Supporting information

Region of bistability for the one-dimensional rate model

The rate model described by Eqs. (1)–(3) of the main text may exhibit bistability. We want to find the range of the gain and the threshold parameters, a and θ, where the system is bistable. This can be done determining, for a given value of a, the interval of values for θ within which two stable states exist. The limits of the interval are found from the bifurcation condition in a one-dimensional system

$$f(x) = 0 \quad \text{(fixed point)}, \tag{A1}$$
$$f'(x) = 0 \quad \text{(bifurcation)}, \tag{A2}$$

which essentially corresponds to finding the values of θ where the bifurcation diagram (θ, x^*) has a turning point (see Figure 1B). Substituting the expression for $f(x)$, Eq. (2), into Eqs. (A1)–(A2) and solving for θ leads to two solutions, θ_l and θ_r. These are the limits of the bistability range

$$[\theta_l, \theta_r] = \big[\ln\{(1/y_- - 1)e^{ay_-}\}/a, \ln\{(1/y_+ - 1)e^{ay_+}\}/a\big],$$

where $y_\pm \equiv (1 \pm \sqrt{1 - 4/a})/2$. The stability diagram 1D is just the plot of the interval of bistability as a function of a.

Network of spiking neurons

We used the network introduced by Brunel and Wang [1], in its particular implementation as a two-choice decision-making network [2].

Architecture The network contains $N_E = 800$ pyramidal cells (excitatory) and $N_I = 200$ interneurons (inhibitory). Excitatory neurons are divided in three subpopulations: $p = 2$ *selective* populations (of $0.15N_E$ cells each, called A and B in the article), whose neurons encode the two possible choices to make, and a population containing the remaining, non-selective neurons ($0.7N_E$). The network is fully connected. Neurons within a selective population are strongly coupled, by a factor $w_+ > 1$ above the baseline connection weight, while neurons from different selective populations are weakly connected, with $w_- < 1$. Connections involving non-selective cells are set to baseline level, $w = 1$. To keep the mean recurrent excitatory weight constant for different values of w_+, w_- has to be set to $1 - f(w_+ - 1)/(1 - f)$.

External inputs To model spontaneous background activity, every neuron in the network is coupled through $N_{\text{ext}} = 800$ synaptic connections to an external source of Poisson-distributed, independent spike trains of rate 3 Hz. The presence of stimuli is modeled by an increase λ in the rate of spikes afferent to the cells in the associated selective population. The total spike rate received by every cell is thus $\nu_{\text{ext}} = 2400\,\text{Hz} + \lambda_{A,B}$, where λ_A and λ_B refer to the specific inputs to cells in populations A and B respectively; for non-selective and inhibitory neurons we set $\lambda = 0$. It is convenient to define the overall input increase in both populations $\lambda = (\lambda_A + \lambda_B)/2$ and the bias $\Delta\lambda = (\lambda_A - \lambda_B)/2$.

Spiking Dynamics The model neuron is a leaky integrate-and-fire (LIF) cell, described in the following. The subthreshold membrane potential V of a LIF neuron evolves according to

$$C_m \frac{dV(t)}{dt} = -g_m(V(t) - V_L) - I_{\text{syn}}(t),$$

where C_m is the membrane capacitance (see numerical values in table 1), g_m is the membrane leak conductance, V_L is the resting potential, and I_{syn} is the synaptic current.

The synaptic current includes glutamatergic excitatory components (mediated by AMPA and NMDA receptors) and inhibitory components (mediated by GABA). External cells contribute to the current only trough AMPA receptors. The total current is

$$I_{\text{syn}}(t) = I_{\text{AMPA,ext}}(t) + I_{\text{AMPA,rec}}(t) + I_{\text{NMDA}}(t) + I_{\text{GABA}}(t),$$

where

$$I_{\text{AMPA,ext}}(t) = g_{\text{AMPA,ext}}(V(t) - V_E) \sum_{j=1}^{N_{\text{ext}}} s_j^{\text{AMPA,ext}}(t),$$

$$I_{\text{AMPA,rec}}(t) = g_{\text{AMPA,rec}}(V(t) - V_E) \sum_{j=1}^{N_E} w_j s_j^{\text{AMPA,rec}}(t),$$

$$I_{\text{NMDA}}(t) = \frac{g_{\text{NMDA}}(V(t) - V_E)}{1 + \gamma \exp(-\beta V(t))} \sum_{j=1}^{N_E} w_j s_j^{\text{NMDA}}(t),$$

$$I_{\text{GABA}}(t) = g_{\text{GABA}}(V(t) - V_I), \sum_{j=1}^{N_I} s_j^{\text{GABA}}(t),$$

with w_j being the synaptic weights, s_j^x the fraction of open channels for each receptor, and g^x the synaptic conductance for receptor $x =$ AMPA, NMDA, GABA. The values for the synaptic conductances and the reversal potentials V_E and V_I are given in table 1. NMDA currents are voltage dependent and controlled by the intracellular magnesium concentration ($[\text{Mg}^{2+}] = 1\,\text{mM}$), with parameters $\gamma = [\text{Mg}^{2+}]/(3.57\,\text{mM}) = 0.280$ and $\beta = 0.062\,(\text{mV})^{-1}$.

