ARTICLE

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Confidence reports in decision-making with multiple alternatives violate the Bayesian confidence hypothesis

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Confidence refers to the “sense of knowing” that comes with a decision

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Confidence affects the planning of subsequent actions

Bayesian Confidence Hypothesis
Confidence is based on the posterior probability of the chosen category : *p(correct)*

e.g.,
Peirce and Jastrow (1884)
Galvin et al. (1959)
Kepecs and Mainen (2012)
Mamassian (2016)
Kiani and Shadlen (2009)
Kepecs, Uchida, Zariwala and Mainen (2009)
Pouget, Drugowitsch and Kepecs (2016)
Sanders, Hangya and Kepecs (2016)

Bayesian Confidence Hypothesis

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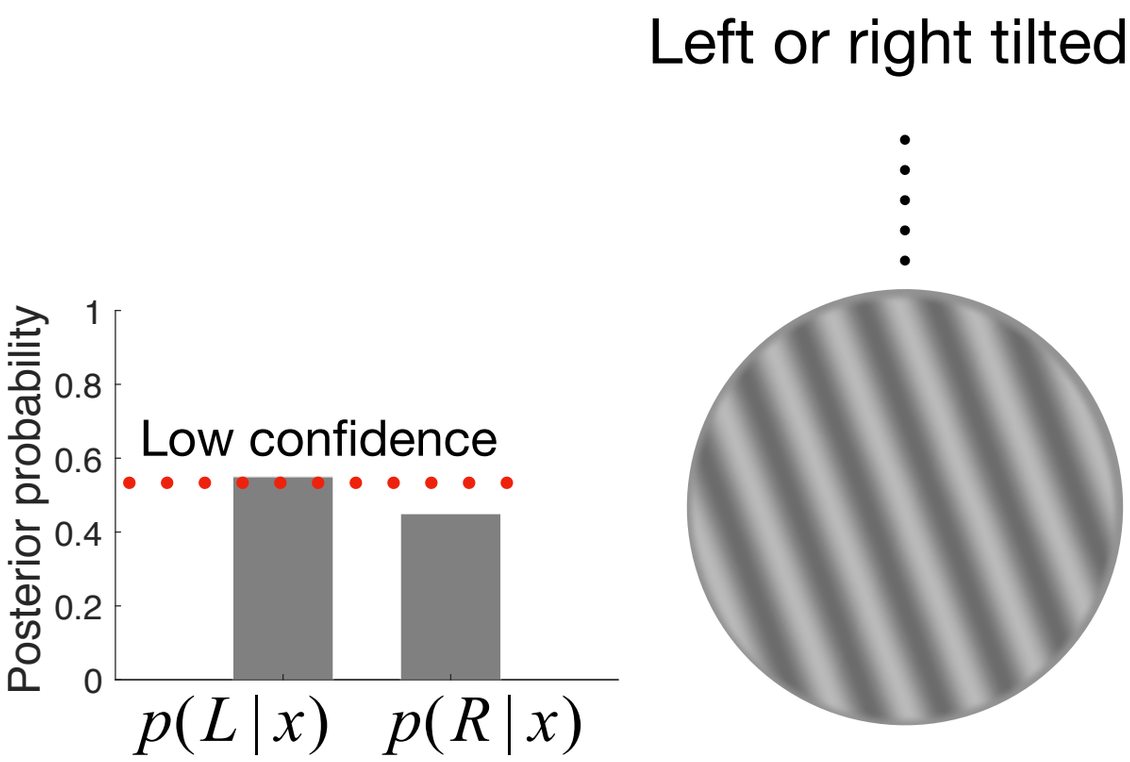

