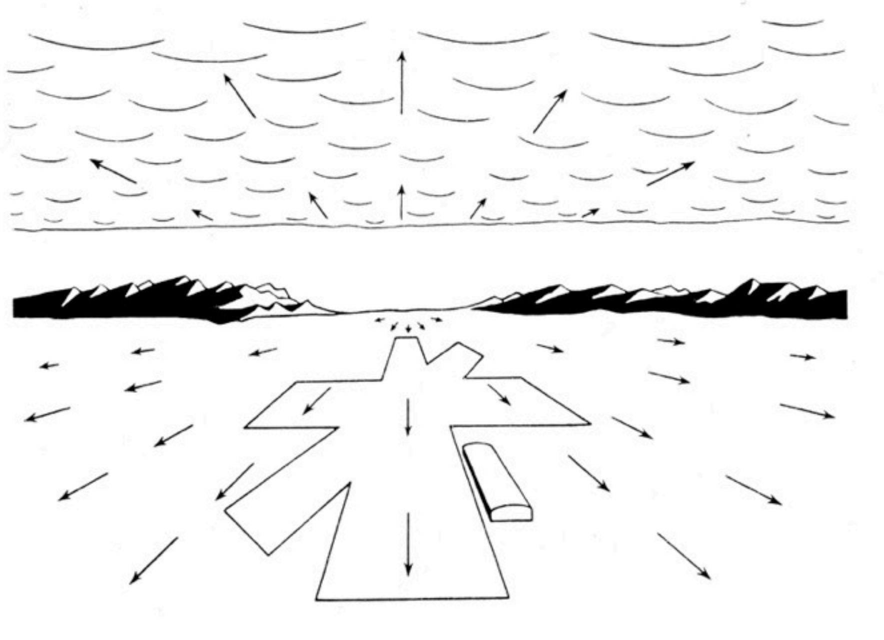
Matching

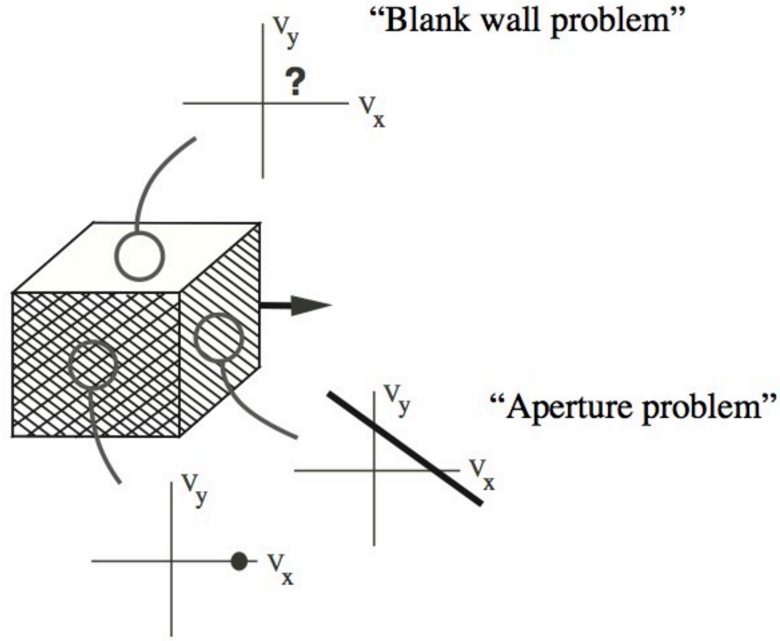
- Template matching: search, classification, recognition, ...
- Correspondence problem: stereo, motion, sensor fusion, ...
- Main issues:
  - ambiguities: blank or oriented regions
  - more generally: template specificity vs. false matches
  - algorithmic: non-Convex optimization
- Differential and coarse-to-fine strategies
- Generalizations to include other invariances

Optic flow



[Gibson, 1950]

Visual motion ambiguity



Differential matching

$$I_2(\tilde{x} + \vec{v}) \approx I_1(\tilde{x} - \vec{v}), \quad \tilde{x} \in \mathcal{N}(\vec{x})$$

$$I_2(\tilde{x}) + \vec{v}^T \vec{\nabla} I_2(\tilde{x}) \approx I_1(\tilde{x}) - \vec{v}^T \vec{\nabla} I_1(\tilde{x})$$

Let $A(\tilde{x}) \equiv I_2(\tilde{x}) + I_1(\tilde{x}), \quad D(\tilde{x}) \equiv I_2(\tilde{x}) - I_1(\tilde{x})$

$$D(\tilde{x}) \approx -\vec{v}^T \vec{\nabla} A(\tilde{x})$$

[Horn & Schunk, 1981]

Differential matching

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$$D(\tilde{x}) \approx -\vec{v}^T \vec{\nabla} A(\tilde{x})$$

One linear constraint for two unknowns ("aperture problem")
=> can only solve for component in gradient direction

[Horn & Schunk, 1981]

