraic method to separate an arbitrary number of sources, given measurements from a larger number of sensors.

THE SOURCE SEPARATION PROBLEM

ICA on image blocks



[Bell/Sejnowski '97]
[example obtained with FastICA, Hyvarinen]

Alt: Sparse representation

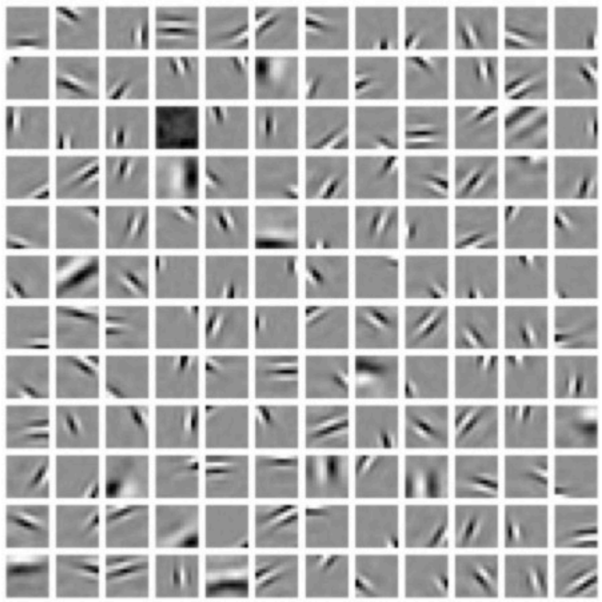
$$E(\vec{c}) = ||\vec{x} - B\vec{c}||^2 + \lambda S_p(\vec{c})$$ [Olshausen & Field '95]

$$S_p(\vec{c}) = \sum |c_k|^p$$

- If p >= 1, the objective function is convex (and thus can solve with descent algorithms)
- The p=1 case is widely used [LASSO - Tibshirani, 1996] [Basis Pursuit - Chen, Donoho, Sanders, 1998]
- Finding efficient solutions, and/or solutions for p<1, has become a major research area

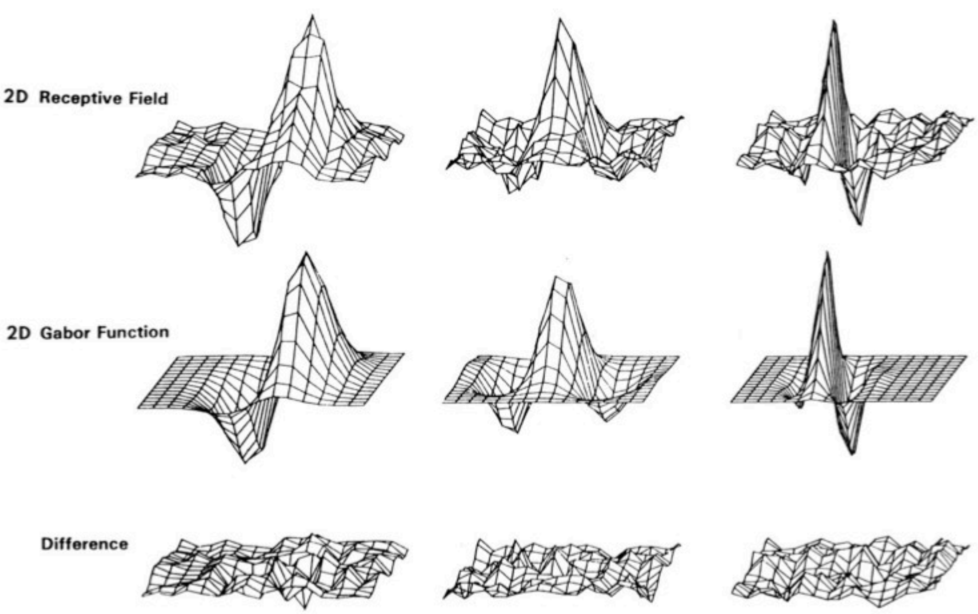
  [e.g., Figueiredo&Nowak 01; Daubechies etal 03; Starck etal 03; Bect etal 04; Elad etal 06; Chartrand 08]

