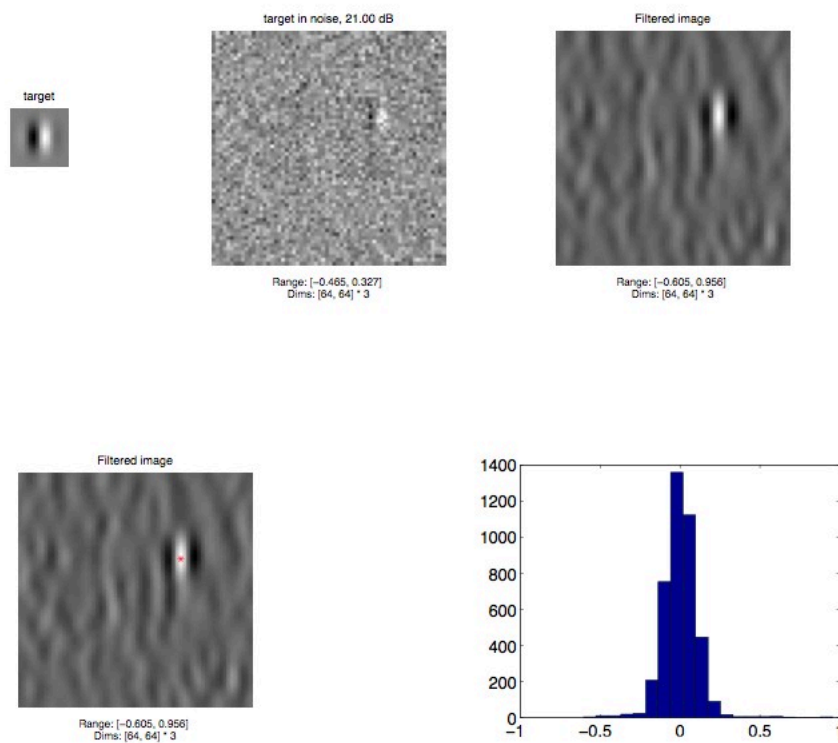


matched filter

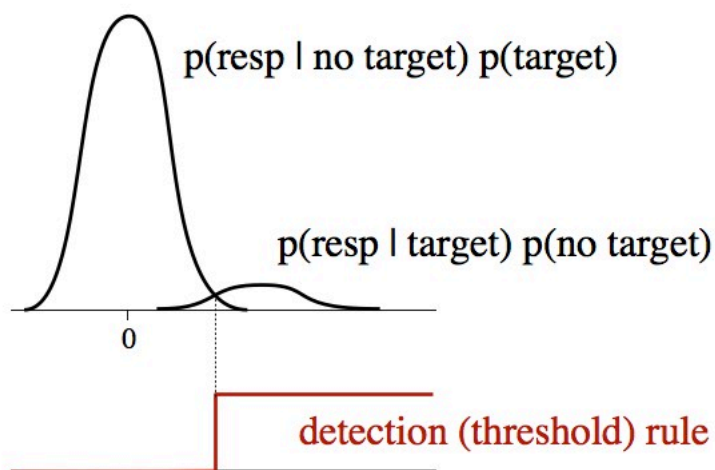
North, D. O. (1943). "An analysis of the factors which determine signal/noise discrimination in pulsed carrier systems". RCA Laboratories, Princeton, NJ, Technical Report PTR-6C.