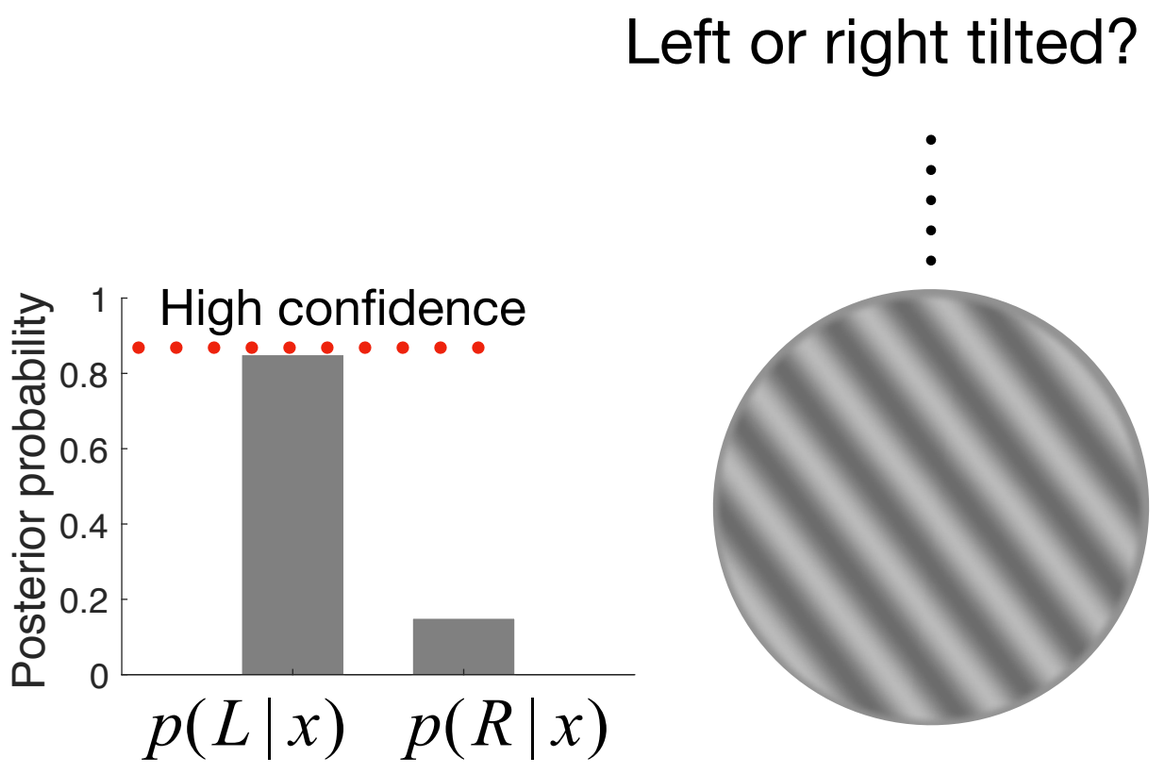

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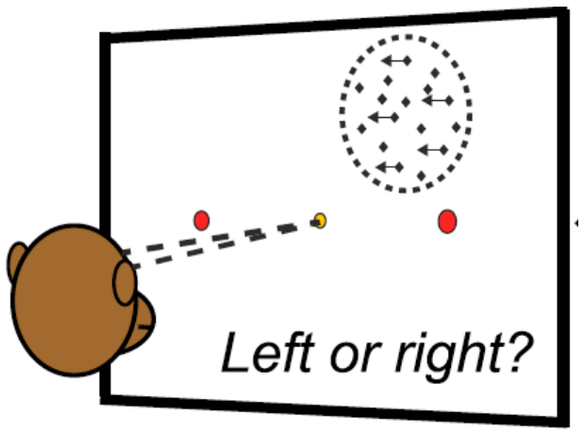
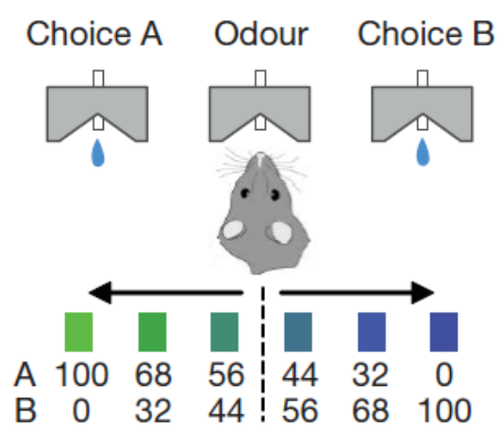
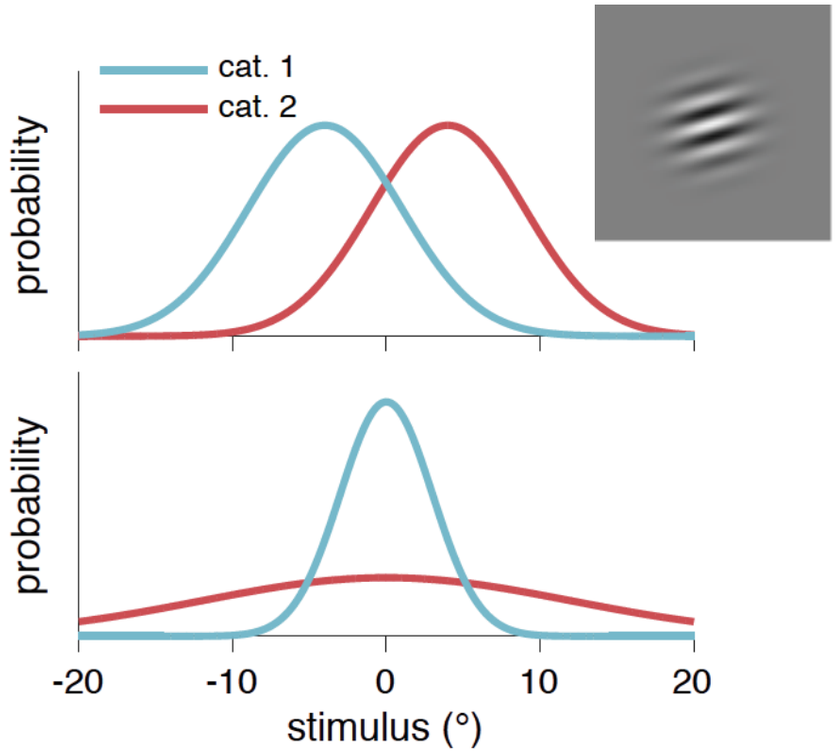


Kiani and Shadlen, 2009
Fetsch et al., 2014



Kepecs et al., 2009



Adler and Ma, 2018

People often face more complex decisions involving multiple choices

People often face more complex decisions involving multiple choices

Distinguish different models of confidence report

A

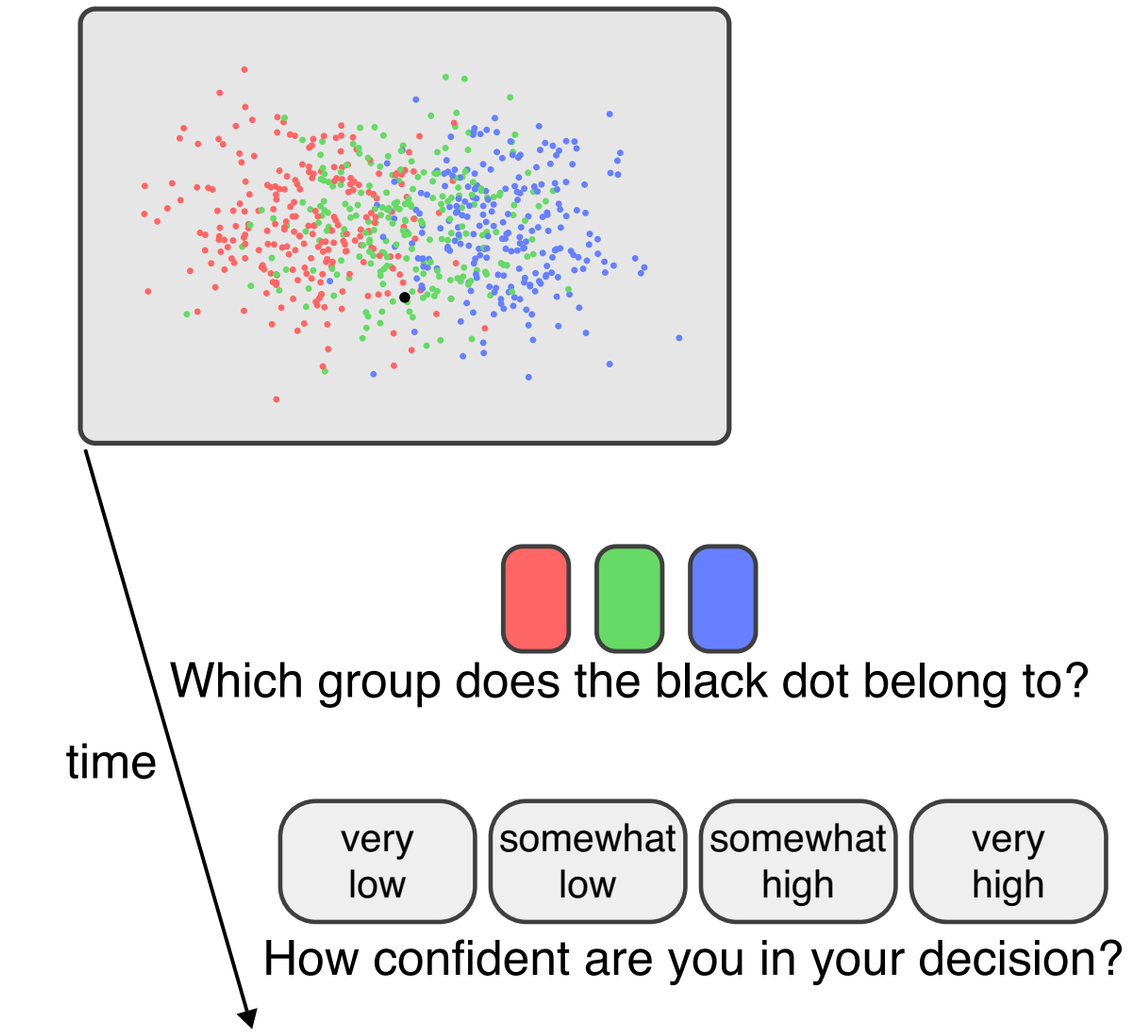
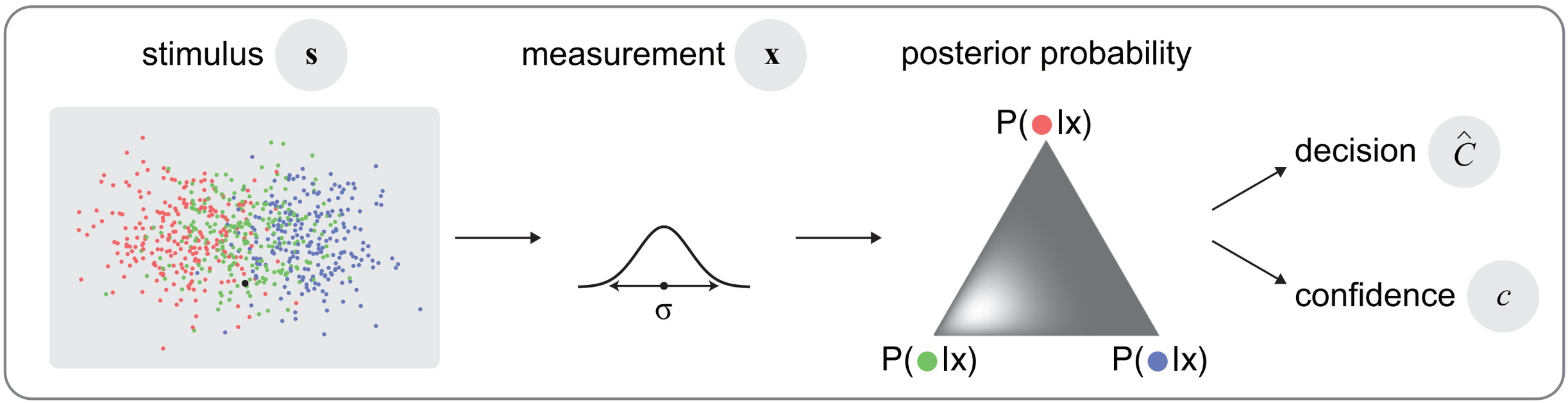