Sparse basis for images



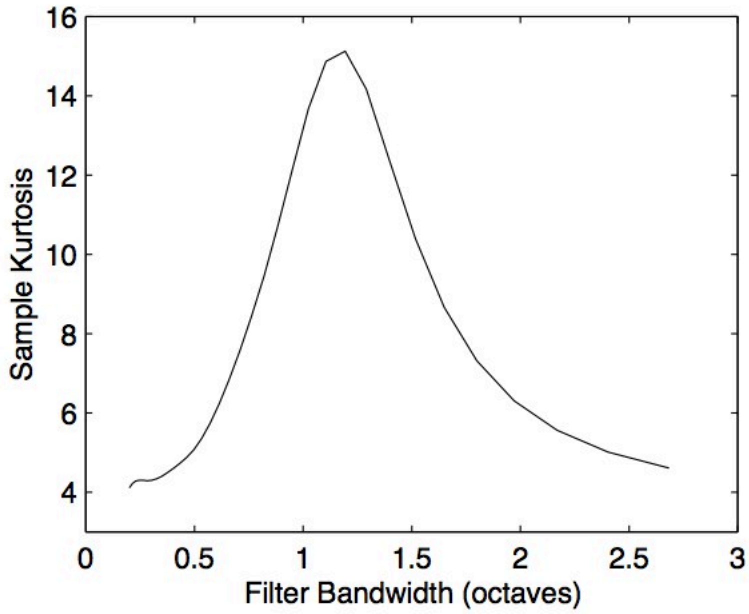
[Olshausen/Field '96]

