Spatial pattern vision and linear systems theory

- Receptive fields and neural images
- Shift-invariant linear systems and convolution
- Fourier transform and frequency response
- Applications of linear systems to spatial vision
  - Contrast sensitivity
  - Spatial frequency and orientation channels
  - Spatial frequency and orientation adaptation
  - Masking

Receptive field

- In any modality: that region of the sensory apparatus that, when stimulated, can directly affect the firing rate of a given neuron
- Spatial vision: spatial receptive field can be mapped in visual space or on the retina
- Examples:

  ![LGN](LGN.png) ![V1](V1.png)

Receptive field

A spatial receptive field is an image:
Neural image: retinal ganglion cell responses

Neural image: simple cell responses

Lots of neural images: V1 simple cells
Lots of neural images

Simple cells  Complex cells

Neural image of a sine wave

For a linear, shift-invariant system such as a linear model of a receptive field, an input sine wave results in an identical output sine wave, except for a possible lateral shift and scaling.

Frequency response

This scaling of contrast by a linear receptive field in the neural image is a function of spatial frequency determined by the shape of the receptive field.
Frequency response

This scaling of contrast by a linear receptive field in the neural image is a function of spatial frequency determined by the shape of the receptive field.

Orientation tuning

If a receptive field is not circularly symmetric, the scaling of contrast is also a function of orientation (for a given spatial frequency) determined by the shape of the receptive field.

Linear systems analysis

**Space/time method**

- Measure the impulse response
- Input stimulus
- Express as sum of shifted and scaled impulses
- Calculate the response to each impulse
- Sum the impulse responses

**Frequency method**

- Measure the frequency response
- Express as sum of shifted and scaled sinusoids
- Calculate the response to each sinusoidal scaled sinusoid
- Sum the sinusoidal responses
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**Functions and Systems**

A function: a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output.

**Examples**

- $f(x) = x^2$
- $f(x) = \cos(x)$
- $f(x,y) = \sqrt{x^2 + y^2}$
Linear Systems Analysis

Systems with signals as input and output

- 1-d: low- and high-pass filters in electronic equipment, fMRI data analysis, or in sound production (articulators) or audition (the ear as a filter)
  \[ y(t) = T\{x(t)\} \]
- 2-d: optical blur, spatial receptive field
  \[ g(x,y) = T\{f(x,y)\} \]
- 3-d: spatio-temporal receptive field
  \[ g(x,y,t) = T\{f(x,y,t)\} \]

Homogeneity (scaling)

Visual stimulus  RGC firing rate

Original input

Output

Original input \text{x} 2

Output \text{x} 2

Additivity

Visual stimulus  RGC firing rate

Input 1

Output 1

Input 2

Output 2

Sum of inputs

Sum of outputs
**Shift invariance**

Visual stimulus

- Original input
  - time
- Original input, later in time
  - time

RGC firing rate

- Output
  - time
- Output, later in time
  - time

**Shift-invariant linear systems and impulses**

- Impulse
- Impulse Response

Each impulse creates a scaled and shifted impulse response

The sum of all the impulse responses is the final system response

**Convolution**

Discrete-time signal: \( x[n] = [x_1, x_2, x_3, \ldots] \)

A system or transform maps an input signal into an output signal:

\[
y[n] = T(x[n])
\]

A shift-invariant, linear system can always be expressed as a convolution:

\[
y[n] = \sum_m x[m] h[n-m]
\]

where \( h[n] \) is the impulse response.
**Convolution as sum of impulse responses**

Input:


Impulse response:

Output:

\[ \begin{align*}
  \text{Input:} & \quad \text{Impulse response:} & \quad \text{Output:} \\
  \text{---} & \quad \text{---} & \quad \text{---} \\
  & \quad + & \quad \text{---}
\end{align*} \]

**Convolution as sequence of weighted sums**

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**Convolution as matrix multiplication**

Linear system ↔ matrix multiplication
Shift-invariant linear system ↔ Toeplitz matrix

\[
\begin{bmatrix}
  \cdots \\
  a \\
  b \\
  c \\
  \cdots \\
\end{bmatrix} \quad \begin{bmatrix}
  \cdots \\
  1 \\
  2 \\
  3 \\
  0 \\
  0 \\
  \cdots \\
\end{bmatrix} = \begin{bmatrix}
  \cdots \\
  1 \\
  2 \\
  0 \\
  0 \\
  \cdots \\
\end{bmatrix}
\]