- Known target
- Additive white Gaussian noise
- Linear + threshold detector

[on board]



Signal detection interpretation



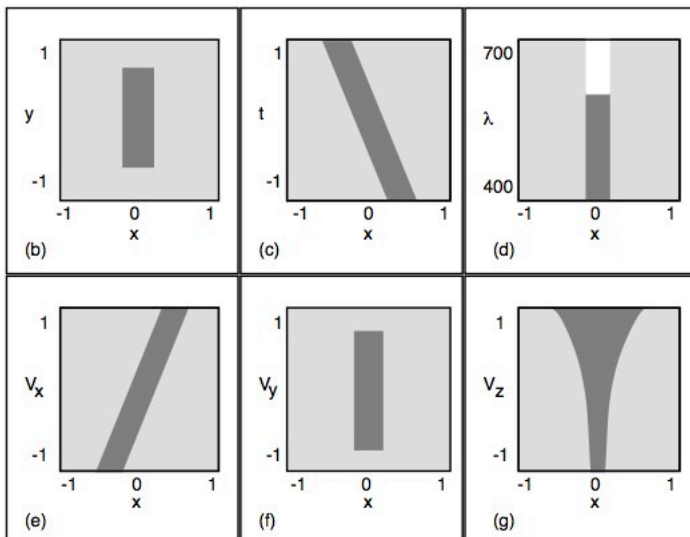
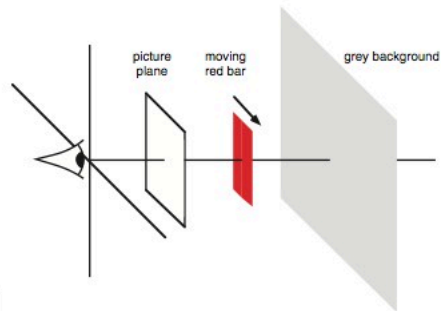
[on board]

matched filter

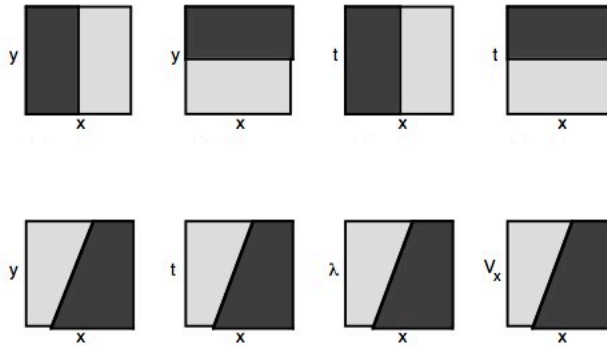
- brute-force solution for finding known template at unknown location
- assumes white Gaussian noise (easily modified for correlated Gaussian noise)
- assumes a single copy of target (not a superposition)
- assumes known, fixed template (no distortions/deformations)

Plenoptic slices

$$I(x, y, \lambda, t, V_x, V_y, V_z)$$

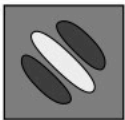


- Visual system is interested in relative quantities
- Machine vision systems often seek boundaries
- Where are the discontinuities, and at what orientation [space, time, etc]?



Derivatives are fundamental...

[Koenderink, 1984;
Adelson & Bergen 1990]

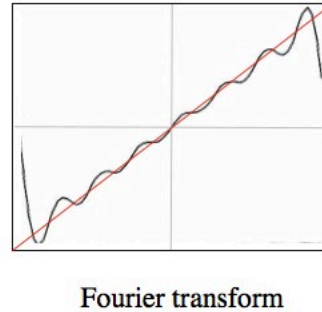
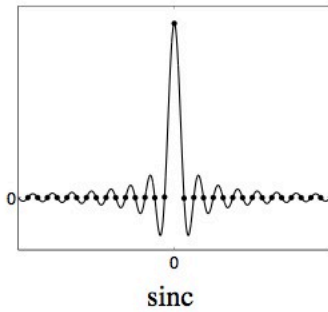
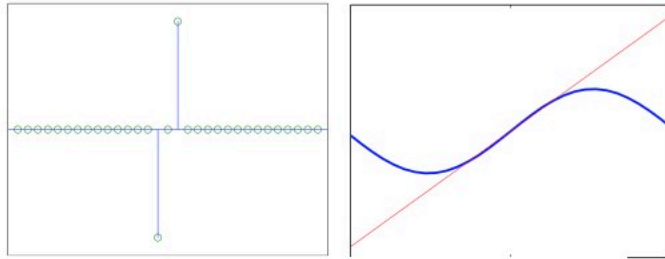


