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Fayet-Iliopoulos terms in 5d theories and their phenomenological implications

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Abstract

Supersymmetric theories in 5d orbifold can have Fayet-Iliopoulos (FI) terms on the boundaries. These terms induce a 5d mass for the bulk matter fields. We analyze this effect using $N = 1$ superfields and show how it can be understood as a mixing between the U(1) gauge boson and the graviphoton. We also study the implications of the FI-terms in the mass spectrum of theories with Scherk-Schwarz supersymmetry breaking. We present a new 5d supersymmetric model that has the massless spectrum of the Standard Model and predicts a Higgs boson with a mass around the present experimental limit.

1 Introduction

The presence of compact extra dimensions allows for new mechanisms of symmetry breaking. One of the simplest idea consists in breaking symmetries by boundary conditions [1]. Recently there has been a renewed interest to apply this idea to obtain the Standard Model (SM) from grand unified theories or supersymmetric theories in higher dimensions.

Here we want to consider the effect of Fayet-Iliopoulos (FI) terms in 5d models. It was shown in Ref. [2] that FI-terms can be present on the boundaries of a 5d orbifold leading to 5d odd masses for the hypermultiplets. We will study this effect in more detail using $N = 1$ superfields [2, 3, 4] and introducing the radion superfield T. This allows to make contact with supergravity and show that the effect of the FI-term can be understood as a mixing between the graviphoton and U(1) gauge boson. We will also show how this mixing can be generated at the one-loop level.

We also want to analyze the modifications in the mass spectrum of the hypermultiplet due to the existence of a 5d odd mass (generated by the FI-term). We will consider the case where supersymmetry is broken by the Scherk-Schwarz mechanism. The presence of 5d odd masses leads to interesting variations on previous models [5, 6, 7]. These effects were first discussed in Ref. [8, 9] where it was shown that the massless spectrum can be modified becoming sensitive to the ultraviolet cutoff. We will present a new SS model where the effect of the 5d mass is used to explain the large ratio m_t/m_b. The massive spectrum is completely determined as a function of the radius R. Although large uncertainties do not allow to predict the scale of electroweak breaking, we find that the model predicts a Higgs boson of mass close to the present experimental bound.

Some of the issues studied here were also discussed recently in Refs. [10, 11].

2 FI-terms in 5d supersymmetric theories

FI-terms are tadpoles of the auxiliary fields of an abelian gauge multiplet. In 4d supersymmetric theories these terms are very important since their presence implies a breaking of either supersymmetry or the gauge symmetry. In 4d they arise from the operator $\int d^4\theta\, V$ where V is the gauge superfield. This operator is invariant under global supersymmetry and gauge symmetry. Nevertheless, if supersymmetry is local (supergravity) such a term can only be gauge invariant if the gauge symmetry is an R-symmetry.

Here we want to analyze the role of FI-terms in 5d theories. For that purpose it is useful to work with $N = 1$ superfields [2, 3, 4]. Following Ref. [3], we can write the 5d supersymmetric abelian theory as

$$S_5 = \int d^5x \left[\frac{1}{4g_5^2} \int d^2\theta \, T W^\alpha W_\alpha + \text{h.c.} + \frac{2}{g_5^2} \int d^4\theta \, \frac{1}{(T+T^\dagger)} \left(\partial_5 V - \frac{1}{\sqrt{2}} (\chi + \chi^\dagger) \right)^2 \right] , \quad (1)$$

where V is a $N=1$ vector supermultiplet and χ a chiral supermultiplet:

$$\begin{aligned} V &= -\theta \sigma^\mu \bar{\theta} A_\mu - i \bar{\theta}^2 \theta \lambda_1 + i \theta^2 \bar{\theta} \bar{\lambda}_1 + \frac{1}{2} \bar{\theta}^2 \theta^2 D \, , \\ \chi &= \frac{1}{\sqrt{2}} \left(\Sigma + i A_5 \right) + \sqrt{2} \theta \lambda_2 + \theta^2 F_\chi \, . \end{aligned} \quad (2)$$

V is given in the Wess-Zumino gauge. Both V and χ form the 5d gauge superfield. Under a gauge transformation, the superfields transform as

$$\begin{aligned} V &\to V + \Lambda + \Lambda^\dagger \, , \\ \chi &\to \chi + \sqrt{2} \partial_5 \Lambda \, , \end{aligned} \quad (3)$$

where Λ is an arbitrary chiral field. The chiral superfield T is the radion multiplet

$$T = R + i B_5 + \theta \Psi_R^5 + \theta^2 F_T \, . \quad (4)$$

It is useful to introduce T to define the 5d gravitational background on which the superfields propagate. It also helps to make the connection with supergravity. Here we are considering that the extra dimension is flat and compact:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2 \, . \quad (5)$$

We leave the case of warped extra dimensions for further publication.