Columns contain shifted copies of the impulse response.
Rows contain time-reversed copies of impulse response.
Convolution as matrix multiplication

\[
\begin{bmatrix}
  \vdots \\
  3 \\
  2 \\
  1 \\
  0 \\
  \vdots \\
\end{bmatrix}
\begin{bmatrix}
  \vdots \\
  1 & 2 & 3 & 0 & 0 \\
  0 & 1 & 2 & 3 & 0 \\
  0 & 0 & 1 & 2 & 3 \\
  \vdots \\
\end{bmatrix}
\begin{bmatrix}
  \vdots \\
  0 \\
  0 \\
  1 \\
  0 \\
  \vdots \\
\end{bmatrix}
\]

Columns contain shifted copies of the impulse response.
Rows contain time-reversed copies of impulse response.

Derivation: shift-invariant linear system \(\Rightarrow\) convolution

Homogeneity:
\[T(ax[n]) = a T(x[n])\]

Additivity:
\[T(x_1[n] + x_2[n]) = T(x_1[n]) + T(x_2[n])\]

Superposition:
\[T(ax_1[n] + bx_2[n]) = a T(x_1[n]) + b T(x_2[n])\]

Shift-invariance:
\[y[n] = T(x[n]) \Rightarrow y[n-m] = T(x[n-m])\]

Convolution derivation (cont)

Impulse sequence:
\[d[n] = 1 \text{ for } n = 0, d[n] = 0 \text{ otherwise}\]

Any sequence can be expressed as a sum of impulses:
\[x[n] = \sum_m x[m] d[n-m]\]

where
\[d[n-m] \text{ is impulse shifted to sample } m\]
\[x[m] \text{ is the height of that impulse}\]

Example:

\[\underline{\text{\_\_\_\_\_\_\_\_\_}} = \underline{\text{\_\_\_\_\_\_\_\_\_}} \ast \underline{\text{\_\_\_\_\_\_\_\_\_}} \ast \underline{\text{\_\_\_\_\_\_\_\_\_}}\]
**Convolution derivation (cont)**

- **x[n]: input**
- **y[n] = T(x[n]): output**
- **h[n] = T(d[n]): impulse response**

1) Represent input as sum of impulses:

\[ y[n] = T\{\sum_{m} x[m] \cdot d[n-m] \} \]

2) Use superposition:

\[ y[n] = \sum_{m} x[m] \cdot T(d[n-m]) \]

3) Use shift-invariance:

\[ y[n] = \sum_{m} x[m] \cdot h[n-m] \]

---

**Matrix multiplication => scaling**

\[
\begin{bmatrix}
  y_1 \\
  y_2 \\
  y_3 \\
  \vdots \\
  y_N
\end{bmatrix} =
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  \vdots \\
  x_N
\end{bmatrix}
\]

\[
\begin{bmatrix}
  a y_1 \\
  a y_2 \\
  a y_3 \\
  \vdots \\
  a y_N
\end{bmatrix} =
\begin{bmatrix}
  a x_1 \\
  a x_2 \\
  a x_3 \\
  \vdots \\
  a x_N
\end{bmatrix}
\]

---

**Matrix multiplication => additivity**

\[
\begin{bmatrix}
  y_1 \\
  y_2 \\
  y_3 \\
  \vdots \\
  y_N
\end{bmatrix} =
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  \vdots \\
  x_N
\end{bmatrix}
\]

\[
\begin{bmatrix}
  z_1 \\
  z_2 \\
  z_3 \\
  \vdots \\
  z_N
\end{bmatrix} =
\begin{bmatrix}
  w_1 \\
  w_2 \\
  w_3 \\
  \vdots \\
  w_N
\end{bmatrix}
\]

\[
\begin{bmatrix}
  y_1 + z_1 \\
  y_2 + z_2 \\
  y_3 + z_3 \\
  \vdots \\
  y_N + z_N
\end{bmatrix} =
\begin{bmatrix}
  x_1 + w_1 \\
  x_2 + w_2 \\
  x_3 + w_3 \\
  \vdots \\
  x_N + w_N
\end{bmatrix}
\]
**Toeplitz matrix => shift invariance**

\[
\begin{bmatrix}
\vdots & 1 & 2 & 3 & 0 & 0 & 0 \\
3 & 2 & 1 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\end{bmatrix}
\]

---

**Neural image and convolution**

A spatial receptive field may also be treated as a linear system, by assuming a dense collection of neurons with the same receptive field translated to different locations in the visual field. In this view, it is a linear, shift-invariant system.

* This is the basis of CNNs (convolutional neural networks).

§ The nervous system doesn’t actually work like this. (It’s not linear and it’s not shift invariant!)

