15.3: *Late-News Paper:* Jagged Edges and Their Visibility

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Abstract

An investigation of visual sensitivity to jagged edges has led to a new conceptualization of the problem. We present preliminary data from a jaggies experiment and describe the pattern along a discrete approximation to an edge in terms of multiple levels of jags.

Introduction

To display text and graphical objects on digital devices such as computer screens and printers, discrete approximations must be generated and presented. Quantization of analog representations may involve spatial, temporal, luminance and/or chromatic channels. For presentations intended for the human eye, visual artifacts that detract from the task to be performed must be identified and, if feasible, avoided or eliminated. Because the human visual system has finite resolving ability in each of these dimensions, we are interested in identifying the conditions under which visual artifacts arise and the attendant requirements to improve display technology.

One of the most commonly-recognized visual artifacts encountered in computer displays is often referred to as *jaggies*: the jagged spatial profile of oblique contours that are intended to be smooth. To attenuate the severity of jaggies, many techniques have been developed for generating so-called 'anti-aliased' lines and edges, whereby a filtering process distributes gray pixels along a contour, giving rise to a perceived improvement in the smoothness of the contour.

Previous work on jaggies has focused on mathematical models of contours as well as on algorithms for their generation [Bresenham 65, Brons 74, Hung 85]. In earlier work, we considered one aspect of filtered edges, the use of grayscale to position edges with sub-pixel precision. We presented results of an experiment delineating the relationship between the luminance of an undetectable gray strip interposed along a straight black/white contour and the perceived position of the black/white contour [Naiman and Makous 93a] as well as limits on the number of bits of grayscale that can be used in this way [Naiman and Makous 93b]. We now consider the image-quality of filtered edges, in terms of the visibility of jaggies.

While there has been some work on the visibility of edge raggedness and blur [Hamerly 81, Hamerly and Springer 81, Tyler and Mitchell 77], we know of no systematic accounts of the effects of spatial and luminance quantization on the image quality of edges and lines. We are particularly interested in issues relating to image quality: 1) Under what conditions can jagged edges be detected? 2) Though detectable, under what conditions are jagged edges 'acceptable'? 3) How does the use of gray pixels along the contour affect the detectability and acceptability of jagged edges? Here, we address the first of these questions.

Our interest here lies primarily with high-contrast – i.e., black and white – edges. For present purposes, we restrict consideration to jagged edges that are specified by integral-coordinate endpoints. The slopes of such edges are rational numbers that can be specified by a pair of integers, which we refer to as n (numerator) and d (denominator). Without loss of generality, we further restrict our discussion to jagged edges in the first octant; i.e., $0 < (n, d)$ and $n \leq d$. We assume that the display has a rectangular grid of pixels, the boundaries of which are aligned with the horizontal and vertical axes (as in an LCD display). We have factored out all display nonlinearities we could identify [Naiman and Makous 92].

Jagged Edge Experiment

To determine the human visual system's ability to detect jagged edges along a straight contour, we have embarked on a series of experiments using stimuli of the sort displayed in Fig. 1. Two parallel edges are presented, differing only in the size of the simulated pixels used to compose the edges; the subject must choose which of the edges appears more jagged. For different edge slopes, blocks of 50 such forced-choice trials were run at different distances, and the results were fit to a psychometric function to estimate the threshold viewing distance, defined as the distance at which responses are 75% correct. We define acuity as the reciprocal of the visual angle (in minutes) that a single pixel subtends at the threshold viewing distance. Note, however, that the orientation of the jaggies relative to the orientation of the edge varies with the slope of the edge, as does the amplitude of the fundamental

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frequency of the jaggies. To compensate for this latter effect, we multiplied the threshold visual angle by the cosine of the angle of the edge relative to horizontal.

Fig. 2 shows acuity for one observer as a function of edge slope (on semi-log axes), for trials in which d was 150 and n was 1, 2, 4, 8, 16, 38, 75, 100, 134, 146, 149 or 150. The straight lines are regression fits to the data clustered around each line, the significance of which is discussed below.

Composition of Jagged Edges

To understand the data presented in Fig. 2, let us examine the jagged profiles for one period of a $d = 16$-edge. In Fig. 3a, $n = 1$; i.e., there is one pixel of vertical displacement (VD) for every 16 pixels of horizontal displacement (HD). This regular pattern of jaggies is repeated for arbitrarily-long edges with a slope of 1/16. From Fig. 2, we know that our sensitivity to this jagged profile is maximal. Notationally, we represent this as {16}, signifying that there are 16 HDs for each VD.