The fraction of open channels in cell j, for all receptors, is described by the following differential equations:

$$\dot{s}_j^{\text{AMPA,ext}}(t) = -s_j^{\text{AMPA,ext}}(t)/\tau_{\text{AMPA}} + \sum_k \delta(t - t_j^k),$$

$$\dot{s}_j^{\text{AMPA,rec}}(t) = -s_j^{\text{AMPA,rec}}(t)/\tau_{\text{AMPA}} + \sum_k \delta(t - t_j^k - \delta),$$

$$\dot{s}_j^{\text{NMDA}}(t) = -s_j^{\text{NMDA}}(t)/\tau_{\text{NMDA,decay}} + \alpha x_j(t)(1 - s_j^{\text{NMDA}}(t)),$$

$$\dot{x}_j(t) = -x_j(t)/\tau_{\text{NMDA,rise}} + \sum_k \delta(t - t_j^k - \delta),$$

$$\dot{s}_j^{\text{GABA}}(t) = -s_j^{\text{GABA}}(t)/\tau_{\text{GABA}} + \sum_k \delta(t - t_j^k - \delta),$$

where the rise time constant for NMDA currents is $\tau_{\text{NMDA,rise}} = 2$ ms, and $\alpha = 1/\tau_{\text{NMDA,rise}} = 0.5(\text{ms})^{-1}$; rise time constant for AMPA and GABA currents are neglected. Decay time constants for AMPA, NMDA, and GABA synapses are $\tau_{\text{AMPA}} = 2$ ms, $\tau_{\text{NMDA,decay}} = 100$ ms, and $\tau_{\text{GABA}} = 10$ ms. The sums over k represent a sum over spikes emitted by pre-synaptic neuron j at time t_j^k. Transmission delays are set to $\delta = 0.5$ ms in all channels.

References

1. Brunel N., Wang X.J. Effects of neuromodulation in a cortical network model of object working memory dominated by recurrent inhibition. *J. Comput. Neurosci.* **11**(1):63–85 (2001).
2. Wang X.J. Probabilistic decision making by slow reverberation in cortical circuits. *Neuron* **36**:955–968 (2002).

Table 1. Parameters used in the network of integrate-and-fire neurons.

Parameter	**Value**	
Network parameters:		
N_E: number of excitatory cells	800	
N_I: number of inhibitory cells	200	
N_{ext}: number of cells in the external module	800	
p: number of selective populations	2	
f: fraction of exc. cells in a particular selective population	0.15	
w_+: relative strength of single potentiated synapses	1.9	
ν_{ext}: spike rate at external synapse	2.4 kHz	
Neuronal parameters:	**excitatory**	**inhibitory**
V_L: leak reversal potential	−70 mV	
V_{thr}: firing threshold	−50 mV	
V_r: reset potential	−55 mV	
C_m: membrane capacitance	0.5 nF	0.2 nF
g_m: membrane leak conductance	25 nS	20 nS
V_E/V_I: resting potential (excitatory/inhibitory)	0 mV	−70 mV
τ_{rp}: refractory period	2 ms	1 ms
Synaptic parameters:	**excitatory**	**inhibitory**
δ: transmission delay	0.5 ms	0.5 ms
$g_{\text{AMPA,ext}}$: external AMPA synaptic conductance	2.08 nS	1.62 nS
$g_{\text{AMPA,rec}}$: recurrent AMPA synaptic conductance	104 pS	81 pS
g_{NMDA}: recurrent NMDA synaptic conductance	327 pS	258 pS
g_{GABA}: recurrent GABA synaptic conductance	1250 pS	973 pS
γ: modulatory factor of magnesium blockade	0.280	
β: gain factor in magnesium blockade	1/(16.1 mV)	
τ_{AMPA}	2 ms	
τ_{GABA}	10 ms	
$\tau_{\text{NMDA,decay}}$	100 ms	
$\tau_{\text{NMDA,rise}}$	2 ms	
α	$\tau^{-1}_{\text{NMDA,rise}} = 0.5\ (\text{ms})^{-1}$	