Fig. 1 Experimental procedure and stimuli.

DEMO

A

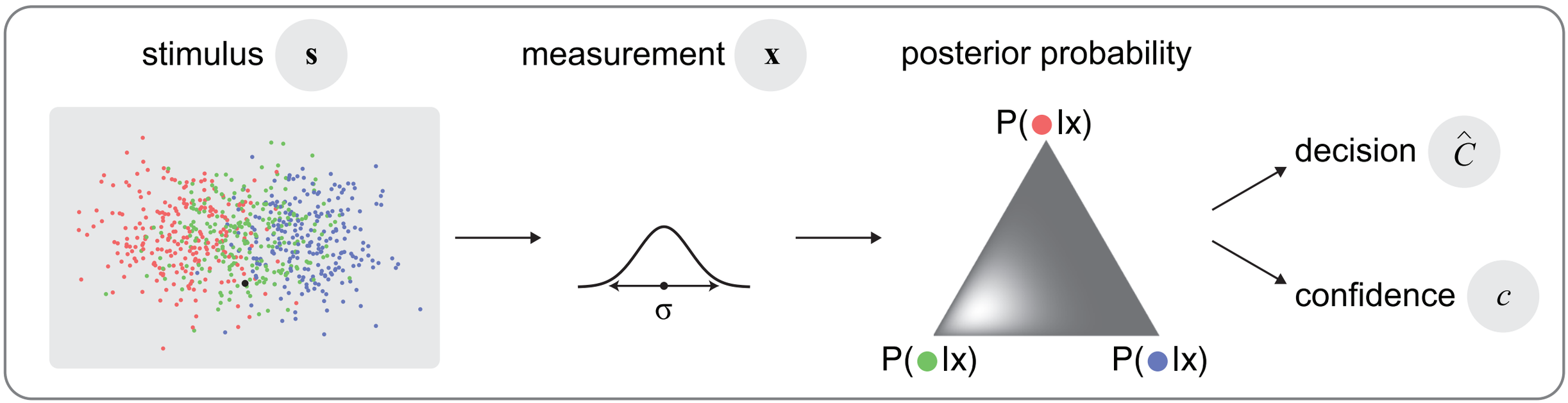


$$p(C \mid \mathbf{x}) = \frac{p(\mathbf{x} \mid C)}{\sum_{C'=1}^{3} p(\mathbf{x} \mid C')} \qquad (1)$$

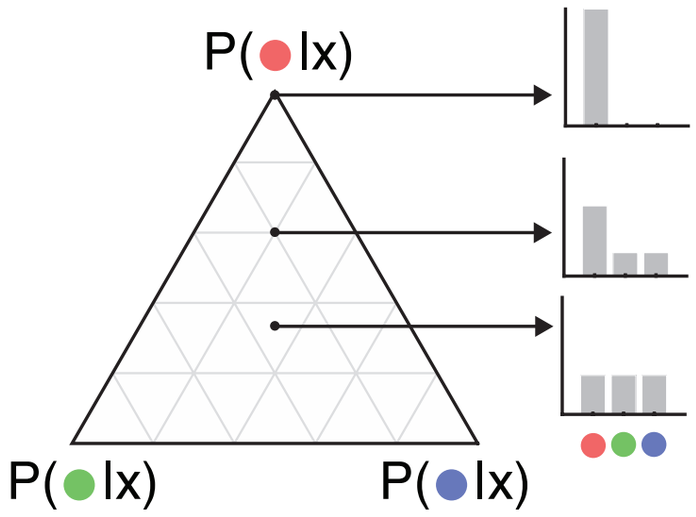
$$p(\mathbf{x} \mid C) = N(\mathbf{m}_C, (\sigma_s + \sigma)\mathbf{I})$$

Fig. 2 Models.