If the theory is compactified in a circle S^1, $0 \le y \le 2\pi$, the following term can also be present in the 5d lagrangian

$$\int d^2\theta \, \chi + \text{h.c.} \, . \quad (6)$$

This term is a tadpole for the auxiliary field F_χ and therefore corresponds to a FI-term in 5d. Notice that if this term is zero in the original theory, it cannot be generated by radiative corrections since it is protected by supersymmetry.

If the extra dimension is compactified in the orbifold $S^1/\mathbb{Z}_2$, $0 \le y \le \pi$, the term (6) is not allowed by gauge invariance. In this case, however, another tadpole term can be present in the 5d lagrangian:

$$-\int d^4\theta \, \xi \, \epsilon(y) \Big[\partial_5 V - \frac{1}{\sqrt{2}} \left(\chi + \chi^\dagger \right) \Big] \, , \quad (7)$$

where $\epsilon(y)$ is a step function, $\epsilon(y) = 1(-1)$ for positive (negative) y. Notice that the presence of χ is necessary to make Eq. (7) gauge invariant under Eq. (3). Integrating by parts, we can write the tadpole of V in Eq. (7) as

$$\int d^4\theta \, 2\xi \left[\delta(y) - \delta(y-\pi)\right] V \,. \tag{8}$$

If the term Eq. (8) is not present in the original theory, it can be generated at the one-loop level. Let us write the induced FI-term as

$$\int d^4\theta \, 2\xi_{in}(y) \, V \,. \tag{9}$$

We will see that Eq. (9) can be generated by either boundary or bulk fields. They give a divergent contribution that renormalizes the operator of Eq. (8).

Boundary fields: Given a field Q of charge q_Q localized on the boundary at $y = y^*$

$$S_5 = \int d^5x \int d^4\theta \, Q^\dagger e^{q_Q V} Q \, \delta(y - y^*) \,, \tag{10}$$

we have (see diagram of Fig. 1) the contribution

$$\xi_{in}(y) = q_Q \frac{\Lambda^2}{16\pi^2} \delta(y - y^*) \,. \tag{11}$$

This contribution is ultraviolet divergent and depends quadratically on the cutoff Λ.

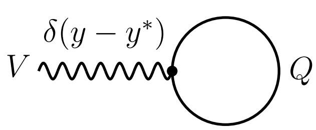

Figure 1

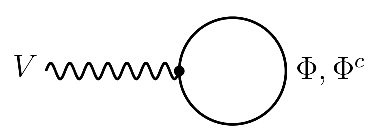

Figure 2

Bulk fields: We will consider a charged hypermultiplet [3]

$$S_5 = \int d^5x \left\{ \int d^4\theta \, \frac{1}{2}(T + T^\dagger) \left(\Phi^\dagger e^{q_\Phi V} \Phi + \Phi^c e^{-q_\Phi V} \Phi^{c\,\dagger}\right) + \int d^2\theta \, \Phi^c \left[\partial_5 + \frac{q_\Phi}{\sqrt{2}} \chi\right] \Phi + \text{h.c.} \right\} \,, \tag{12}$$

with two possible boundary conditions for the superfields: [1]

[1]In the $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ orbifold [7] the case (a) corresponds to the parity assignment $++$ and $--$ to Φ and Φ^c respectively, while case (b) corresponds to the parity assignment $+-$ and $-+$.

a) Φ is even with respect to the two boundaries at $y=0$ and $y=\pi$, while Φ^c is odd. We will denote them by Φ^{++} and $\Phi^{c\,--}$.

b) Φ is even (odd) with respect to the boundary at $y=0$ ($y=\pi$), while Φ^c is odd (even). We will denote them by Φ^{+-} and $\Phi^{c\,-+}$.

The cases (a) and (b) correspond to impose respectively periodic and antiperiodic boundary conditions for the superfields under a 2π rotation of the extra dimension. Let us now calculate the contribution from Φ and Φ^c (Fig. 2) to the operator of Eq. (9). This is given by

$$\xi_{in}(y) = q_\Phi R \int \frac{d^4p}{(2\pi)^4} \left[G_p^{\phi}(y,y) - G_p^{\phi^c}(y,y) \right] , \tag{13}$$

where $G_p^{\phi,\phi^c}(y,y)$ are the scalar propagators of the hypermultiplet. It is instructive (to understand the origin of $\xi_{in}(y)$) to calculate these propagators starting from an infinite extra dimension. In this case the propagator of a 5d scalar with 4d Euclidean momentum p is given by [2]

$$G_p(y,y') = \frac{e^{-p|y-y'|R}}{2p} . \tag{14}$$

Compactifying in a circle means identifying $y \leftrightarrow y+2\pi n$ where $n\in Z$. Therefore in the tadpole contribution (Fig. 2) the fields can propagate from y to $y+2\pi n$. If we now orbifold the space, we identify $y \leftrightarrow -y$. The fields can propagate from y to $-y+2\pi n$. Therefore the propagator in Eq. (13) reads, for the case (a),

$$G_p^{\phi^{++},\phi^{c\,--}}(y,y) = \sum_{n=-\infty}^{\infty} \Big[G_p(y, y+2\pi n) \pm G_p(y,-y+2\pi n) \Big] . \tag{15}$$