---

\[ \text{See code on next slide} \]
Neural image and convolution

% Make a Difference of Gaussian receptive field using fspecial
DoG = fspecial('gaussian', 20,2) -  fspecial('gaussian', 20,5);

% Make a neural image by convolution
neuralim = conv2(double(im), DoG);

% Show the image, the RF, and the neural image
figure, subplot(1,3,1), imshow(im), subplot(1,3,2), surf(DoG)
subplot(1,3,3), imagesc(neuralim); colormap gray, axis image off

Continuous-time derivation of convolution

A system (or transform) converts (or maps) an input signal into an output signal:
\[ y(t) = T[x(t)] \]

A linear system satisfies the following properties.
1) Homogeneity (scalar rule):
\[ T[a x(t)] = a y(t) \]
2) Additivity:
\[ T[x_1(t) + x_2(t)] = y_1(t) + y_2(t) \]

Often, these two properties are written together and called superposition:
\[ T[a x_1(t) + b x_2(t)] = a y_1(t) + b y_2(t) \]

Linear systems requirements

- Make a Difference of Gaussian receptive field using fspecial
- Show the image, the RF, and the neural image
- Continuous-time derivation of convolution
- Linear systems requirements
Pulses and impulses

\[ \delta(t) = \begin{cases} \infty & \text{if } t = 0 \\ 0 & \text{otherwise} \end{cases} \]

\[ \delta_{\Delta}(t) = \begin{cases} \frac{1}{\Delta} & \text{if } 0 < t < \Delta \\ 0 & \text{otherwise} \end{cases} \]

\[ \delta(t) = \lim_{\Delta \to 0} \delta_{\Delta}(t). \]

Staircase approximation to continuous time signal

\[ x(t) = \sum_{k=-\infty}^{\infty} x(k\Delta) \delta_\Delta(t - k\Delta) \Delta. \]

\[ x(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{\infty} x(k\Delta) \delta_{\Delta}(t - k\Delta) \Delta. \]

\[ x(t) = \int_{-\infty}^{\infty} x(s) \delta(t - s) \, ds. \]

Shift invariance

For a system to be shift-invariant (or time-invariant) means that a time-shifted version of the input yields a time-shifted version of the output:

\[ y(t) = T[x(t)] \]

\[ y(t - s) = T[x(t - s)] \]

The response \( y(t - s) \) is identical to the response \( y(t) \), except that it is shifted in time.
**Convolution**

Representing the input signal as a sum of pulses:

\[ y(t) = T[x(t)] = T \left[ \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) d\tau \right] \]

Using additivity,

\[ y(t) = \sum_{k=-\infty}^{\infty} T[x(t-k\Delta)] \delta(t-k\Delta) \Delta_k \]

Taking the limit,

\[ y(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{\infty} T[x(t-k\Delta)] \delta(t-k\Delta) \Delta_k \]

Using homogeneity (scalar rule),

\[ y(t) = \int_{-\infty}^{\infty} T[x(t-s)] \delta(t-s) ds \]

Defining \( h(t) \) as the impulse response,

\[ y(t) = \int_{-\infty}^{\infty} x(s) h(t-s) ds \]

---

**Linear systems analysis**

- **Space/time method**
  - Measure the impulse response
  - Input stimulus
  - Express as sum of shifted and scaled impulses
  - Calculate the response to each impulse
  - Sum the impulse responses

- **Frequency method**
  - Measure the frequency response
  - Express as sum of shifted and scaled sinusoids
  - Calculate the response to each sinusoidal
  - Sum the sinusoidal responses

---

**Spatial pattern vision and linear systems theory**

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Fourier transform and frequency response: summary

- Signals can be represented as sums of sine waves.
- Linear, shift-invariant systems operate "independently" on each sine wave, and merely scale and shift them.
- A simplified model of neurons in the visual system, the linear receptive field, results in a neural image that is linear and shift-invariant.
- Psychophysical models of the visual system might be built of such mechanisms.
- It is therefore important to understand visual stimuli in terms of their spatial frequency content.
- The same tools can be applied to other modalities (e.g., audition) and other signals (EEG, MRI, MEG, etc.).