A



B

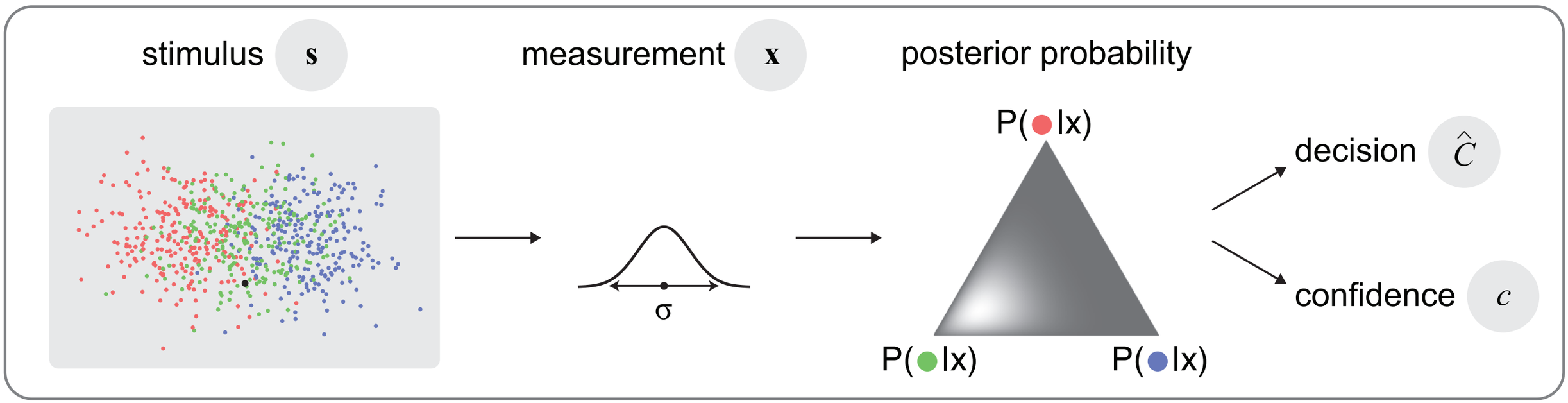


Categorical decision:
Always pick the group with the highest posterior

Confidence report:
Three candidate models
Max, Difference, Entropy

Fig. 2 Models.

A



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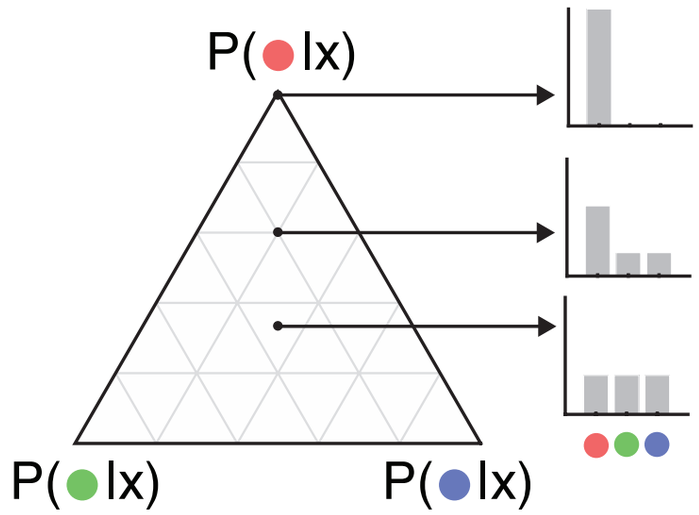


C

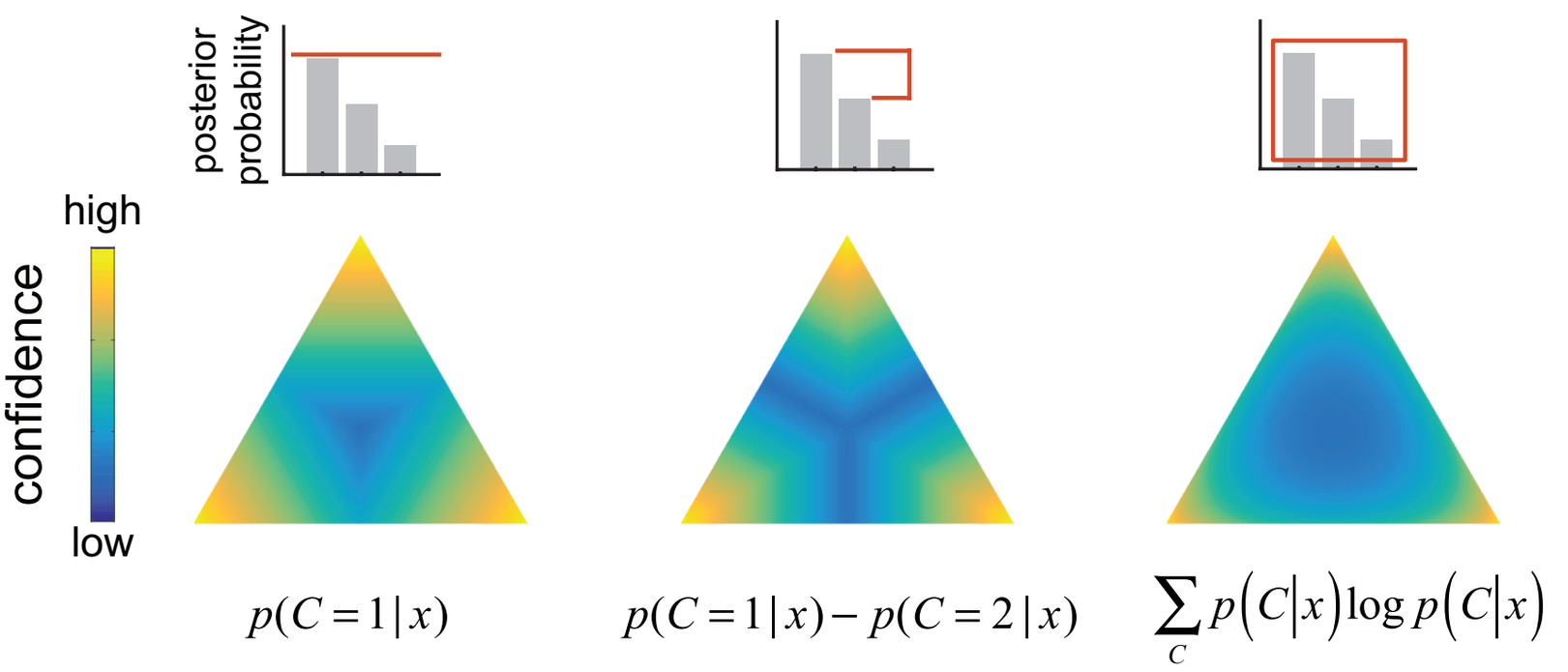


Fig. 2 Models.

Max (Maximum a posteriori)