Only the second term in Eq. (15) gives a nonzero contribution to Eq. (13). This is given by

$$\xi_{in}(y) = q_\Phi R \sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{e^{-2p|y-\pi n|R}}{p} . \tag{16}$$

This contribution is convergent away from the fixed points at $y=n\pi$ due to the exponential suppression. At the fixed points $y=n\pi$, however, the integral diverges. The divergence depends on how we regularize the boundaries. In the thin-boundary limit, we get for the divergent contribution (restricting now to the interval [0,π]):

$$\xi_{in}(y) = q_\Phi \frac{\Lambda^2}{32\pi^2} \Big[\delta(y) + \delta(y-\pi) \Big] + q_\Phi \frac{\ln(\Lambda/\mu)}{64\pi^2 R^2} \Big[\delta''(y) + \delta''(y-\pi) \Big] , \tag{17}$$

[2] Here we include R to keep y dimensionless.

where μ is an infrared cutoff. For the case (b), where the fields are antiperiodic under a 2π rotation, a factor $(-1)^n$ must multiply the propagator

$$G_p^{\phi^{+-},\phi^{c-+}}(y,y) = \sum_{n=-\infty}^{\infty} (-1)^n \Big[G_p(y, y+2\pi n) \pm G_p(y, -y+2\pi n) \Big] \,, \tag{18}$$

and, consequently, also in the induced FI-term. Restricted to the interval $[0,\pi]$, we have

$$\xi_{in}(y) = q_\Phi \frac{\Lambda^2}{32\pi^2} \Big[\delta(y) - \delta(y-\pi) \Big] + q_\Phi \frac{\ln(\Lambda/\mu)}{64\pi^2 R^2} \Big[\delta''(y) - \delta''(y-\pi) \Big] \,. \tag{19}$$

Only Eq. (11) and the first term of Eqs. (17) and (19) (the quadratic divergent part) contribute to the renormalization of the operator Eq. (8). Nevertheless, these contributions do not seem to have the advocated form Eq. (8). This is because we have not yet demanded that the theory is anomaly free. Demanding that the effective 4d theory is free of gravitational anomalies requires $\sum_i q_i = 0$, where the sum is over the massless spectrum. In 5d this implies

$$\sum_i q_{Q_i} + \sum_i q_{\Phi_i^{++}} = 0 \,. \tag{20}$$

The condition Eq. (20) leads to a total FI-term (sum of Eq. (11) and the first term of Eqs. (17) and (19)) of the form Eq. (8). Notice that there is no restriction on the charges of the hypermultiplet Φ^{+-} and $\Phi^{c\,-+}$, since they do not contain massless states (and therefore they do not contribute to the 4d gravitational anomaly). The absence of 4d anomalies does not guarantee, in general, a 5d anomaly-free theory [12, 10]. Nevertheless the anomalies of the 5d theory can always be cancelled if a proper Chern-Simons term is added [12, 10].

The second term of Eqs. (17) and (19) that diverges logarithmically and involve two derivatives with respect to the extra dimension renormalizes the operator

$$\int d^4\theta \, \frac{\hat{\xi}_i}{|T|^2} \partial_5 \Big[\partial_5 V - \frac{1}{\sqrt{2}} \left(\chi + \chi^\dagger \right) \Big] \delta(y - y_i^*) \,, \tag{21}$$

where $y_i^* = 0, \pi$. Eq. (21) is a brane operator subleading with respect to Eq. (8) in an expansion in derivatives ∂_5^2/Λ^2. We will only be interested in the phenomenological implications of Eq. (8) since it is more sensitive to the cutoff scale.

3 Phenomenological implications of the FI-terms

What are the implications of the existence of the FI-term Eq. (8)? Contrary to the 4d case, the presence of this term does not necessary lead to either supersymmetry or gauge symmetry breaking [2]. This is because the field equation of the auxiliary field D is given by

$$D = -\frac{\partial_5 \Sigma}{R^2} - \frac{g_5^2}{R}\xi\Big[\delta(y) - \delta(y-\pi)\Big]\,, \tag{22}$$

and the singlet Σ can adjust to guarantee $D = 0$ (a supersymmetric vacuum):

$$\langle\Sigma\rangle = -g_5^2 R\xi \int dy\Big[\delta(y) - \delta(y-\pi)\Big] = -\frac{g_5^2 R}{2}\xi\epsilon(y)\,. \tag{23}$$

The FI-terms induce a step-function profile for Σ. Since Σ couples to the charged hypermultiplets, these fields get a 5d mass given by $q_\Phi\langle\Sigma\rangle/2$ that, written in superfields, corresponds to a T-dependent mass:

$$-\int d^2\theta\, M\,\epsilon(y)\,T\Phi^c\Phi + \text{h.c.}\,, \qquad M = \frac{q_\Phi g_5^2}{4}\xi\,. \tag{24}$$