Least squares differential matching

Combine constraints over local neighborhood:

$$\begin{aligned}
\hat{v}(\vec{x}) &= \arg\min_{\vec{v}} \sum_{\tilde{x} \in \mathcal{N}(\vec{x})} \left[I_2(\tilde{x} + \vec{v}) - I_1(\tilde{x} - \vec{v})\right]^2 \\
&\approx \arg\min_{\vec{v}} \sum_{\tilde{x} \in \mathcal{N}(\vec{x})} \left[I_2(\tilde{x}) + \vec{v}^T \vec{\nabla} I_2(\tilde{x}) - I_1(\tilde{x}) + \vec{v}^T \vec{\nabla} I_1(\tilde{x})\right]^2 \\
&= \left[\sum_{\tilde{x} \in \mathcal{N}(\vec{x})} \vec{\nabla} A(\tilde{x}) \vec{\nabla}^T A(\tilde{x})\right]^{-1} \left[\sum_{\tilde{x} \in \mathcal{N}(\vec{x})} \vec{\nabla} A(\tilde{x}) D(\tilde{x})\right]
\end{aligned}$$

[Lucas & Kanade, 1981]

Least squares differential matching

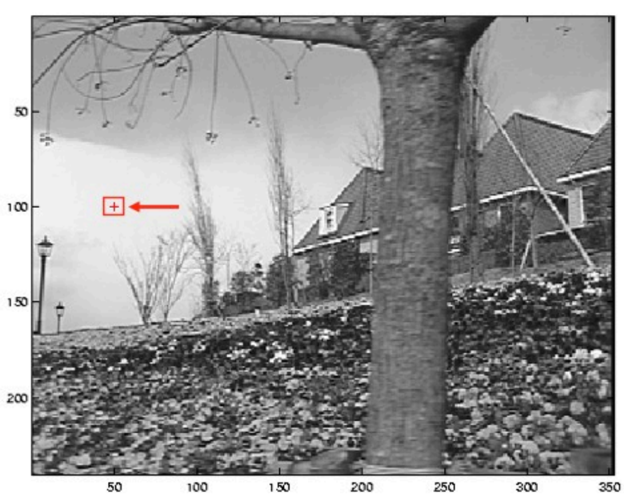
Combine constraints over local neighborhood:

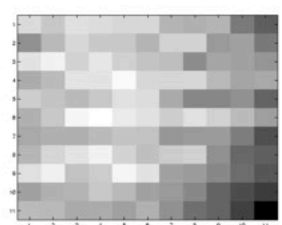
$$
\begin{aligned}
\hat{v}(\vec{x}) &= \arg\min_{\vec{v}} \sum_{\tilde{x} \in \mathcal{N}(\vec{x})} \left[I_2(\tilde{x} + \vec{v}) - I_1(\tilde{x} - \vec{v}) \right]^2 \\
&\approx \arg\min_{\vec{v}} \sum_{\tilde{x} \in \mathcal{N}(\vec{x})} \left[I_2(\tilde{x}) + \vec{v}^T \vec{\nabla} I_2(\tilde{x}) - I_1(\tilde{x}) + \vec{v}^T \vec{\nabla} I_1(\tilde{x}) \right]^2 \\
&= \left[\sum_{\tilde{x} \in \mathcal{N}(\vec{x})} \vec{\nabla} A(\tilde{x}) \vec{\nabla}^T A(\tilde{x}) \right]^{-1} \left[\sum_{\tilde{x} \in \mathcal{N}(\vec{x})} \vec{\nabla} A(\tilde{x}) D(\tilde{x}) \right]
\end{aligned}
$$

Structure tensor, again!

[Lucas & Kanade, 1981]

Blank wall problem

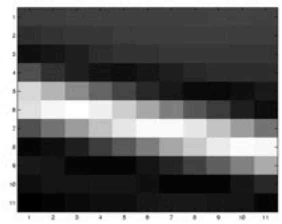




All gradients have small magnitude
=> Both eigenvalues of structure matrix are small

[example from Szeliski, 2010]

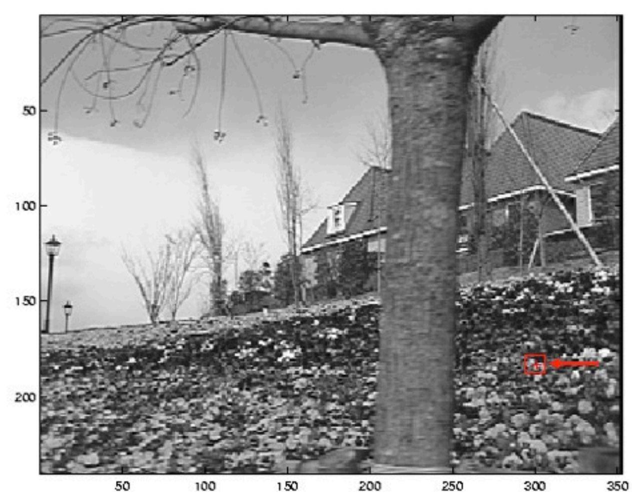
Aperture problem

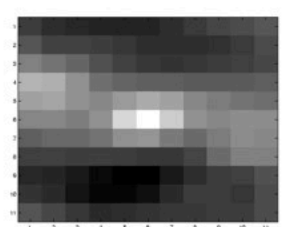


All gradient directions are matched (or opposite)
=> Structure matrix has one large, one small eigenvalue

[example from Szeliski, 2010]

No problem [pascal]





Healthy mixture of gradient directions
=> Structure matrix has two large eigenvalues (and is non-singular)

[example from Szeliski, 2010]