10.1: The Visibility of Higher-Level Jags

A. C. Naiman
Hong Kong University of Science and Technology, Kowloon, Hong Kong

W. Makous
University of Rochester, Rochester, NY

Abstract

We have extended our preliminary investigation of visual sensitivity to jagged edges. Agreement amongst data from three subjects allows us to predict sensitivity and leads to a model based on *jag levels*. We find that only first- and second-level jags play a significant role in the visibility of jaggedness.

Introduction

Depiction of oblique edges by discrete pixels usually results in visible stair steps, often called *jaggies*. While there has been some work carried out on the visibility of edge raggedness and blur [Hamerly 81, Hamerly and Springer 81, Tyler and Mitchell 77], we know of no systematic accounts of the effects of spatial and luminance quantization on the image quality of edges and lines. We have previously presented preliminary data from a jaggies experiment [Naiman and Makous 94a] and described the pattern along a discrete approximation of an edge in terms of multiple levels of jags [Naiman and Makous 94b]. Here we extend and refine our initial observations: three subjects made forced-choice observations of the visibility of the jaggedness of 34 edges varying in slope.

Jagged Edge Experiment

To determine the human visual system's ability to detect jagged edges along a straight contour, we have embarked on a series of experiments using stimuli of the sort displayed in Fig. 1. Two parallel edges are presented, differing only in polarity and size of the simulated pixels used to compose the edges. The subject must choose the edge that is more jagged.

For each of 34 slopes between 0.007 and 1.0 (inclusive), subjects first estimated the range of viewing distances that would likely result in performance – in terms of percentage of correct responses – ranging from just above 50% to just below 100%. After a suitable amount of practice for each slope, subjects were presented with blocks of 50 such forced-choice trials at each of a variety of distances spanning the estimated range. To calculate the threshold viewing distance – defined as the distance at which responses are 75% correct – for a particular subject and a particular slope, a cumulative Gaussian function was fit to the subject's proportions correct at the various distances.

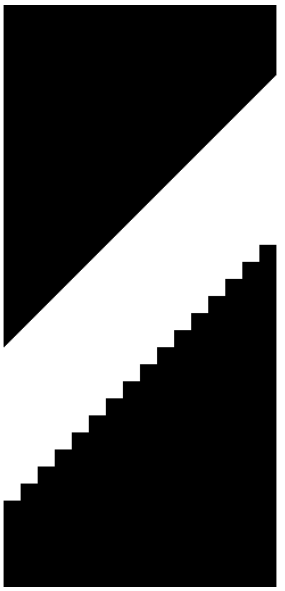

Figure 1: Stimulus. A representative edge stimulus with a slope of 1 is shown.

Results

We define *acuity* as the reciprocal of the visual angle (in minutes) that a single pixel subtends at the threshold viewing distance. Fig. 2 shows mean acuity for three observers as a function of edge slope (on semi-log axes); for clarity, we have not shown the standard errors, which are fairly uniform, at about the size of the plotted symbols. The straight, diagonal lines are regression fits to the data clustered around each line, the significance of which is discussed below.

Generally-affordable soft-copy displays today have addressable resolutions between 25 and 40 pixels per centimeter. Even at the high-end of this range, at a standard viewing distance of 40 centimeters from the display, a pixel subtends 2.15 arc minutes of visual angle, corresponding to an 'acuity' of 0.465. As this number is well below the bottom of the plotted region in Fig. 2, jagged edges will be visible at *all* orienta-

ISSN0097-0966X/95/2601-0113-$1.00 + .00 © 1995 SID

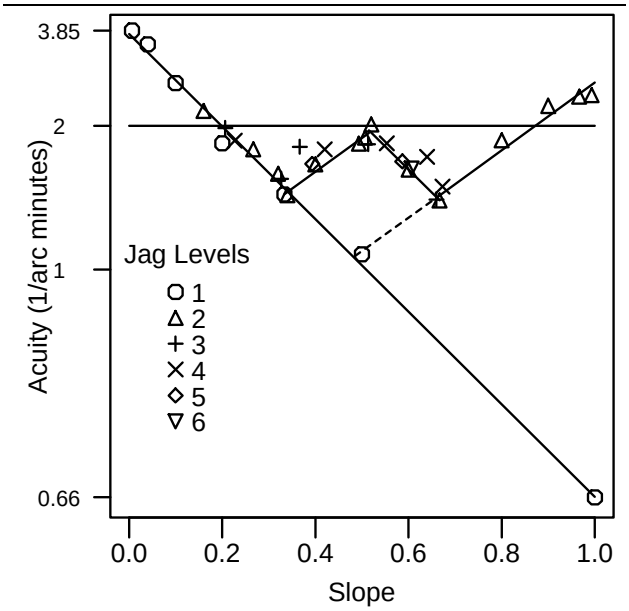

Figure 2: Jag Detectability. Mean acuity of jaggies is plotted on a log axis for three subjects. The significance of the regression fits is discussed in the text.