This is the net result of the FI-terms Eq. (8) in 5d orbifolds. From Eq. (4) we see that T contains the fifth-component of the graviphoton B_5. This implies that a hypermultiplet with an induced 5d mass is coupled to the graviphoton. This is expected since a 5d mass is possible in 5d supergravity if the hypermultiplet is charged under the graviphoton. Therefore the effect of the FI-term can also be understood as a mixing between the graviphoton B_M and the U(1)-photon A_M $[M = (\mu, 5)]$. This can be explicitly seen in Eq. (1) where a $\langle\Sigma\rangle \neq 0$ leads to a mixing between the kinetic term of A_5 and B_5. Going to the canonical basis corresponds to redefine A_5 that in superfields means $\chi \to \chi - (\sqrt{2}M\epsilon(y)/q_\Phi)T$. One can check that by performing this redefinition in Eq. (1), the FI-term Eq. (7) can be eliminated from the theory. The mixing between A_μ and B_μ comes from the induced operator

$$\int d^2\theta\, \xi\epsilon(y)\,TW_B^\alpha W_\alpha + \text{h.c.}\,, \tag{25}$$

where W_B^α is the superfield strength of the graviphoton B_μ (up to some normalization factor). This operator contains a Chern-Simons term $\varepsilon_{5\mu\nu\rho\sigma}B^5 F_B^{\mu\nu}F_A^{\rho\sigma}$ that is known not to be renormalized beyond one-loop. One can then expect that the FI-term will be only renormalized at the one-loop level [10].

To analyze the implications of the FI-terms, we must study the effects of adding a 5d odd mass to the hypermultiplets. The equations of motion of the scalar fields of the hypermultiplet are given by

$$\Big(\frac{1}{R^2}\partial_5^2 + \partial_\mu^2 - M^2 - 2\frac{M}{R}\big[\delta(y) - \delta(y-\pi)\big]\Big)\phi = 0\,, \tag{26}$$

$$\Big(\frac{1}{R^2}\partial_5^2 + \partial_\mu^2 - M^2 + 2\frac{M}{R}\big[\delta(y) - \delta(y-\pi)\big]\Big)\phi^c = 0\,. \tag{27}$$

To obtain the 4d mass spectrum, we can calculate their propagators and look at their poles. These are given in the Appendix. For ϕ^{++}, we obtain a massless state of wave-function

$$f(y) = \frac{1}{N_0} e^{MRy} , \tag{28}$$

where N_0 is a normalization constant. For negative (positive) M, this mode is localized toward the boundary at $y = 0$ ($y = \pi$). The Kaluza-Klein (KK) spectrum have masses $m_n^2 = M^2 + n^2/R^2$. This mass spectrum is paired up with that of $\phi^{c\,--}$.

For the hypermultiplet ϕ^{+-}, we do not have massless states. The KK states have masses $m_n^2 = q_n^2 + M^2$, where q_n is given by $M \tan(q_n \pi R) = -q_n$. For $-M\pi R > 1$, there is an imaginary solution for q_n that, in the limit $-MR\pi \gg 1$, leads to a KK mass that is exponentially suppressed

$$m^2 \simeq 4M^2 e^{2M\pi R} . \tag{29}$$

The full mass spectrum is paired up with that of $\phi^{c\,-+}$. From Eq. (29) we see an interesting effect. Large 5d odd masses can lead to new light states in the 4d theory.

4 Implications in theories with Scherk-Schwarz supersymmetry breaking

In this section we want to analyze the implications of a 5d odd mass in theories of Scherk-Schwarz (SS) supersymmetry breaking [1]. The SS mechanism in superfields consists in breaking supersymmetry by turning on a nonzero F-term for the radion T in Eqs. (1), (12) and (24). This gives new mass terms to the hypermultiplet scalars. To derive the mass spectrum, it is more convenient to absorb these mass terms by redefining the scalar fields (but not the fermion fields):

$$\begin{pmatrix} \phi_{SS} \\ \phi^{c*}_{SS} \end{pmatrix} = e^{-iF_T \sigma_2 y/2} \begin{pmatrix} \phi \\ \phi^{c*} \end{pmatrix} . \tag{30}$$

We will consider the particular case $F_T = 1$ (this is the value determined dynamically [13]). The boundary terms are not in general invariant under the redefinition Eq. (30). For example at $y = \pi$, where Eq. (30) gives

$$\phi_{SS}(\pi) = -\phi^{c*}(\pi) , \qquad \phi^{c*}_{SS}(\pi) = \phi(\pi) , \tag{31}$$

we have that a boundary mass term becomes

$$M\delta(y-\pi)(|\phi|^2 - |\phi^c|^2) \ \rightarrow\ M\delta(y-\pi)(|\phi^c_{SS}|^2 - |\phi_{SS}|^2) . \tag{32}$$