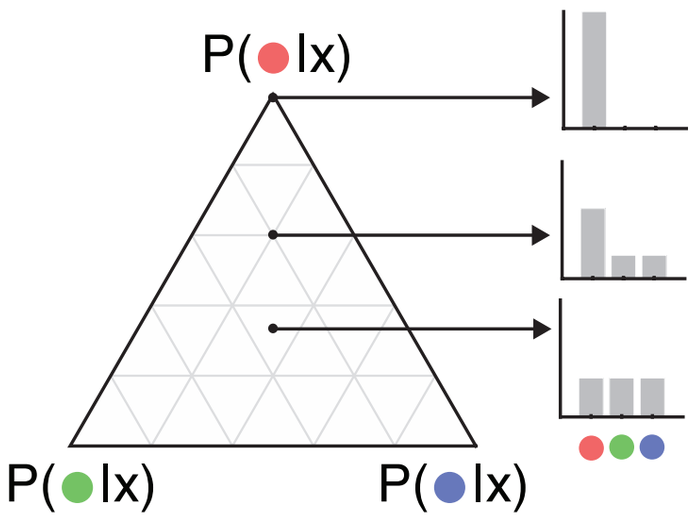
$$p(C=1\,|\,x)$$

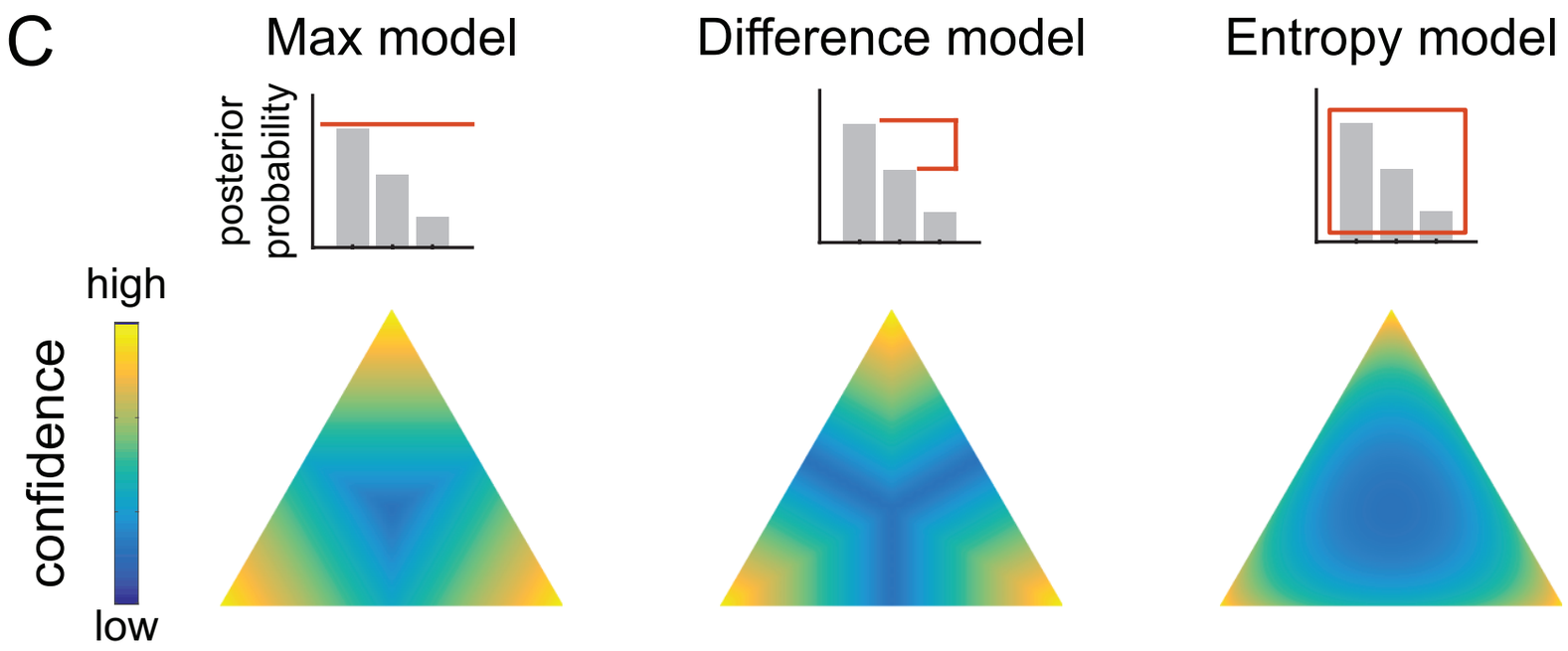
Diff (Difference)

$$p(C=1\,|\,x)-p(C=2\,|\,x)$$

Ent (Negative entropy)

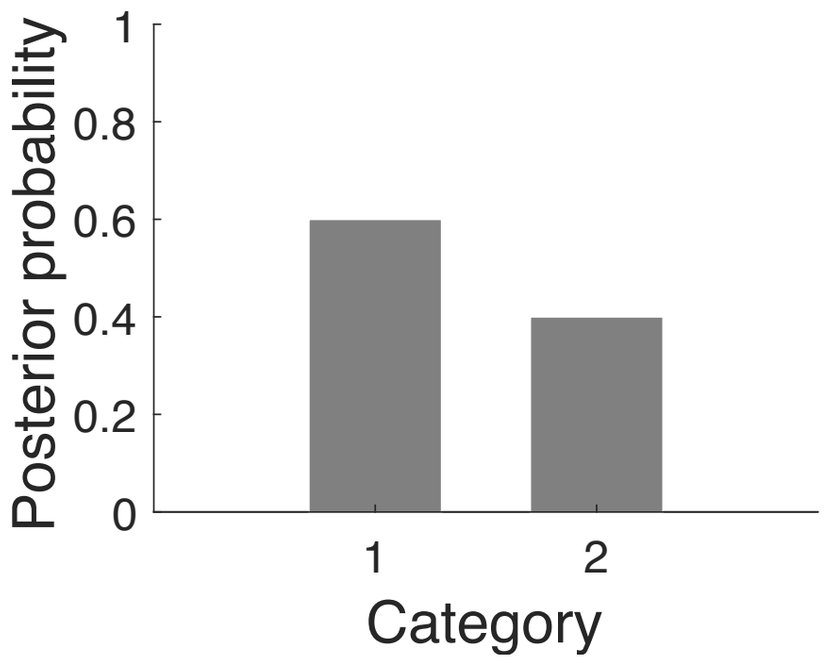
$$\sum_C p\left(C\middle|x\right)\log p\left(C\middle|x\right)$$





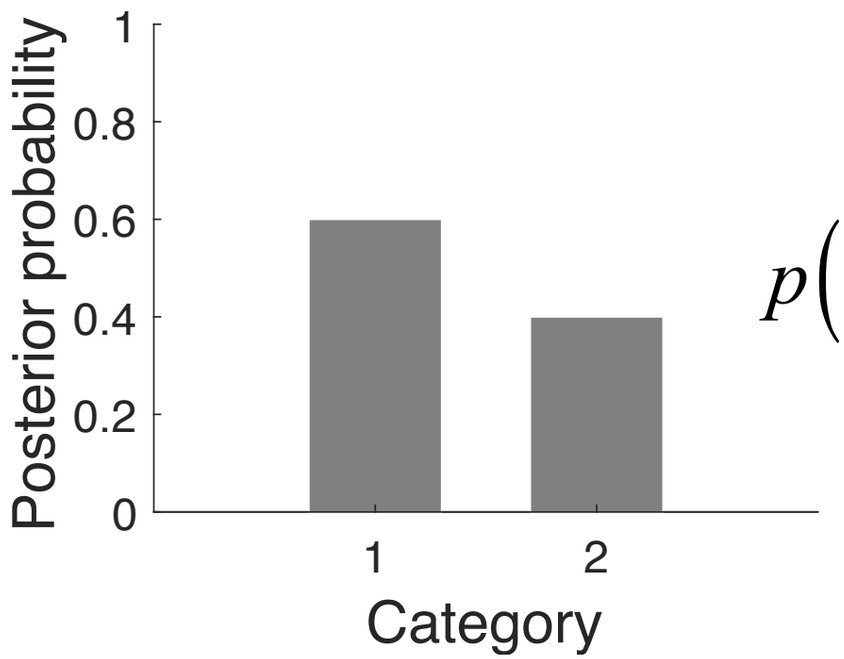
Confidence

Max	$p(C=1\,\vert\,x)$
Diff	$p(C=1\,\vert\,x)-p(C=2\,\vert\,x)$
Entropy	$\sum_C p\left(C\middle\vert x\right)\log p\left(C\middle\vert x\right)$