V1 receptive fields



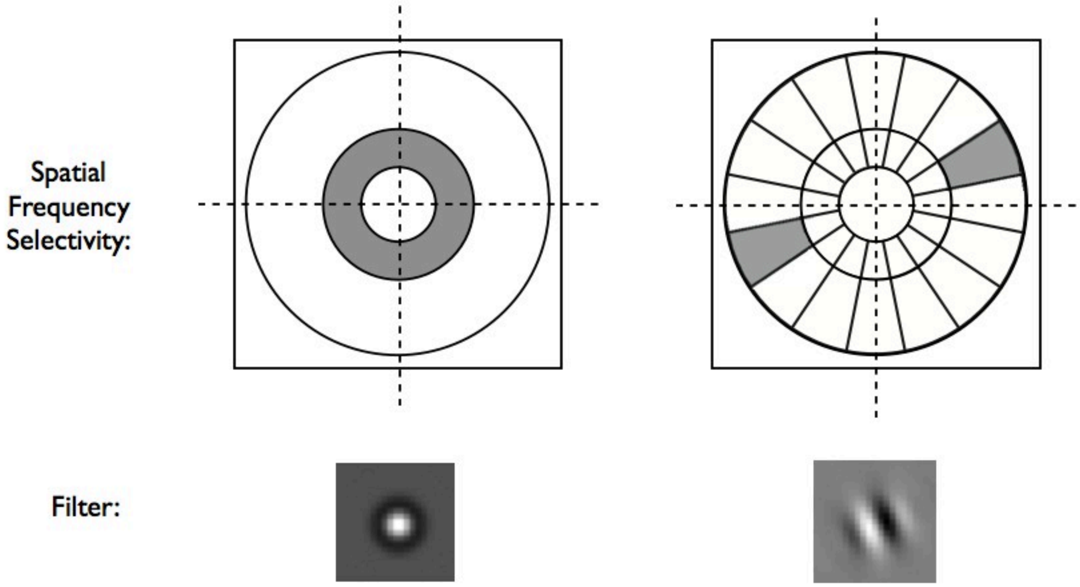
Daugman '89

Kurtosis vs. bandwidth

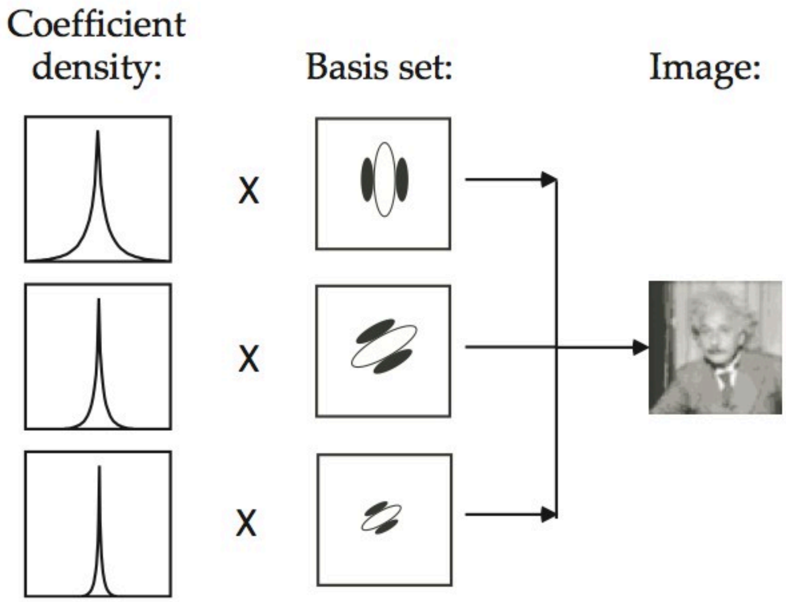


[after Field 87]

Octave-bandwidth representations



Model II (LTF)

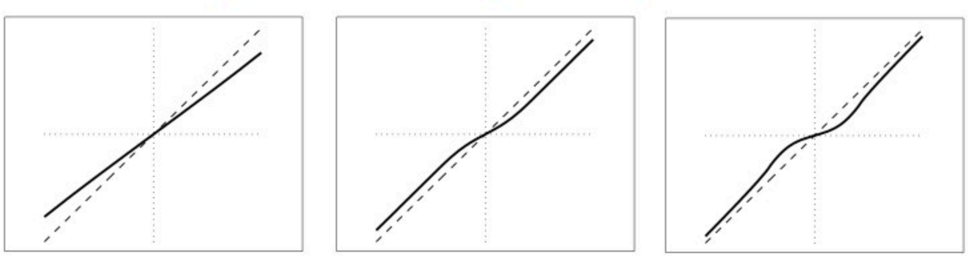


II. BLS for non-Gaussian prior

- Assume marginal distribution [Mallat '89]:

$$P(x) \propto \exp -|x/s|^p$$

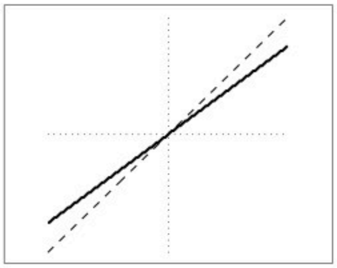
- Then Bayes estimator is generally nonlinear:



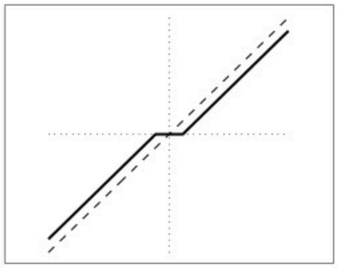
$p = 2.0$ $p = 1.0$ $p = 0.5$

[Simoncelli & Adelson, '96]

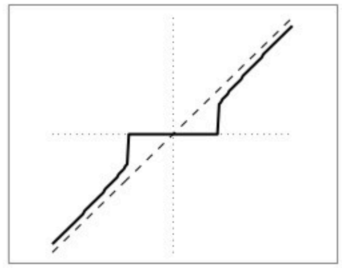
MAP shrinkage



p=2.0



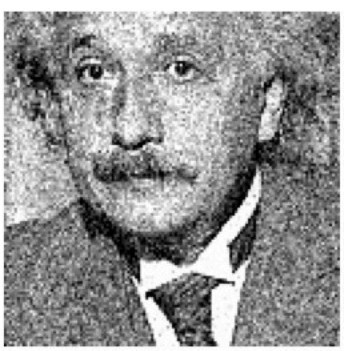
p=1.0

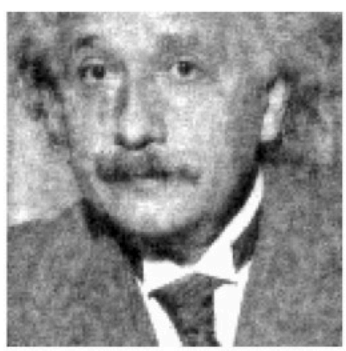


p=0.5

[Simoncelli 99]

noisy
(4.8)





I-linear
(10.61)

II-marginal
(11.98)