tions on such devices. To make the jaggies completely disappear from black/white edges for all slopes, pixels must be no larger than that dictated by the highest acuity. For the slopes tested, the highest acuity was 3.85 (at a slope of 0.007), prescribing pixels no larger than 15.6 seconds (60/3.85). At a viewing distance of 40 cm, the display would require approximately 330 pixels per centimeter for the slopes we tested! From the literature on Vernier acuity [Westheimer and McKee 77], one can infer that the use of slopes lower than the minimum tested here would not increase the visibility of jaggedness, as long as at least one jag is present in the display.

The horizontal line in Fig. 2, corresponding to an acuity of 2, demonstrates the visibility of jagged edges on a display with a resolution of 170 pixels per cm (when viewed from 40 cm). On such a display, high-contrast edges will appear jagged only for those slopes for which acuity is above the line; all slopes below the line will be indiscriminable from straight contours of the same orientations.

Jag Levels

To interpret the pattern of results visible in Fig. 2, we have introduced the notion of *jag level* [Naiman and Makous 94a]. The concept is explained fully elsewhere [Naiman and Makous 94b]; a qualitative understanding is sufficient for present purposes.

Fig. 3 shows three patterns of jagged edges. The left side shows a regular pattern that can be described as going ‘up one and over one’, ad infinitum. In the notation we have adopted, this is expressed simply as {1}, indicating the number of horizontal steps after each and every vertical step. Such a uniformly-regular pattern is considered to have *first-level* jags. In the middle part of the figure, this regular pattern is interrupted at one location, where the pattern has one instance of ‘up one and over two’; notationally this is $\{1^6, 2, 1^8\}$, which, for an arbitrarily long edge of the same slope, can be denoted equivalently as $\{1^{14}, 2\}$, indicating 14 repetitions of a single horizontal step for each vertical step, along with one occurrence of a double horizontal step after a vertical step. This interruption of regularity raises the jag level to 2. The third example in Fig. 3 is even more complex and can be expressed as $\{2^4, 1, 2^3, 1\}$; this edge has a jag level of 3.

We suggest that, as slope changes, the introduction of a single higher-level jag can increase the visibility of jaggedness, sometimes quite substantially. This can be verified by viewing the left and middle edges of Fig. 3 from a sufficient distance: the jag in the middle edge is visible at distances at which no jaggedness is visible in the left edge of the figure.

In Fig. 2, the jag level of the pattern corresponding to the slope of each jagged edge is denoted symbolically. We can characterize the important features of this plot as follows:

- The points corresponding to all of the first-level jagged edges fall along a single, straight line with a negative slope.
- All of the points corresponding to edge slopes less than 1/3 – regardless of jag-level – fall along this same line.
- All of the points corresponding to edge slopes between 2/3 and 1.0 – regardless of jag-level – lie along the positively-sloped, straight line branching off from the point corresponding to an edge slope of 0.5. (Note that, although most of the points along this line are from second-level jags, the point corresponding to an edge slope of 101/150 is from a fourth-level jagged edge.) Consider, for a moment, only the points along this line that correspond to second-level jagged edges. As the slope decreases from 1, the spacing between second-level jags decreases and so does their visibility, in a way that is directly analogous to the effect of decreased spacing of first-level jags on their visibility.
- All of the points corresponding to edge slopes between 1/3 and 0.5 – regardless of jag-level – fall along a line parallel to the previously-mentioned, positively-sloped line in the figure.
- All of the points corresponding to edge slopes between 0.5 and 2/3 – regardless of jag-level – fall along a line parallel to the previously-mentioned, negatively-sloped line in the figure.