In Figs. 3b, 3c and 3d, $n = 2$, 4 and 16, respectively, with one VD for every 8, 4 and 1 HD, respectively. Notationally, this can be represented as $\{8^2\}$, $\{4^4\}$ and $\{1^{16}\}$ respectively (where the exponent indicates a repetition factor); however, as the subpatterns are all identical, an equivalent, simpler notation is {8}, {4} and {1}, respectively. In all of these cases, there is an integral number of HDs for each VD. Consequently, the qualitative characteristic of regularity in the jaggies is maintained, and our sensitivity to the jagged profile diminishes exponentially (cf. Fig. 2). The solid line in Fig. 4 simulates acuity for these n (as well as for $n = 8$).

Now let us consider a slope of $n = d - 1$. The trend plotted along the solid line in Fig. 4 suggests that sensitivity to this profile would be relatively minimal. However, examine the profile of this edge (for $n = 15$) in Fig. 3e. As only $d - 1$ VDs must be made for d HDs, there is a single occurrence of a two-pixel HD, seen in the middle of the figure. In words, the profile pattern consists of 7 repetitions of 1 HD for every VD, followed by a single instance of 2 HDs for 1 VD, followed by another 7 repetitions of 1 HD for every VD. When more than one period of an edge with this slope is presented, the two end runs of 7 repetitions become consecutive, resulting in a pattern that consists of 14 repetitions of 1 HD for every VD, followed by a single instance of 2 HDs for 1 VD. In our notation, this is expressed as $\{1^{14}, 2\}$. Thus, the regular pattern that can be described as going up one and over one is interrupted at one location by going up two and over one. We regard any such interruption of a regular pattern of jags as a *higher-level* jag, in this case, a *second-level* jag. This second-level jag makes the jaggedness of the $n = d - 1$ profile much more visible than that of the $n = d$ profile, as is seen in the data of Fig. 2. To appreciate this, the reader is invited to view Fig. 3 from a sufficient distance to notice that the jagged edge in Fig. 3d can be undetectable while the one in Fig. 3e is clearly visible.

In Fig. 3f, $n = 14$, there are 2 two-pixel HDs, and the pattern could be expressed as $\{1^6, 2, 1^6, 2\}$. However, as there is a subpattern that is repeated an

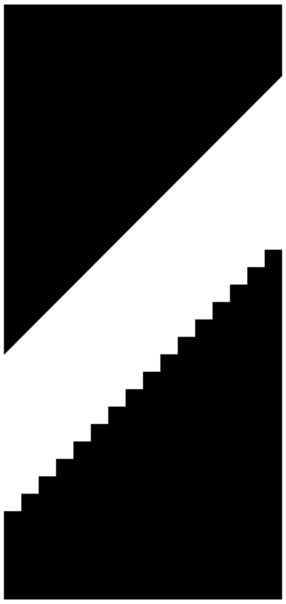


Figure 1: Stimulus. A representative edge stimulus with a slope of 1 is shown. In this case, the top edge appears continuous, while, on the bottom, simulated pixels present an edge with $n = d = 16$.

