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Subband Image Coding with Hexagonal Quadrature Mirror Filters

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Subband coding schemes, originally developed for the compression of one-dimensional signals, are becoming increasingly popular as a means of compressing images and image sequences. In particular, critically-sampled filterbank systems using Quadrature Mirror filters have been found to be quite effective [1, 2, 3].

In one dimension, a pair of QMF filters is formed by modulating and shifting a single baseband filter, $H(\omega)$. The formation of the highpass filter through shifts and modulations ensures that the system aliasing (produced when the subband signals are subsampled) will cancel when the signal is reconstructed. The remaining linear shift-invariant (LSI) system response may be written in terms of the baseband filter:

$$\frac{1}{2}\Big[|H(\omega)|^2 + |H(\omega+\pi)|^2\Big]. \tag{1}$$

The advantages of these systems are many. A QMF system provides an orthogonal subband decomposition without the arbitrary block boundaries of the blocked DCT. Moreover, the ringing and computational inefficiency of a sharp subband (sinc filter) system are reduced through the use of relatively small filters with soft transitions (this is possible due to the cancellation of the overall system aliasing). The filter design process is simplified by the relationships between the various subband filters. Since the system aliasing cancels, the only constraint on the baseband filter $H(\omega)$ is that expression (1) have a value of unity for all ω. Finally, these systems may be cascaded uniformly or non-uniformly (to form pyramid structures). Unlike traditional Gaussian or Laplacian pyramids, the representation is not overcomplete and may contain oriented subbands.

We have been exploring the use of nonseparable QMF's with hexagonal symmetry. The basic framework is outlined in [4], and a more thorough description may be found in [5]. On a hexagonal sampling lattice system it is possible to construct a pyramid decomposition in which all of the subbands are selective for orientation – something that is not possible with the separable systems that are commonly used. An idealized illustration of the frequency subdivision performed by such a system is given in figure 1.

The hexagonal QMF's are strictly analogous to the one-dimensional case, in that the subband filters are formed by shifting and modulating a baseband filter. The system aliasing cancels exactly, and the remaining LSI system response is in the form

$$\frac{1}{4}\sum_{i=0}^{3}|H(\omega+\tilde{\kappa}_i)|^2, \tag{2}$$

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where the $\tilde{\kappa}_i$ consist of the zero vector and a set of three *modulation* vectors aligned with the hexagonal axes in the frequency domain. In order to design the filters, we used a simplex method to minimize a weighted error function which was a combination of the maximum absolute value deviation of the value of expression (2) from unity and the maximal intra-band aliasing error. The impulse response of a hexagonal kernel of radius four is given in table 1, and the power spectrum of one of the filters is plotted in figure 2.

We have found that the filters described above are excellent for data compression. Figure 3 gives an example of an image coded to 0.5 bits/pel. After resampling the image on a hexagonal sampling lattice, we built a 4-level pyramid using the hexagonal QMF given in table 1, quantized the subbands according to a simple log-variance rule, and reconstructed.

References

[1] D. Esteban and C. Galand. Application of quadrature mirror filters to split band voice coding schemes. In *Proceedings ICASSP*, pages 191–195, 1977.

[2] Martin Vetterli. Multi-dimensional sub-band coding: some theory and algorithms. *Signal Processing*, 6(2):97–112, February 1984.

[3] John W. Woods and Sean D. O'Neil. Subband coding of images. *IEEE Trans. ASSP*, ASSP-34(5):1278–1288, October 1986.

[4] Edward H. Adelson, Eero Simoncelli, and Rajesh Hingorani. Orthogonal pyramid transforms for image coding. In *Proceedings of SPIE*, October 1987.

[5] Eero P. Simoncelli and Edward H. Adelson. *Non-separable Extensions of Quadrature Mirror Filters to Multiple Dimensions.* Vision Science Technical Report 119, Massachusetts Institute of Technology, 1989. In press, Proceedings of the IEEE (1990).

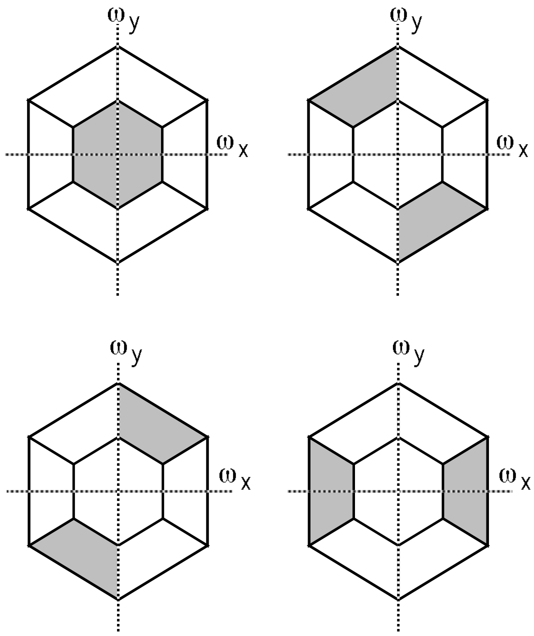

Figure 1: Frequency plots indicating the idealized frequency spectra of each of the hexagonal subband filters.

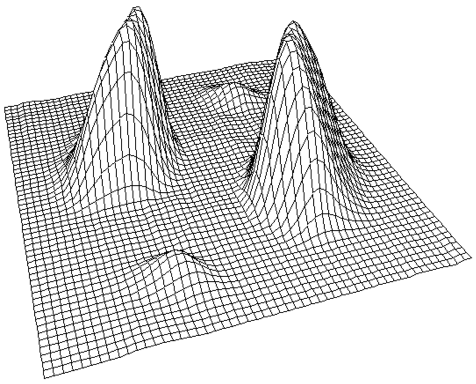

Figure 2: The power spectrum for one of the "4-ring" set of hexagonal QMF filters. The filter kernels are given in table 1.

				g		h		i		h		g				
			h		e		f		f		e		h			
		i		f		c		d		c		f		i		
	h		f		d		b		b		d		f		h	
g		e		c		b		a		b		c		e		g
	h		f		d		b		b		d		f		h	
		i		f		c		d		c		f		i		
			h		e		f		f		e		h			
				g		h		i		h		g				

a	=	0.6066799
b	=	0.3162482
c	=	-0.028019974
d	=	-0.0016289932
e	=	-0.02741341
f	=	-0.038143888
g	=	-0.005958891
h	=	0.019682134
i	=	0.016045252

Table 1: A hexagonal filter. Only the low-pass filter is shown. The three highpass filters are formed by modulating and shifting the low-pass.

Figure 3: At the top is the original "Lena" image at 256×256 pixels. Below is a reconstruction that was compressed using the 4-ring hexagonal filter bank. The compressed pyramid occupied 4096 bytes or 0.5 bits per pixel.