The scalar equations of motion are therefore given by

$$\Big(\frac{1}{R^2}\partial_5^2+\partial_\mu^2-M^2-2\frac{M}{R}\big[\delta(y)+\delta(y-\pi)\big]\Big)\phi_{SS}=0\,, \tag{33}$$

$$\Big(\frac{1}{R^2}\partial_5^2+\partial_\mu^2-M^2+2\frac{M}{R}\big[\delta(y)+\delta(y-\pi)\big]\Big)\phi^c_{SS}=0\,. \tag{34}$$

Eq. (31) also tell us that ϕ_{SS} must have at $y=\pi$ the boundary condition of that of $\phi^{c\,*}$, and ϕ^c_{SS} the boundary condition at $y=\pi$ of that of ϕ^*. For the hypermultiplet of type (a), this means that ϕ_{SS} (ϕ^c_{SS}) is odd (even) with respect to the boundary at $y=\pi$. We will denote them as ϕ^{+-}_{SS} and $\phi^{c\,-+}_{SS}$. For the hypermultiplet (b), ϕ_{SS} (ϕ^c_{SS}) is even (odd) with respect to the boundary at $y=\pi$. We will denote them as ϕ^{++}_{SS} and $\phi^{c\,--}_{SS}$.

The scalar ϕ^{++}_{SS} has a mass spectrum given by the poles of its propagator calculated in the Appendix. There is no massless state. For $|M|R\pi\ll 1$ we have the 4d mass-squared spectrum

$$\frac{2M}{\pi R},\ \frac{1}{R^2},\ \frac{2^2}{R^2},\dots, \tag{35}$$

and for $-MR\pi\gg 1$

$$4M^2e^{MR\pi},\ M^2+\frac{1}{R^2},\ M^2+\frac{2^2}{R^2},\dots\,. \tag{36}$$

Therefore in these two limits, we have a light scalar (of mass smaller than $1/R$). The spectrum of the other scalars, ϕ^{--}_{SS}, ϕ^{-+}_{SS} and ϕ^{+-}_{SS} is given in the Appendix.

4.1 A new Scherk-Schwarz model of electroweak symmetry breaking

Models of SS supersymmetry breaking have been subject of recent research [5, 6, 7]. Nevertheless, they present some phenomenological difficulties, either because they predict a Higgs too light [6] or a compactification scale too low [7].

The presence of FI-terms (that, as we said, results in 5d odd masses for the hypermultiplets) allows to construct new models of SS supersymmetry breaking. Here we will present a new 5d supersymmetric model similar to those of Refs. [6, 7] but with the advantage of being able to accommodate a larger R^{-1} and a Higgs boson of mass close to the experimental bound. The model consists in a 5d supersymmetric extension of the SM. The extra dimension will be compactified in a $S^1/\mathbb{Z}_2$ orbifold. The gauge bosons will be living in the 5d bulk, belonging to a 5d vector supermultiplet. As in Ref. [7], the Higgs boson superfields, $\Phi=\{\phi,\widetilde{\phi}\}$ and $\Phi^c=\{\phi^c,\widetilde{\phi}^c\}$, will belong to a 5d hypermultiplet of type (b) with zero 5d mass. We will discuss later the implication of a 5d mass. For simplicity, we will only consider the third family

5d field	Boundary at $y=0$	Boundary at $y=\pi$
A_μ, q_3, ϕ	$+$	$+$
Σ, A_5, q_3^c, ϕ^c	$-$	$-$
λ_1, $\widetilde{Q}_3$, $\widetilde{\phi}$	$+$	$-$
λ_2, $\widetilde{Q}_3^c$, $\widetilde{\phi}^c$	$-$	$+$

Table 1 : Boundary conditions, even (+) or odd (−), for the bulk superfields.

of quarks. The left-handed quark superfield $Q_3 = \{q_3, \widetilde{Q}_3\}$ will be assumed to live in the bulk in a hypermultiplet of type (a) and 5d mass M_3. The right-handed quark superfields, U_3 and D_3, will be respectively localized on the boundaries at $y = 0$ and $y = \pi$ [3]. This allows the Yukawa couplings

$$\mathcal{L}_Y = \int d^2\theta \left[Y_t\, \Phi Q_3 U_3 \delta(y) + Y_b\, \Phi^c Q_3 D_3 \delta(y-\pi) \right] + \text{h.c.}\,. \tag{37}$$