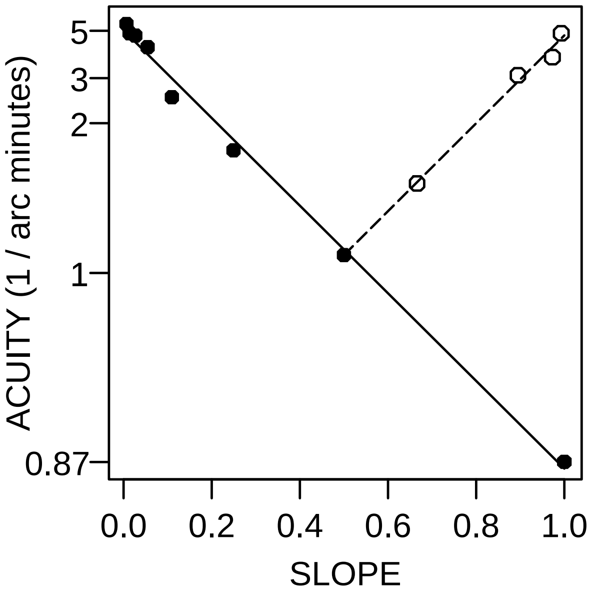
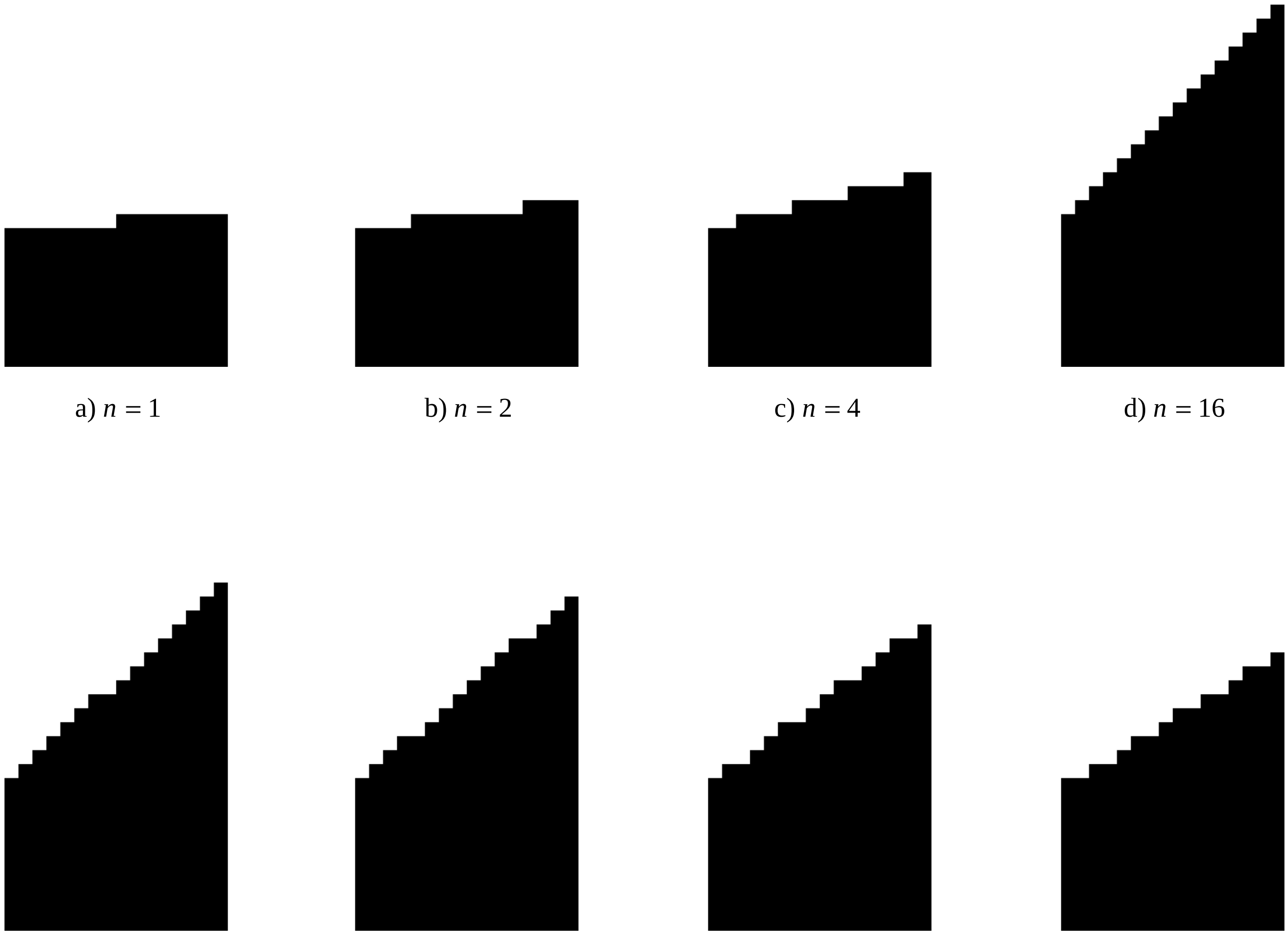


Figure 2: Detectability Data. Acuity of jaggies is plotted on a log axis for one subject, where $d = 150$. The filled symbols correspond to slopes where n is 1, 2, 4, 8, 16, 38, 75, or 150, and the open symbols correspond to n = 100, 134, 146, or 149. The significance of the regression fits to the two sets of data is discussed in the text.