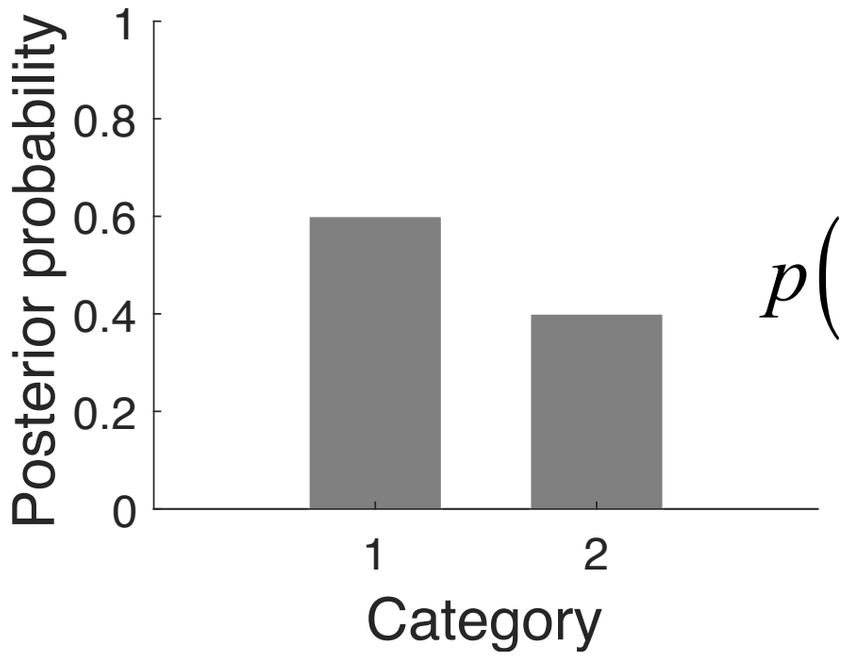
Confidence

Max	$p(C=1\,\|\,x)$
Diff	$p(C=1\,\|\,x)-p(C=2\,\|\,x)$
Entropy	$\sum_C p\left(C\middle\|x\right)\log p\left(C\middle\|x\right)$



$$p\left(C=1\middle|x\right)+p\left(C=2\middle|x\right)=1$$

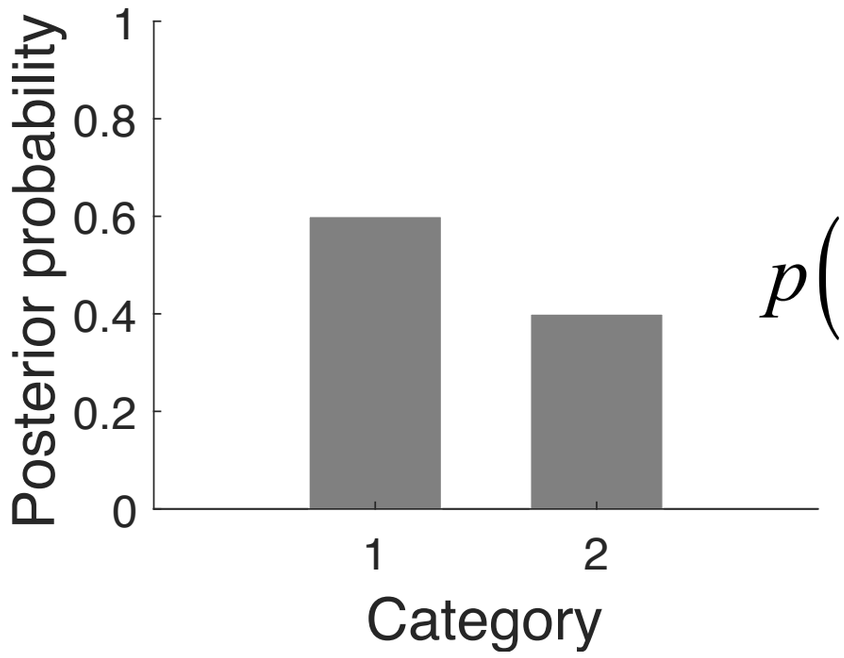
	Confidence
Max	$p(C=1\mid x)$
Diff	$p(C=1\mid x)-(1-p(C=1\mid x))$
Entropy	$p\left(C=1\middle\vert x\right)\log p\left(C=1\middle\vert x\right)+\left(1-p\left(C=1\middle\vert x\right)\right)\log\left(1-p\left(C=1\middle\vert x\right)\right)$



$$p\left(C=1\middle\vert x\right)+p\left(C=2\middle\vert x\right)=1$$

Confidence

$$p(C = 1 \mid x)$$



$$p\left(C = 1 \middle| x\right) + p\left(C = 2 \middle| x\right) = 1$$

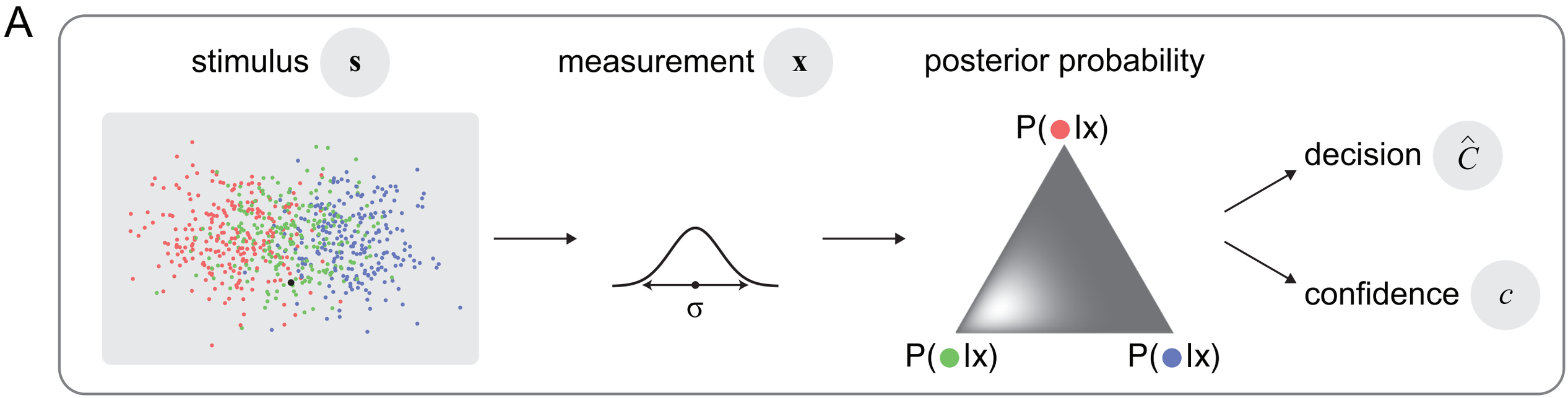


Fig. 2 Models.

