

Hodgkin-Huxley Summary

Professor David Heeger

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- Membrane equation:

$$C \frac{dV}{dt} + I_{Na}(V, t) + I_K(V, t) + I_{leak} - I_{inject} = 0$$

- Leak current:

$$I_{leak}(V) = g_{leak}[V - E_{leak}]$$

- Ionic currents:

$$I_K(V, t) = g_K(V, t) [V(t) - E_K]$$

$$I_{Na}(V, t) = g_{Na}(V, t) [V(t) - E_{Na}]$$

- Ionic conductances:

$$g_K(V, t) = \overline{g_K} [n(V, t)]^4$$

$$g_{Na}(V, t) = \overline{g_{Na}} [m(V, t)]^3 h(V, t)$$

- Potassium activation:

$$\frac{dn}{dt} = \alpha_n(V) (1 - n) - \beta_n(V) n$$

$$\alpha_n(V) = .01 \frac{(10 - V)}{e^{(10-V)/10} - 1}$$

$$\beta_n(V) = 0.125 e^{-V/80}$$

- Sodium activation:

$$\frac{dm}{dt} = \alpha_m(V) (1 - m) - \beta_m(V) m$$

$$\alpha_m(V) = 0.1 \frac{(25 - V)}{e^{(25-V)/10} - 1}$$

$$\beta_m(V) = 4 e^{-V/18}$$

- Sodium inactivation:

$$\frac{dh}{dt} = \alpha_h(V) (1 - h) - \beta_h(V) h$$

$$\alpha_h(V) = .07 e^{-V/20}$$

$$\beta_h(V) = \frac{1}{e^{(30-V)/10} + 1}$$

- Potassium activation time constant (likewise for m and h):

$$\tau_n(V) = \frac{1}{\alpha_n(V) + \beta_n(V)}$$

- Steady state potassium activation (likewise for m and h):

$$n_\infty(V) = \frac{\alpha_n(V)}{\alpha_n(V) + \beta_n(V)}$$

- Parameters:

$$\begin{aligned} V_{rest} &= 0 \text{ mV} \\ g_{leak} &= 8.4 \times 10^{-3} \text{ uS} \\ C &= 28 \times 10^{-3} \text{ nF} \\ E_{Na} &= 115 \text{ mV} \\ E_K &= -12 \text{ mV} \\ \overline{g_K} &= 1.008 \text{ uS} \\ \overline{g_{Na}} &= 3.36 \text{ uS} \end{aligned}$$

- Leak potential:

$$E_{leak} = - \frac{\overline{g_{Na}} m_\infty^3(0) h_\infty(0) E_{Na} + \overline{g_K} n_\infty^4(0) E_K}{g_{leak}} = 10.6$$

- Pseudo-code for Euler's method implementation:

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V[1]=0

n[1]=ninf(0)
and likewise for m[1] and h[1]

loop for i = 1 to Nsamples

    alpha_n = .01 * (10-V[i]) / (exp((10-V)/10)-1)
    and likewise for beta_n, alpha_m, beta_m, alpha_h, and beta_h

    n[i+1] = n[i] + deltaT*(alpha_n*(1-n[i])-beta_n*n[i])
    and likewise for m[i+1] and h[i+1]

    gK = gK_max * n^4
    gNa = gNa_max * m^3 * h

    IK = gK*(V[i]-EK)
    INa = gNa*(V[i]-ENa)
    Ileak = gLeak*(V[i]-Eleak)

    V[i+1] = V[i] + (deltaT/C)*(Iinject[i]-IK-INa-Ileak)

endloop

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