x	—				
y	diag. "bar" static achromatic no dispar	—			
t	vert. "bar" leftward achromatic no dispar	hor. "bar" downward achromatic no dispar	—		
λ	vertical static hue-sweep no dispar	horiz. static hue-sweep no dispar	full-field sequential hue-sweep no dispar	—	
V _x	vert. "bar" static achromatic hor. dispar	hor. "bar" static achromatic vert. dispar	full-field sequential achromatic eye-order	full-field static hue-shift luster	—
	x	y	t	λ	V _x

[Adelson & Bergen 91]

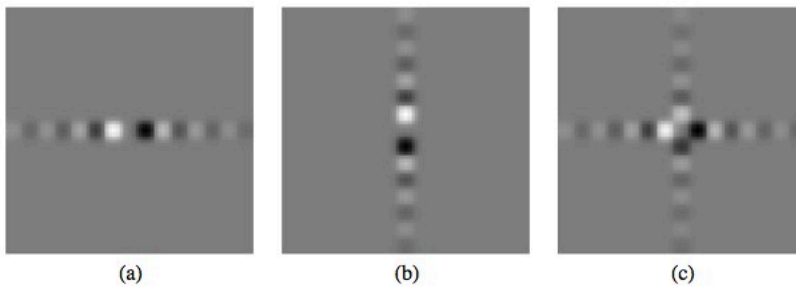
Discrete derivatives

- calculus definition
- differences?
- sinc derivative?

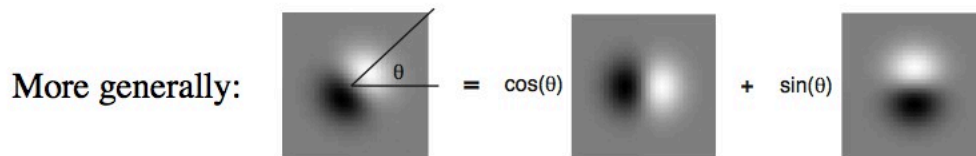
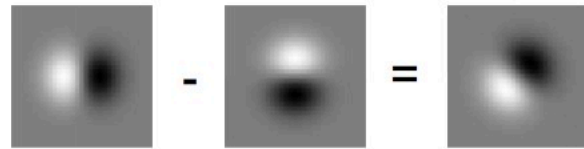
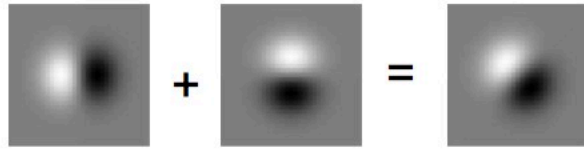


2D derivatives?

- vector calculus => need rotation invariance!
- differences, and sinc derivative, don't work
- Compromise: incorporate prefiltering