Temporal frequency and Fourier decomposition

Shift-invariant linear systems & sinusoids

Measure the scaling and shifting for each sinusoid.
**Shift-invariant linear systems & sinusoids**

Frequency response

Measure the impulse response

- Space/time method
  - Input stimulus
  - Measure the impulse response
  - Express as sum of shifted and scaled impulses
  - Calculate the response to each impulse
  - Sum the impulse responses

- Frequency method
  - Measure the frequency response
  - Express as sum of shifted and scaled sinusoids
  - Calculate the response to each sinusoidal
  - Sum the sinusoidal responses

**Linear systems analysis**

**Convolution and multiplication**

Convolution:

\[ x_1[n] \cdot x_2[n] \rightleftharpoons X_1[k] X_2[k] \]

Multiplication:

\[ x_1[n] x_2[n] \rightleftharpoons (1/N) X_1[k] \cdot X_2[k] \]
Linear filter example

Miles Davis

“Half Nelson”

Bass only:

Treble only:

Time

Signal

Filter

Frequency content

Low-pass or Bass Filter

High-pass or Treble Filter

Time

Prev

Next
Linear filter example

Low-pass or Bass Filter

High-pass or Treble Filter

Miles Davis
"Half Nelson"

Bass only:

Treble only:

Time

Frequency content

Input image (cornea)

Convolution with impulse response

"Neural image" (V1)

Fourier spectrum of input image

Fourier transform

Multiplication with frequency response

Fourier spectrum of neural image

Linear filter

Fourier Analysis

Signals as sums of sine waves
- 1d: time series
  - fMRI signal from a voxel or ROI
  - mean firing rate of a neuron over time
  - auditory stimuli
- 2d: static visual image, neural image
- 3d: visual motion analysis
Auditory example: Pure tones

Pure tones can be described by 3 numbers:
Frequency = rate of air pressure modulation (related to pitch)
Amplitude = sound pressure level (related to loudness)
Phase = sin vs. cosine vs. another horizontal shift

Frequency and amplitude

Fourier spectrum representation of sound
Fourier spectra of some sounds

- Pure tone
- White noise
- Violin note

Frequency (Hz)

Amplitude

Fundamental frequency and harmonics

(a) 400 Hz tone plus harmonics

(b) 800 Hz tone plus harmonics

Lots of Fourier transforms

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<tr>
<th>Name</th>
<th>Time domain</th>
<th>Freq domain</th>
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<td>discrete, finite</td>
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FFT algorithm

- Computes DFT of finite length input.
- Efficient for inputs of length $N = m^n$.
- Produces 2 outputs, each of size/length equal to that of the input: real part (cosine coeffs), imaginary part (sine coeffs).

DFT of a cosine

$$\cos\left(\frac{2\pi k}{N}\right)$$

$k = 4$ cycles/image
$N = 32$ pixels

Real and imaginary parts

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<td>imaginary part</td>
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<td>phase</td>
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<td>$\pi/2$</td>
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</table>
DFT of impulse signal and constant signal

Uncertainty principle

Multiplication and convolution
Discrete Fourier Transform (DFT)

Analysis:
\[X[k] = \sum_{n=0}^{N-1} x[n] e^{-j(2\pi/N)kn} \quad 0 \leq k \leq N - 1\]
\[0 \quad \text{otherwise}\]

Synthesis:
\[x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j(2\pi/N)kn} \quad 0 \leq n \leq N - 1\]
\[0 \quad \text{otherwise}\]

\(x[n]\): discrete, finite

\(X[k]\): discrete, finite

Complex numbers and complex exponentials

\[z = a + jb = Ae^{j\phi} = A[\cos(\phi) + j\sin(\phi)]\]

amplitude
\[A = \sqrt{a^2 + b^2}\]
\[\phi = \tan^{-1}(b/a)\]

phase
\[j = \sqrt{-1}\]

Why bother with complex exponentials?

\[(A_1e^{j\phi_1})(A_2e^{j\phi_2}) = A_1A_2e^{j(\phi_1 + \phi_2)}\]

Discrete Fourier Transform matrix

Analysis:
\[X[k] = \sum_n x[n] \exp(-j2\pi kn/N)\]

For real valued inputs:
\[X_R[k] = \sum_n x[n] \cos(\ldots) \quad X_I[k] = \sum_n -x[n] \sin(\ldots)\]

\[
\begin{pmatrix}
X_R[k] \\
X_I[k]
\end{pmatrix} =
\begin{pmatrix}
cosines \\
sines
\end{pmatrix}
\begin{pmatrix}
x[n]
\end{pmatrix}
\]

Rows of \(P\) called projection functions:
\[
\frac{1}{N} P^T P = I
\]
Discrete Fourier transform matrix

Synthesis: \[ x[n] = \sum_k X[k] \exp(j2\pi kn/N) \]

\[
\begin{pmatrix}
X[n]
\end{pmatrix} = \frac{1}{N} \begin{pmatrix}
\text{cosines} & \text{sines}
\end{pmatrix} \begin{pmatrix}
X_0[k]
X_1[k]
\end{pmatrix}
\]

Cols of \( B \) called basis functions. \( B = \mathbf{p}^T, \frac{1}{N} B B^T = I \)

Sine wave gratings and spatial frequency

Measured in cycles per degree (cpd or c/deg or c/º) of visual angle.