We will break supersymmetry by the SS mechanism. This means, as said above, a change of boundary conditions for the hypermultiplet scalars according to Eq. (31). We present in Table 1 the resulting boundary conditions of the bulk fields. The Higgs hypermultiplet will have a massless scalar, H, that we will associate to the SM Higgs. Using Eq. (28) and (37), we can deduce the top and bottom mass ratio

$$\frac{m_t}{m_b} = \frac{Y_t f_{q_3}(0) f_H(0)}{Y_b f_{q_3}(\pi) f_H(\pi)} = \frac{Y_t}{Y_b} e^{-M_3 \pi R}\,, \tag{38}$$

where f_{q_3} is the wave-function of the zero-mode left-handed quark (given by Eq. (28)) and $f_H = 1/\sqrt{\pi R}$ is the wave-function of H. From the experimental value $m_t/m_b \simeq 60$, we determine $M_3 \simeq -1.3/R$ for $Y_t \simeq Y_b$. Therefore the effect of the 5d mass M_3 is to localize the left-handed quark toward the boundary at $y = 0$ and, as a consequence, give a large m_t/m_b ratio. Fixed M_3, the model has only one parameter R. All the mass spectrum is determined as a function of R. Also the SM Higgs potential

$$V(H) = m_H^2 H^2 + \lambda H^4\,, \tag{39}$$

is determined as a function of R. Let us calculate it. Contributions to m_H^2 arise at the loop level from gauge and top interactions. They can be calculated following Ref. [6]. We obtain

$$\Delta m_H^2 = \left[3g_2^2 + \frac{3}{5} g_1^2 \right] \Pi_V + 6 h_t^2\, \Pi_{Q_3}\,, \tag{40}$$

[3] They could also be considered to be living in the bulk with large 5d supersymmetric masses such that, as we said above, localize the zero mode toward the corresponding boundaries.

where g_i and h_t are the 4d gauge and top couplings and

$$\Pi_V = \pi R \int \frac{d^4p}{(2\pi)^4} \left[G_V^{++} - G_V^{+-}\right] , \tag{41}$$

$$\Pi_{Q_3} = \frac{1}{f_{q_3}^2(0)} \int \frac{d^4p}{(2\pi)^4} \left[G_{Q_3}^{+-} - G_{Q_3}^{++}\right] , \tag{42}$$

and the propagators G_V and G_{Q_3} are given in the Appendix. In the limit $-M_3 \gg 1/(\pi R)$, the top contribution becomes small. This is because in this limit the KK states of Q_3 become infinitely massive and only a massless chiral superfield, localized on the boundary, remains in the theory. For our value $M_3 \simeq -1.3/R$ we are close to this regime. Hence the quark sector of our model is similar to that of Ref. [6] where the quarks are localized on the boundary. It is known that in this case one-loop corrections give masses to the squark on the boundary

$$m_{\tilde{Q}_3}^2 = \left[\frac{16}{3}g_3^2 + 3g_2^2 + \frac{1}{15}g_1^2\right] \Pi_V + 2h_t^2 \Pi_V , \tag{43}$$

$$m_{\tilde{U}_3}^2 = \left[\frac{16}{3}g_3^2 + \frac{16}{15}g_1^2\right] \Pi_V + 4h_t^2 \Pi_V , \tag{44}$$

and, at the two-loop level, a sizeable contribution to m_H^2 [6]:

$$\Delta m_H^2 \simeq \frac{3h_t^2}{16\pi^2}(m_{\tilde{Q}_3}^2 + m_{\tilde{U}_3}^2) \ln[R^2 m_{\tilde{Q}_3}^2] . \tag{45}$$

Up to now we have assumed that the 5d mass of the Higgs hypermultiplet is zero. Nevertheless, we already saw that it can be generated by radiative corrections due to a FI-term of the $\mathrm{U}(1)_Y$. From Eqs. (24) and (35) we see that a 5d mass for the Higgs hypermultiplet implies a mass for H. This is given by

$$\Delta m_H^2 \simeq \frac{2M}{\pi R} = \frac{g_1^2 Y_H}{2}\xi \simeq \frac{g_1^2 Y_H}{32\pi^2}\Lambda^2 , \tag{46}$$

where Y_H is the Higgs hypercharge. This contribution depends on the ultraviolet cutoff Λ and therefore cannot be predicted. For $\Lambda \sim 1/R$ this is around a 25% of the others. This tell us that, if this contribution is present, we will have large uncertainties in our calculation of the electroweak scale. Nevertheless, we must say that since the presence of a 5d mass M for the Higgs is related to the anomalies of the theory, it could be possible that the contribution Eq. (46) is zero, as it happens in certain models [6].

Finally, the quartic coupling in $V(H)$ is fixed by supersymmetry to be $\lambda = (g_2^2 + 3g_1^2/5)/8$. It also receives sizeable loop corrections that in our case, where $M_3 \simeq -1.3/R$, are approximately given by

$$\Delta\lambda = \frac{3h_t^4}{16\pi^2} \ln\left(\frac{m_{\tilde{Q}_3}^2 + m_t^2}{m_t^2}\right) . \tag{47}$$

Combining all the contributions to the Higgs potential, we can study the breaking of the electroweak symmetry. We see that the negative contribution to m_H^2, Eq. (45), dominates over the positive gauge contribution triggering the electroweak symmetry breaking. Fixing the vacuum expectation value of H to be 174 GeV and neglecting Eq. (46), we obtain a value of $R^{-1} = 3 - 4$ TeV. This large value of the compactification scale $1/R$ guarantees small corrections (from the KK) to the electroweak observables. This prediction, however, turns to be very sensitive to the value of m_t and g_3 which have large experimental uncertainties. This is because the positive and negative contributions to m_H^2 are very close in size, cancelling each other for some values of the top mass inside the experimental window. This accidental cancellation does not allow us to get a precise determination of R (even when Eq. (46) is neglected). Nevertheless, the physical Higgs mass is not very much sensitive to R and can be determined with more precision. We obtain $m_{Higgs} \simeq 110$ GeV, with a theoretical uncertainty that we estimate of $5 - 10$ GeV. We see then that the Higgs is around the present experimental bound. Therefore this model can be confirmed or excluded in the near future.