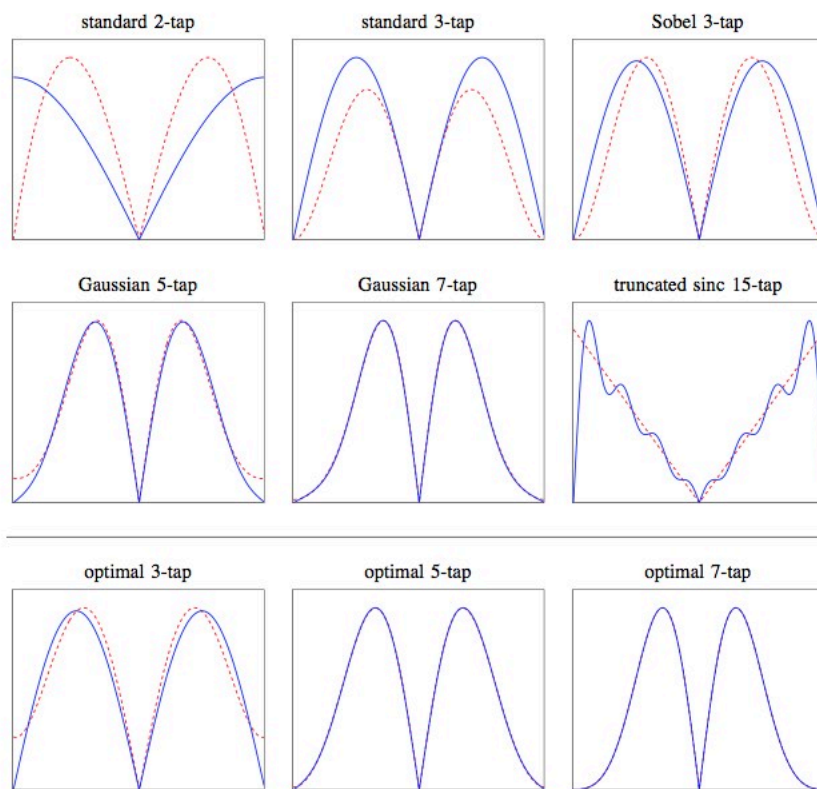


Rotation-invariance of gradient operators



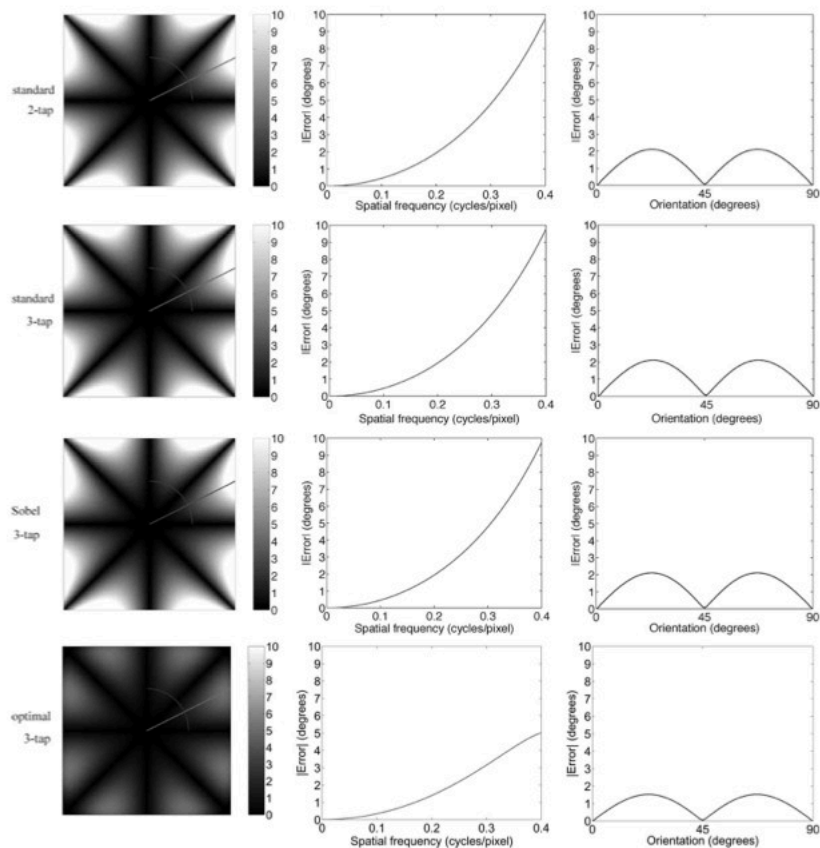
Multi-D differentiator criteria

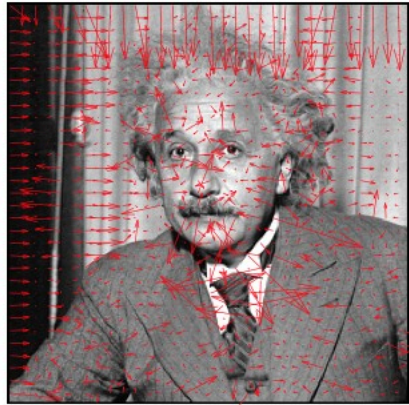
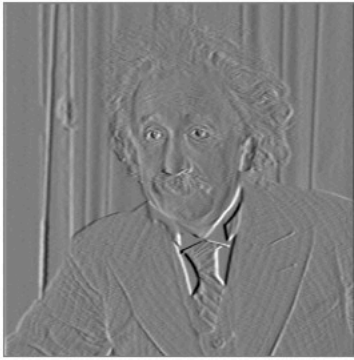
- *Two* 1D filters [interpolator, differentiator]
- Differentiator = interpolator .* ramp
- **Rotation-invariant gradient !**
- Localized



Fourier amplitudes, discrete differentiators (blue)
compared to ramp times interpolator (red dashed)

[Farid & Simoncelli 03]





orientation undefined at local maxima/minima

Local orientation analysis

1) Find direction with largest summed projection:

$$\max_{\hat{u}} \sum_{\vec{x}_k \in \mathcal{N}} w_k \vec{\nabla} I(\vec{x}_k)^T \hat{u}$$

2) Find direction that captures most energy [better]:

