Response to "Reply to Van den Berg and Ma: Robust decision-makers are not omniscient"

We thank De Gardelle and Summerfield for their reply (PNAS 2012, doi:10.1073/pnas.1120640109), but it has not addressed our main concern (PNAS, 2012, doi:10.1073/pnas.1119078109). In contrast to their statement "Rather, an ideal (but not omniscient) observer who simply computed the stimulus response association through feedback in the simplest possible way would perform exactly as our LPR model", an unweighted averaging rule is optimal even if the subject has not learned the exact values of μ and σ used in the experiment. The proof goes as follows. Suppose the observer assumes some distribution $q(\mu,\sigma)$. Then the likelihood of class C (+1 or −1) is

$$\begin{aligned} p(\mathbf{x}\,|\,C) &= \iint q(\mu,\sigma)\, p(\mathbf{x}\,|\,\mu,\sigma,C)\, d\mu d\sigma \\ &= \iint q(\mu,\sigma) \prod_{i=1}^{N} p(x_i\,|\,\mu,\sigma,C) d\mu d\sigma \\ &= \iint q(\mu,\sigma) \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_i - C\mu)^2}{2\sigma^2}} d\mu d\sigma \\ &= \iint q(\mu,\sigma) \left(2\pi\sigma^2\right)^{-\frac{N}{2}} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{N}(x_i - C\mu)^2} d\mu d\sigma \end{aligned}$$

The optimal decision is to report C=1 if $p(\mathbf{x}|C=1) > p(\mathbf{x}|C=-1)$. In other words, we are interested in the sign of the difference,

$$\begin{aligned} &\operatorname{sgn}\left(p(\mathbf{x}\,|\,C=1) - p(\mathbf{x}\,|\,C=-1)\right) = \\ &= \operatorname{sgn}\left(\iint q(\mu,\sigma)\left(2\pi\sigma^2\right)^{-\frac{N}{2}} \left(e^{-\frac{1}{2\sigma^2}\sum_i (x_i-\mu)^2} - e^{-\frac{1}{2\sigma^2}\sum_i (x_i+\mu)^2}\right) d\mu d\sigma\right) \end{aligned}$$

For given μ and σ, the sign of the integrand is equal to the sign of $\left(e^{-\frac{1}{2\sigma^2}\sum_i (x_i-\mu)^2} - e^{-\frac{1}{2\sigma^2}\sum_i (x_i+\mu)^2}\right)$, which, as an easy calculation shows, is equal to the sign of $\sum_i x_i$. This means that the sign of every term in the integral is the same, and the sign of the whole integral is equal to the sign of $\sum_i x_i$. This shows that the optimal observer uses a simple unweighted averaging rule regardless of his belief about the distribution of μ and σ. The LPR rule advocated by De Gardelle and Summerfield is severely suboptimal; it would correspond to the observer using the incorrect belief that for a given C, the stimuli are drawn from the same, non-Gaussian distribution on each trial. Such a belief would not be supported by the feedback received by human subjects. Suboptimal behavior by humans would be of great interest in its own right but the burden of proof for such suboptimality is not met by De Gardelle and Summerfield's paper. Our concern is intuitively obvious: it is unclear why an observer would consider some stimuli to provide evidence *too strong* to be taken into account.

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