Sources of variability:
(1) sensory noise (noisy measurement of the target dot location)
(2) inference noise (decision noise)

surement $\mathbf{x}$

posterior probability

P(●|x)

decision $\hat{C}$

confidence c

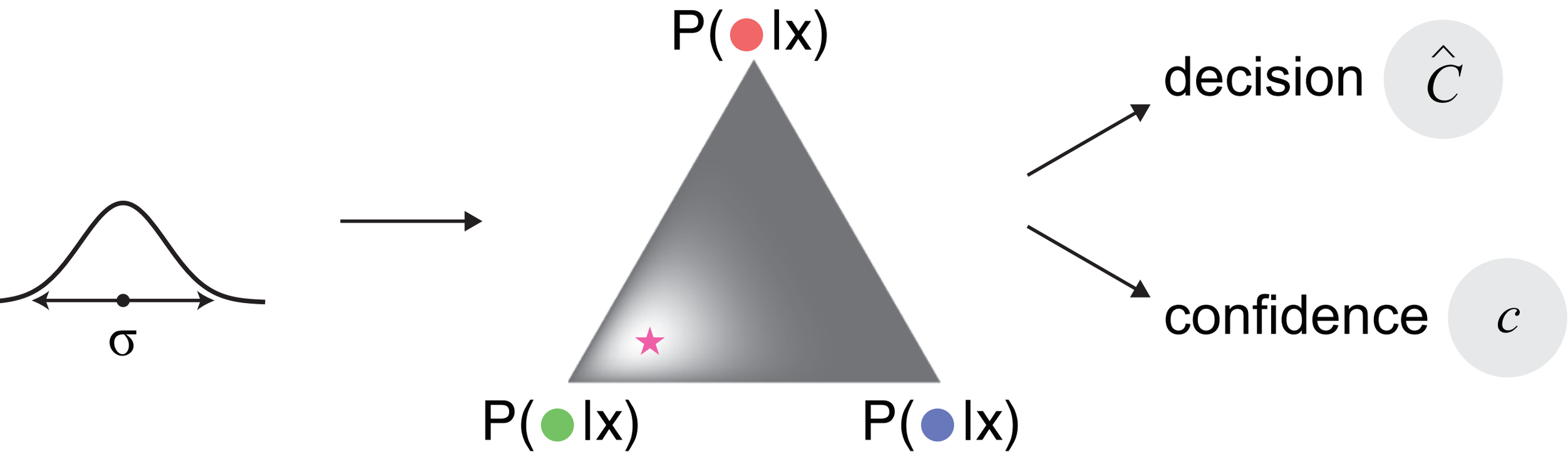
σ

P(●|x)

P(●|x)