$$\max_{\hat{u}} \sum_{\vec{x}_k \in \mathcal{N}} w_k \|\vec{\nabla} I(\vec{x}_k)^T \hat{u}\|^2$$

[on board]

The structure tensor

$$M(\vec{x}) = \sum_{\vec{x}_k \in \mathcal{N}(\vec{x})} w_k \vec{\nabla} I(\vec{x}_k) \vec{\nabla} I(\vec{x}_k)^T$$

Let eigenvalues/eigenvectors be $\{\lambda_1, \lambda_2\} \{\hat{e}_1, \hat{e}_2\}$

- Total energy: $\lambda_1 + \lambda_2$
- “Orientedness”: $(\lambda_1 - \lambda_2)/(\lambda_1 + \lambda_2)$
- Dominant orientation: $\hat{e}_1$

[Bigun & Granlund 87; Knutsson 89; many others]

sqrt(energy)



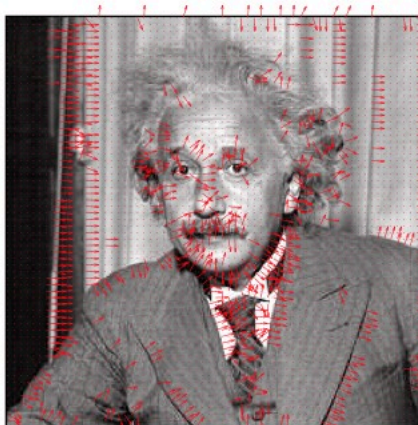
Orientedness



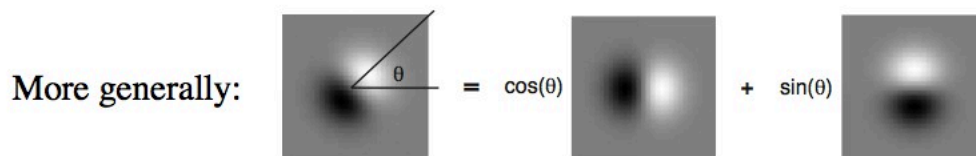
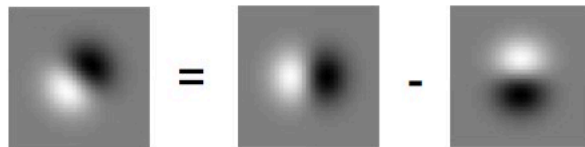
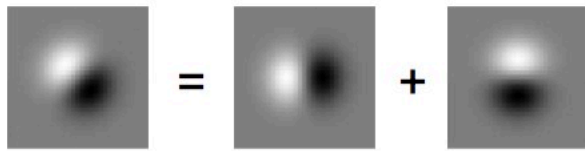
Oriented vs. non-oriented, large-energy regions