5 Conclusions

We have studied the effect of FI-terms in 5d orbifold theories. These terms can be induced at the one-loop level by boundary or bulk fields. Their form is dictated by the gauge symmetry and supersymmetry and can be easily derived using $N = 1$ superfields. By introducing the radion superfield T we were able to understand the effect of the FI-term as a mixing between the graviphoton and the U(1) gauge boson. In superfields this corresponds to a mixing between χ and T, and between W^α and W_B^α. This allows us to relate the FI-term to the Chern-Simons term graviphoton-graviphoton-photon [10]. The net effect of the FI-term is to induce 5d odd masses for the hypermultiplets.

We have then analyzed how the 4d mass spectrum is modified if hypermultiplets have 5d odd masses. For large values of these masses, we have seen that new light states can be present in the effective 4d theory. We have also considered the modifications in the mass spectrum of theories with Scherk-Schwarz supersymmetry breaking. The existence of 5d odd masses allows for variations on previous models [6, 7]. We have presented a new model where the left-handed quark is in the bulk and has a 5d mass M_3. The value of M_3 can be fixed to explain the m_t/m_b ratio. The model has only one parameter R and it is a priori very predictive. We find however accidental cancellations in the contributions to the Higgs potential that makes difficult to determine the electroweak scale. Higher loop contributions must be considered. Nevertheless, the massive spectrum is completely determined as a function of R. The massless spectrum corresponds to the one of the SM. The model also predicts a Higgs boson with a mass around the present experimental bound.

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Appendix. Propagators of the 5d fields

In this Appendix we will derive the propagators of the hypermultiplet scalars, ϕ and ϕ^c, with a 5d odd mass $M\epsilon(y)R$. Some of them have also been derived in Ref. [10].

1) Supersymmetric case: The propagators in the mixed position-momentum space are defined by

$$\Big(\frac{1}{R^2}\partial_5^2 - p^2 - M^2 - 2\frac{M}{R}\big[\delta(y)-\delta(y-\pi)\big]\Big)G_p^{\phi}(y,y') = -\frac{1}{R}\delta(y-y')\,, \tag{48}$$

$$\Big(\frac{1}{R^2}\partial_5^2 - p^2 - M^2 + 2\frac{M}{R}\big[\delta(y)-\delta(y-\pi)\big]\Big)G_p^{\phi^c}(y,y') = -\frac{1}{R}\delta(y-y')\,, \tag{49}$$

where p is the Euclidean 4d momentum. Below we will show the propagators for the scalar ϕ for different boundary conditions. The propagators for ϕ^c can be obtained from those of ϕ just by replacing $M \to -M$.

Even-Even: For the case (a) defined in section 2 the scalar ϕ is even at both boundaries, and therefore the two source terms in (48) will affect the propagator behaviour through boundary conditions. The propagator is given by

$$G_p^{\phi^{++}}(y,y') = \frac{\cosh\big[p_M R y_<\big] + \frac{M}{p_M}\sinh\big[p_M R y_<\big]}{p_M\Big([1-M^2/p_M^2]\sinh\big[p_M\pi R\big]\Big)} \times\left(\cosh\big[p_M R(\pi-y_>)\big] - \frac{M}{p_M}\sinh\big[p_M R(\pi-y_>)\big]\right), \tag{50}$$

with $p_M \equiv \sqrt{p^2+M^2}$ and where $y_<$ ($y_>$) denote the lesser (greater) of y and y'. The pole at $p_M = M$ corresponds to the massless mode, with a wave-function given by

$$f(y) = \big[\mathrm{Res}_{p_M^2=M^2}\, G_p(y,y)\big]^{1/2} = \frac{1}{\sqrt{(e^{2MR\pi}-1)/(2MR)}}e^{MRy}\,, \tag{51}$$

The poles at $p_M = in/R, n = 1,2,\dots$ lead to a spectrum of massive modes given by $m^2 = -p^2 = M^2 - p_M^2 = (n/R)^2 + M^2$.

Odd-Odd: In this case the propagator must vanish at both boundaries. We have

$$G_p^{\phi^{--}}(y,y') = \frac{\sinh\big[p_M y_<\big]\sinh\big[p_M R(\pi-y_>)\big]}{p_M\sinh\big[p_M\pi R\big]}\,. \tag{52}$$

The mass spectrum is the same as the massive sector of the even-even case.