Why is it that the visibility of second-level jagged edges between 0.5 and 2/3 *decreases* with increasing

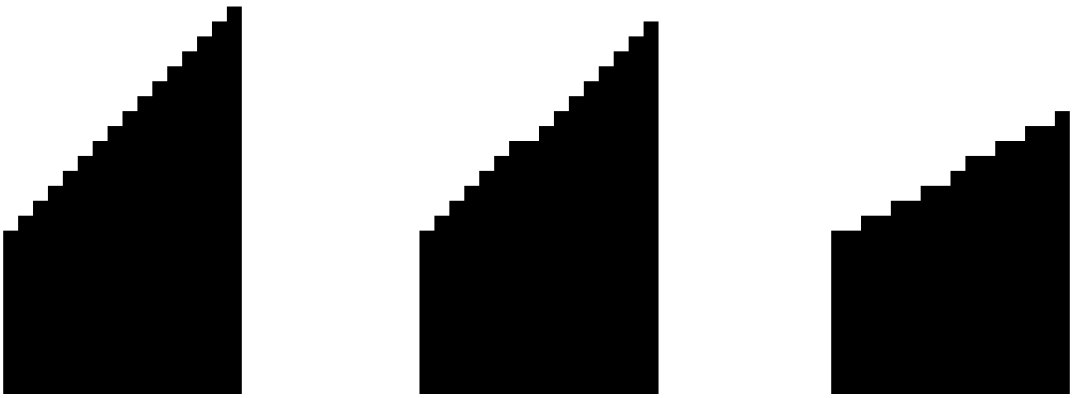

Figure 3: Jag Levels. From left to right, first-, second- and third-level jagged edges are shown.

slope, while the visibility of second-level jagged edges between 2/3 and 1.0 *increases* with increasing slope? In other words, why don't the points corresponding to second-level jagged edges between 0.5 and 2/3 fall on the dashed portion of the positively-sloped line rather than on the negatively-sloped line above it? Let us examine the edge patterns for these slopes, as listed in the following table.

Slope	Basic Pattern	Number of Repetitions	Jag Level
75/150	2^{75}	1	1
76/150	2^{37}, 1^{1}	2	2
78/150	2^{12}, 1^{1}	6	2
90/150	2^{2}, 1^{1}	30	2
100/150	2^{1}, 1^{1}	50	2
120/150	2^{1}, 1^{3}	30	2
135/150	2^{1}, 1^{8}	15	2
145/150	2^{1}, 1^{28}	5	2
149/150	2^{1}, 1^{148}	1	2
150/150	1^{150}	1	1

First, consider the slope of 75/150: this is a first-level jagged edge, with a basic pattern of 75 2's. At a slope of 76/150, this regular pattern is interrupted with a second-level jag – the '1' – which occurs twice over the course of this edge. The introduction of a higher-level jag increases the visibility substantially (cf. Fig. 2). For a slope of 78/150, a total of 6 second-level jags are introduced, while, for a slope of 90/150, there a total of 30 second-level jags. That is, the frequency of the second-level jags – the 1's in the patterns – increases from 2 to 30 and, not surprisingly, there is an attendant decrease in sensitivity.

Now consider the slope of 150/150: this, too, is a first-level jagged edge, with a basic pattern of 150 1's. At a slope of 149/150, this regular pattern is interrupted with one second-level jag – the '2'. The frequency of this second-level jag increases to 5, 15 and 30, for slopes of 145/150, 135/150 and 120/150, respectively. Here, too, the increase in frequency of the second-level jag results in decreased sensitivity.

At a slope of 100/150, there are an equal number of 1's and 2's – 50 of each. This represents both a local maximum in spatial frequency of the 1's serving as second-level jags for edges with slopes between 0.5 and 2/3 as well as a local maximum of the 2's serving as second-level jags for edges with slopes between 2/3 and 1.0. Hence, sensitivity to an edge with a slope of 2/3 is a local minimum.

It is conceivable that sensitivity to jagged edges follows a pattern suggested by the branches shown in Fig. 2, but replicated at finer detail for higher jag-levels. However, our data – which includes purposely-chosen edges with jag-levels as high as 6 – tend to cluster along only the four branches shown, suggesting that whatever micro-structure might actually exist is not important for characterizing sensitivity to jagged edges.

Fourier Analysis

As the slope of the approximated edge increases from 0 to 1/3, the spacing among first-level jags decreases (i. e., their spatial frequency increases) and the spacing between any second-level jags increases. This complementary behavior of first- and second-level jags is exact, so that $f_1(s) = f_2(1-s)$, where f_1 is the spatial frequency of first-level jags, f_2 is the spatial frequency of second-level jags, and s is the slope of the approximated edge.