Contrast

Contrast = 1
Contrast = 0.5
Two-dimensional Fourier spectra

Space domain

Frequency domain

Input image (cornea)

"Neural image" (V1)

Convolution with impulse response

Multiplication with frequency response

Intensity in the Fourier spectrum at each location indicates amount of contrast (in the original image) for each spatial frequency and orientation.

Linear systems analysis

Fourier spectrum of input image

Fourier spectrum of neural image
Spatial pattern vision and linear systems theory

- Receptive fields and neural images
- Shift-invariant linear systems and convolution
- Fourier transform and frequency response
- Applications of linear systems to spatial vision
  - Contrast sensitivity
  - Spatial frequency and orientation channels
  - Spatial frequency and orientation adaptation
  - Masking

Spatial contrast sensitivity

Contrast sensitivity

Spatial frequency (c/deg)

Spatial contrast sensitivity

Spatial frequency (cycles/degree)

Threshold contrast

Sensitivity (threshold contrast)

Spatial frequency (cycles/mm on retina)
Spatial frequency channels

Each channel is sensitive to a narrow range of frequencies. Overall contrast sensitivity depends on all of them together.

Theory of spatial pattern analysis by the visual system

Low sf filters encode coarse-scale information (large objects, overall shape)

High sf filters encode fine-scale information (small objects, detail)

Multi-resolution model
Psychophysical/perceptual evidence for spatial-frequency and orientation selective channels

Orientation and spatial frequency selective adaptation: appearance

Emma by Chuck Close
Spatial frequency selective adaptation: appearance

(a) No adaptation

(b) Low frequency adaptation

(c) High frequency adaptation

Figure 1.13: A multiresolution model can explain certain aspects of pattern adaptation.

(a) In normal viewing, the bar width is inferred from the relative responses of a collection of component-images, each responding best to a selected spatial frequency band. The spatial frequency selectivity of each component-image is shown above and the amplitude of the component-image encoding of the test stimulus is shown in the bar graph below.

(b) Following adaptation to a low frequency stimulus (shown in inset), the sensitivity of the neurons comprising certain component-images is reduced. Considering the responses of all the component-images, the response to the test is similar to the unadapted response to a high frequency target.

(c) Following adaptation to a high frequency pattern (shown in inset), the neural representation is consistent with the unadapted response to a low frequency target.
Spatial frequency selective adaptation: sensitivity

Contrast sensitivity before & after adaptation

Figure 3.22
Squares and solid curve: Contrast sensitivity function for a sine-wave grating. (From Campbell & Robson, 1968.) Dotted curve: Contrast sensitivity measured after adaptation to a 7.5 cycle/degree grating.
Orientation selective adaptation

Applications: Optical Line Spread Function (Campbell & Gubisch, 1966)

Fig. 1. Examples of optical line spread function and modulation transfer function.

- (a) Grating response
- (b) Modulation transfer function
- (c) Linespread function

Optical quality. As the fineness of a black-and-white illumination modulation of its image decreases, the decrease of contrast is the modulation transfer function. This distribution in the image of a thin line is termed the linespread function; it is also obtained as the Fourier transform of the modulation transfer function.
Applications: Optical Line Spread Function  
(Campbell & Gubisch, 1966)

Applications: Optical Line Spread Function  
(Campbell & Gubisch, 1966)

Applications: Optical Line Spread Function  
(Westheimer, 1986)
Applications: Optical Line Spread Function (Westheimer, 1986)

Modulation transfer function measurements of the optical quality of the lens made using visual interferometry (Williams et al., 1995; described in Chapter 3). The data are compared with the predictions from the linespread suggested by Westheimer (1986) and a curve fit through the data by Williams et al. (1995).

Applications: Optical Line Spread Function, Wavelength Dependency

OTF of Chromatic Aberration: Two views of the modulation transfer function of a model eye at various wavelengths. The model eye has the same chromatic aberration as the human eye (see Figure 2.23) and a 3.0mm pupil diameter. The eye is in focus at 580nm; the curve at 580nm is diffraction limited. The retinal image has no contrast beyond four cycles per degree at short wavelengths. (From Marimont and Wandell, 1993).