Even-Odd: In the case (b), since ϕ (and $G_p^{\phi^{+-}}$) vanishes at the $y = \pi$ boundary it cannot feel the effect of the source term at $y = \pi$. Solving (48) with these boundary conditions gives rise to

$$G_p^{\phi^{+-}}(y,y') = \frac{\Big(\cosh\big[p_M R y_<\big] + \frac{M}{p_M}\sinh\big[p_M R y_<\big]\Big)\sinh\big[p_M R(\pi - y_>)\big]}{p_M\Big(\cosh\big[p_M \pi R\big] + \frac{M}{p_M}\sinh\big[p_M \pi R\big]\Big)} \,. \tag{53}$$

There are non-trivial poles located at the imaginary axis $p_M = iq_n$, where q_n are the solutions of $M\tan(q_n\pi R) = -q_n$. The mass spectrum is then given by $m_n^2 = q_n^2 + M^2$. When $-M\pi R > 1$ there is also a pole at the real axis. In the limit $-M\pi R \gg 1$ this corresponds to a state with an exponentially-suppressed mass, $m^2 \simeq 4M^2 e^{2M\pi R}$.

Odd-Even: We have

$$G_p^{\phi^{-+}}(y,y') = \frac{\sinh\big[p_M R y_<\big]\Big(\cosh\big[p_M R(\pi - y_>)\big] - \frac{M}{p_M}\sinh\big[p_M R(\pi - y_>)\big]\Big)}{p_M\Big(\cosh\big[p_M \pi R\big] - \frac{M}{p_M}\sinh\big[p_M \pi R\big]\Big)} \,, \tag{54}$$

For $\phi^{c\,-+}$ we must replace $M \to -M$ which leads to the same 4d mass spectrum as that of ϕ^{+-}.

2) Scherk-Schwarz supersymmetry breaking: The boundary mass terms at $y = \pi$ in the equations of motion change as a result of the SS mechanism as explained above. We have now

$$\Big(\frac{1}{R^2}\partial_5^2 - p^2 - M^2 - 2\frac{M}{R}\big[\delta(y) + \delta(y-\pi)\big]\Big)G_p^{\phi_{SS}}(y,y') = -\frac{1}{R}\delta(y-y') \,, \tag{55}$$

$$\Big(\frac{1}{R^2}\partial_5^2 - p^2 - M^2 + 2\frac{M}{R}\big[\delta(y) + \delta(y-\pi)\big]\Big)G_p^{\phi^c_{SS}}(y,y') = -\frac{1}{R}\delta(y-y') \,. \tag{56}$$

The only propagators that are modified with respect to the supersymmetric case are those which are even at $y = \pi$.

Even-Even:

$$G_p^{\phi_{SS}^{++}}(y,y') = \frac{\cosh\big[p_M R y_<\big] + \frac{M}{p_M}\sinh\big[p_M R y_<\big]}{p_M\Big(\big[1 + (M^2/p_M^2)\big]\sinh\big[p_M \pi R\big] + 2\frac{M}{p_M}\cosh\big[p_M \pi R\big]\Big)} \times \Big(\cosh\big[p_M R(\pi - y_>)\big] + \frac{M}{p_M}\sinh\big[p_M R(\pi - y_>)\big]\Big) \,. \tag{57}$$

There are poles at the real axis in the p_M complex plane, given by the solutions of the equation

$$\tanh(p_M \pi R) = -\frac{2p_M/M}{1+(p_M/M)^2} \,. \tag{58}$$

For $|M|R\pi \ll 1$ we get a scalar of mass-squared $m^2 \simeq 2M/(\pi R)$, while for $-MR\pi \gg 1$ there are two light scalars with mass-squared given by $m^2 \simeq \pm 4M^2 e^{M\pi R}$. There are also poles at the imaginary axis, placed at $p_M = iq_n$, where q_n is the solution to the equation $\tan(p_M\pi) = -(2M/p_M)/[1-(M/p_M)^2]$. For $MR\pi \ll 1$ the KK mass spectrum is $m_n^2 \sim n^2/R^2$.

Odd-Even:

$$G_p^{\phi_{SS}^{-+}}(y,y') = \frac{\sinh\big[p_M R y_<\big]\Big(\cosh\big[p_M R(\pi - y_>)\big] + \frac{M}{p_M}\sinh\big[p_M R(\pi - y_>)\big]\Big)}{p_M\Big(\cosh\big[p_M \pi R\big] + \frac{M}{p_M}\sinh\big[p_M \pi R\big]\Big)} \,. \tag{59}$$

The pole structure is the same as that in Eq. (53).

From the above propagators we can derive the propagators of the vector and quark sector that are used in section 4.1. They are simply given by $G_V^{++} = G_p^{\phi^{++}}$ and $G_V^{+-} = G_p^{\phi^{+-}}$ in the limit $M = 0$. For Q_3, we have $G_{Q_3}^{++} = G_p^{\phi^{++}}$ and $G_{Q_3}^{+-} = G_p^{\phi^{+-}}$ with $M = M_3$.

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