a) $n = 1$ b) $n = 2$ c) $n = 4$ d) $n = 16$

e) $n = 15$ f) $n = 14$ g) $n = 12$ h) $n = 10$

Figure 3: Jagged Edges. Jagged edge profiles are shown for $d = 16$ and various n.

integral number of times, we can simplify this expression to $\{1^6, 2\}$. In Fig. 3g, $n = 12$, there are 4 two-pixel HDs and the pattern is expressed as $\{1^3, 2\}$. Although the number of two-pixel HDs has changed in these last two instances, there is a regularity to the second-level jaggies superimposed on the first-level jaggies, as there was to the first-level jaggies superimposed on the zeroth-level jaggies (i.e., a straight line).

Now consider $n = 10$, as shown in Fig. 3h. Here, there are 10 VDs for every 16 HDs and the profile pattern is $\{2^2, 1, 2, 1\}$, which is repeated a second time within the period. Without the initial 2 in the sequence, the pattern would be $\{2, 1\}$ (repeated a second time), which consists entirely of first- and second-level jags. However, the initial 2 interrupts this regular pattern, and this interruption represents a *third-level* jag.

If the visibility of jaggedness along a straight contour is determined uniquely by the amplitude of the fundamental spatial frequency [Hamerly and Springer 81], then second-level jags would be just as visible as first-level jags of equal spatial frequency. Therefore, we would expect a plot of relative acuity (on a log axis) to look like the theoretical data portrayed in Fig. 4 for $d = 16$; the numbers along the x-axis correspond to the first- and second-level n discussed above. The solid dots represent the hypothetical data, the solid line segment connects data points of edges with first-level jags, and the dashed line connects data points of edges with second-level jags that lie between the two data points corresponding to $n = 8$ and $n = 16$.

The fit of the data to the lines in Fig. 2 (where $d = 150$) supports the assumption on which the

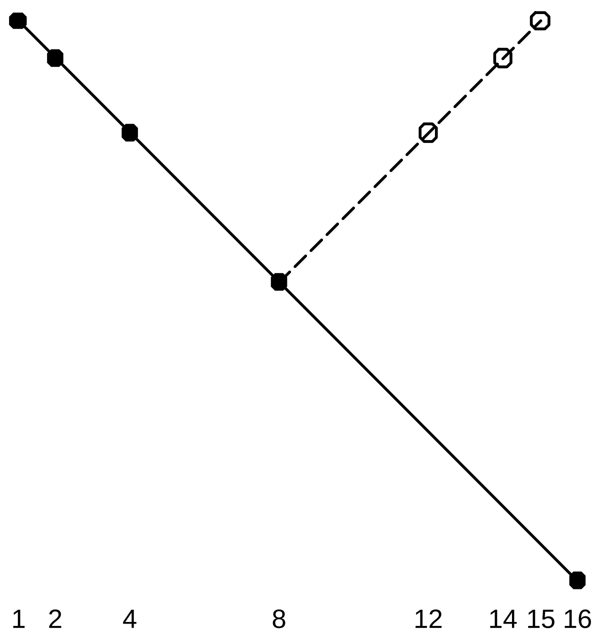


Figure 4: First- and Second-Level Jags. The hypothetical acuity data for $d = 16$ and $n = 1, 2, 4, 8, 12, 14, 15$ and 16 are plotted (on semi-log axes).

theoretical plot in Fig. 4 is based. This was, in part, due to our deliberate choice of n, so that only first- and second-level jags (between $n = 75$ and $n = 150$) were present in the experiment. As noted above, the edge corresponding to $d = 16$ and $n = 10$ is composed of first-, second- and third-level jags. Determination of its visibility – and that of all edges with higher-level jags – awaits the results of additional data collection.

Conclusions

Initial investigations into the visibility of jagged edges have resulted in quantitative data and in a novel conceptualization of their visibility in terms of multiple levels of jags. A theoretical graphical portrayal of acuity and jag levels is supported by the data collected so far. The utility of both the data and the conceptualization will be in posing and answering future questions about sensitivity to jaggies and guiding both manufacturers and users of digital display technologies in choosing edge orientations that have visibility appropriate to given tasks. For example, changing the slope from 15/16 to 16/16 can change the visibility of an edge's jaggedness from maximum to minimum.

Acknowledgements

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