Orientation

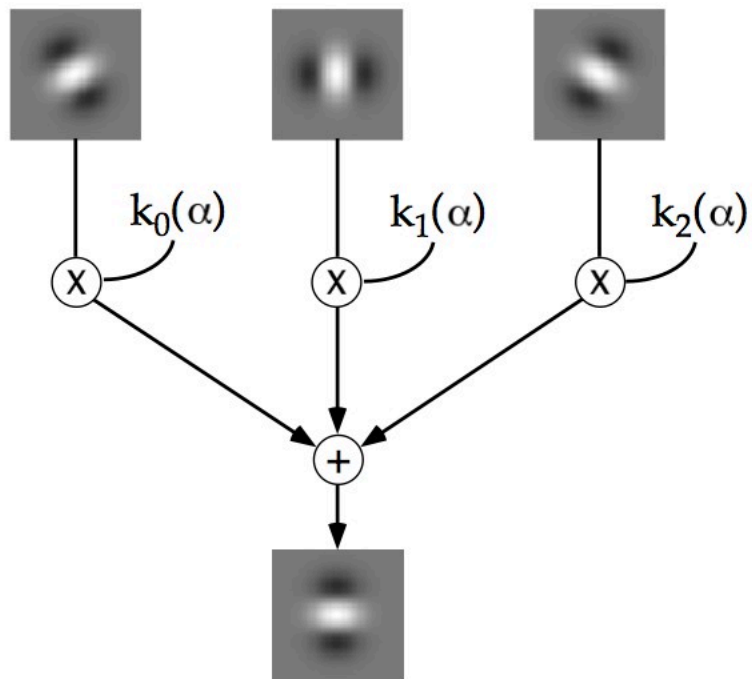


Non-separable from separable

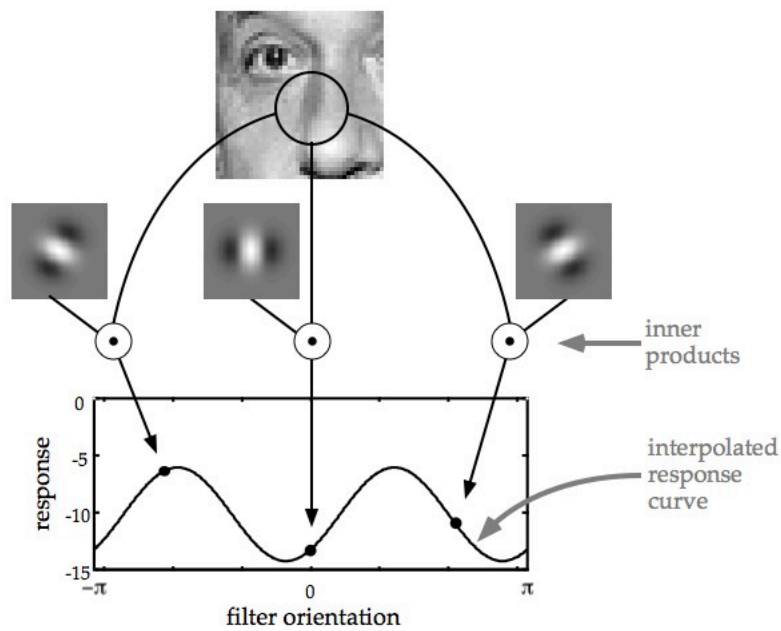


Generalizations [on board]

- Directional Nth-order derivatives [need N+1]
- Polar frequency interpretation
- Steerability: the angular Nyquist theorem
- Orientation estimation



Steerability



[Freeman&Adelson 91]