Fig. 2 shows that sensitivity to jags decreases exponentially with their spatial frequency. An exponential decay has also been observed for the visibility of sinusoidal perturbations from straightness above 4 cycles per degree [Hamerly and Springer 81], and for contrast sensitivity at corresponding spatial frequencies [Yang and Makous 94, Yang *et al.* 1995]. The similarity of the visibility of jags to that of sinusoidally-perturbed edges can be explained on the assumption that the visibility of jags is determined by the fundamental Fourier component of the jags [Hamerly and Springer 81]. The similarity of both sets of data to contrast sensitivity can be explained on the basis of the excitation that these edges supply to the spatial frequency and orientation selective channels known to underlie contrast sensitivity [Olzak and Thomas 86]. At low frequencies, contrast sensitivity curves and sensitivity to sinusoidal edge perturbations depart from the exponential decay in Fig. 2, showing instead a maximum near 5 c/deg. Although the jag data of Fig. 2 differ from the shapes of the other two curves only at a single point, the difference is highly reliable; we do not yet know what accounts for this difference.

Conclusions

Our investigations have led us to conclude that:

- the visibility of the jaggedness of an edge depends on the slope of the edge;
- the precise relationship between the two is complex and depends on the concept of jag level;
- rendering the jaggedness of edges invisible at all edge orientations requires pixels that subtend less than 15 arc seconds at the distance viewed;
- if the orientation of edges is restricted to 0, 45, and 90 degrees, pixels that subtend as much as 1.5 arc minutes will not produce visible jaggedness;
- pixels larger than 1.5 arc minutes – half the size of commonly-available CRT and flat panel displays at normal viewing distances – will produce visibly jagged edges at all orientations;
- changing the orientation of an edge by as little as 18 arc minutes from 45 degrees increases the visibility of its jaggedness four-fold, from minimum to almost maximum;
- the jaggedness of edges formed by pixels 30 arc seconds in subtense will be invisible over 70% of possible orientations;
- at pixel sizes greater than 30 arc seconds, there is an abrupt increase in the range of orientations at which jaggedness is visible; and
- only first- and second-level jags play a significant role in the visibility of jaggedness.

Acknowledgements

This research was supported by US Public Health Service Grants EY-01319 and EY-04885 and Hong Kong's RGC DAG93/94.EG01 and CERG HKUST600/94E. We thank Ada Yuen, Ivan Wong and Desamanya Peramunetilleke for serving as subjects in these experiments.

References

Hamerly 81
James R. Hamerly, ''An Analysis of Edge Raggedness and Blur,'' *Journal of Applied Photographic Engineering*, Volume 7, Number 6, December 1981, pp. 148-151.

Hamerly and Springer 81
James R. Hamerly and Robert M. Springer, ''Raggedness of Edges,'' *Journal of the Optical Society of America*, Volume 71, Number 3, March 1981, pp. 285-288.

Naiman and Makous 94a
Avi C. Naiman and Walter Makous, ''Jagged Edges and Their Visibility,'' *Society for Information Display 94 Digest*, Volume 25, June 1994, pp. 205-208.

Naiman and Makous 94b
Avi C. Naiman and Walter Makous, ''Computing the Jaggedness of Edges,'' *Proceedings of the 4th International Conference on Computer-Aided Drafting, Design and Manufacturing Technology*, August 1994, pp. 41-45. International Conference Proceedings: Pacific Graphics'94 / CADDM'94, 26-29 August 1994, Beijing, China.

Olzak and Thomas 86
Lynn A. Olzak and James P. Thomas, ''Seeing Spatial Patterns,'' in *The Handbook of Perception and Human Performance, Volume 1: Sensory Processes and Perception*, Edited by Kenneth R. Boff, Lloyd Kaufman and James P. Thomas, John Wiley and Sons, New York, 1986, pp. 7.1-7.56.

Tyler and Mitchell 77
Christopher W. Tyler and Donald E. Mitchell, ''Orientation Differences for Perception of Sinusoidal Line Stimuli,'' *Vision Research*, Volume 17, 1977, pp. 83-88.

Westheimer and McKee 77
Gerald Westheimer and Suzanne P. McKee, ''Spatial Configurations for Visual Hyperacuity,'' *Vision Research*, Volume 17, 1977, pp. 941-947.

Yang and Makous 94
Jian Yang and Walter Makous, ''Spatiotemporal Separability in Contrast Sensitivity,'' *Vision Research*, Volume 34, Number 19, October 1994, pp. 2569-2576.

Yang *et al.* 95
Jian Yang, Xiaofeng Qi, and Walter Makous, ''Zero Frequency Masking and a Model of Contrast Sensitivity,'' *Vision Research*, In Press.