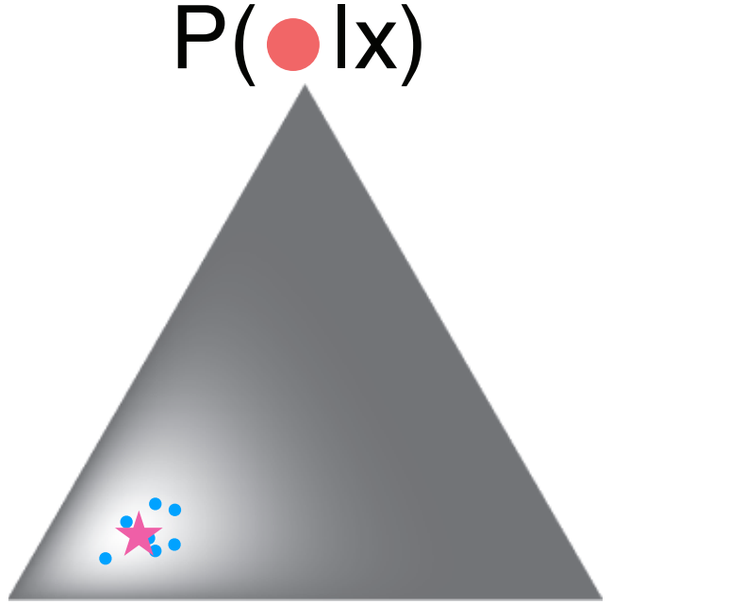
surement $\mathbf{x}$

posterior probability



surement $\mathbf{x}$

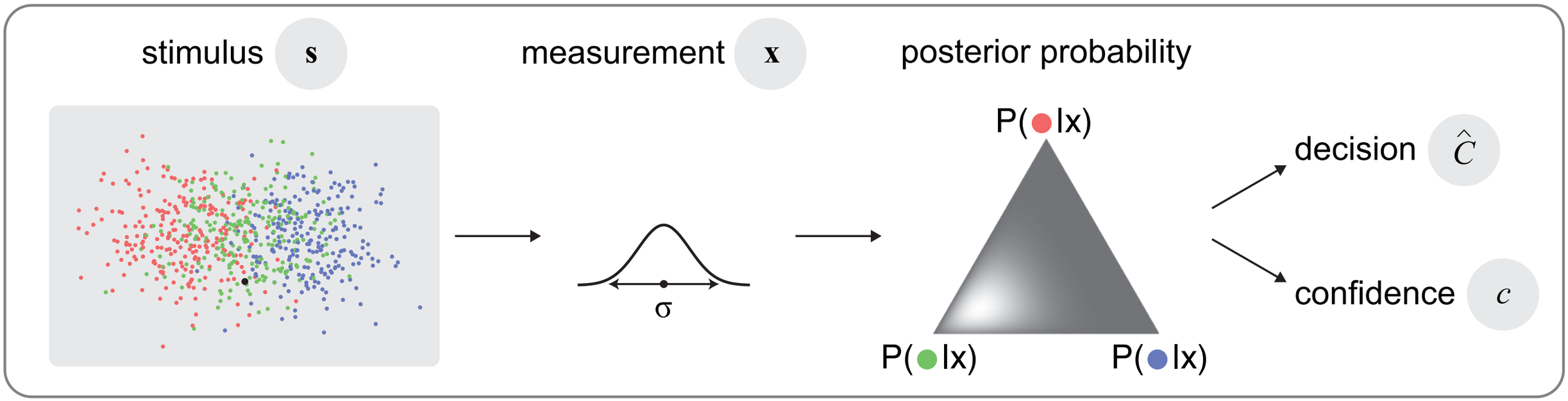
posterior probability



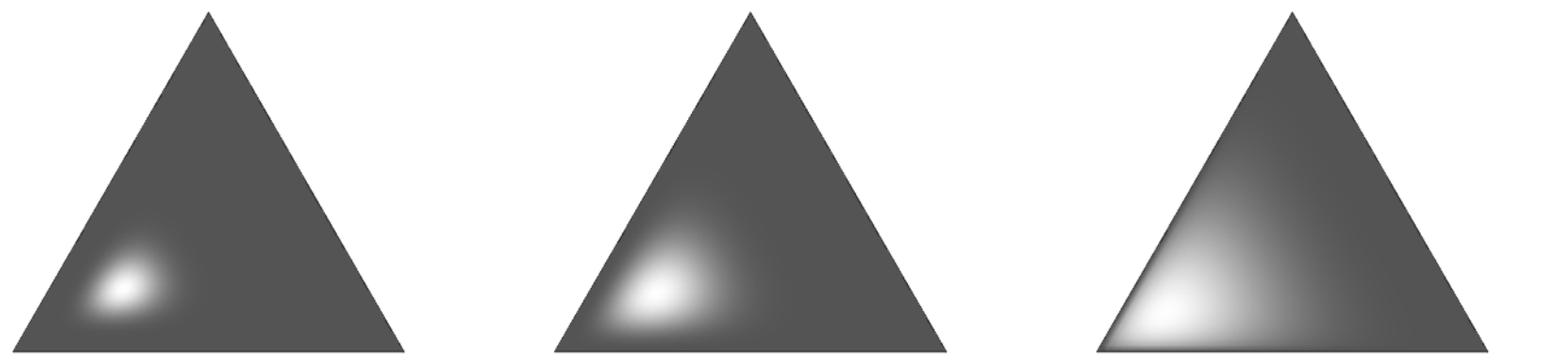
decision $\hat{C}$

confidence c

A



$$\mathbf{q} \sim \text{Dirichlet}(\mathbf{p}; \alpha)$$



low decision noise

high decision noise

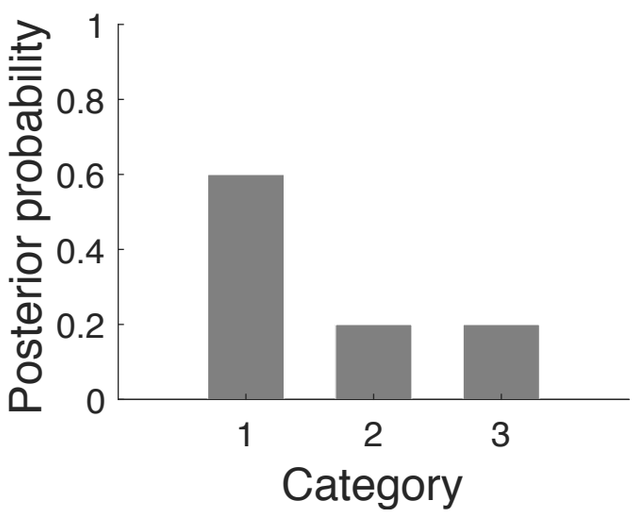
The expected value of $\mathbf{q}$ is $\mathbf{p}$.
The concentration parameter α, a scalar that determines the magnitude of the decision noise

Free parameters

Criteria: b_1, b_2, b_3

Noise: σ, α

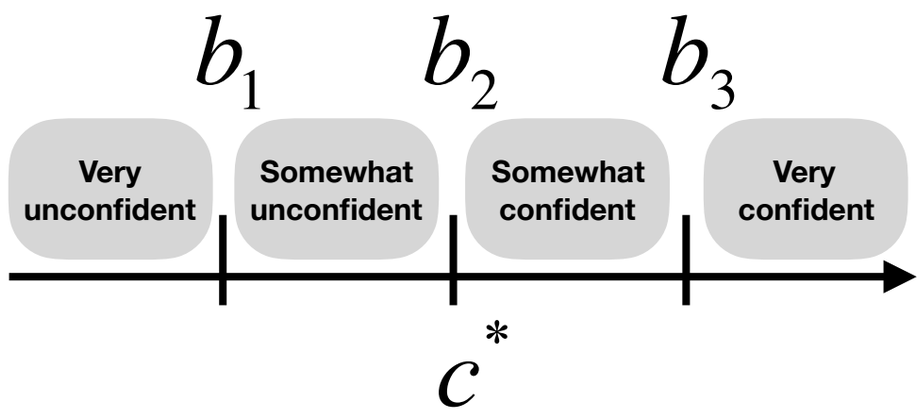
Lapse rate: λ



Max model

Diff model

Ent model



Maximum likelihood estimation

$$L_M(\theta) = \prod_{i=1}^{n} p(d_i, c_i \mid s_i, b_1, b_2, b_3, \sigma, \alpha, \lambda)$$

Free parameters

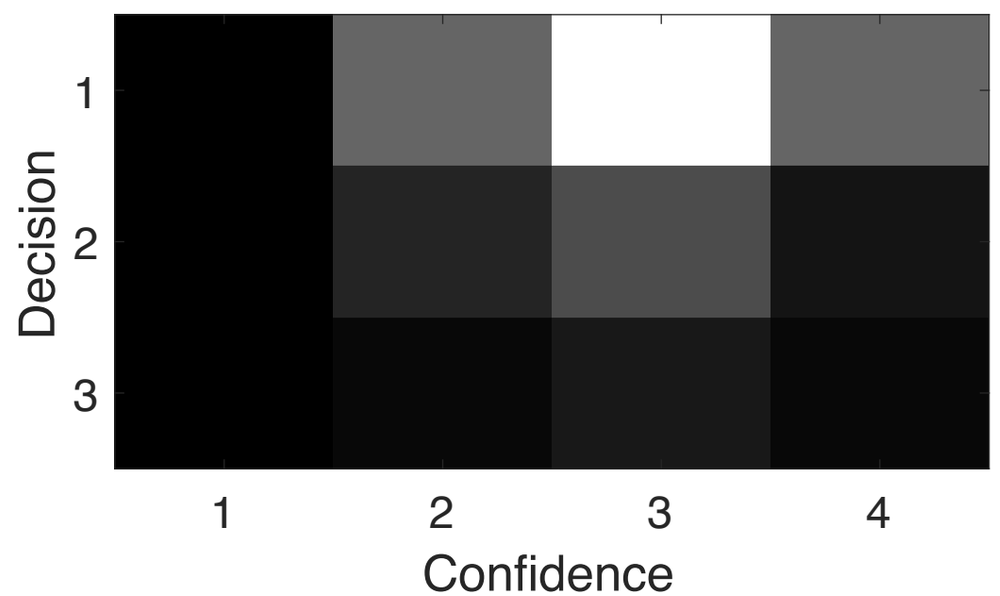
Criteria: b_1, b_2, b_3

Noise: σ, α

Lapse rate: λ

joint probability for trial i

$$p(d_i, c_i \mid s_i, b_1, b_2, b_3, \sigma, \alpha, \lambda)$$



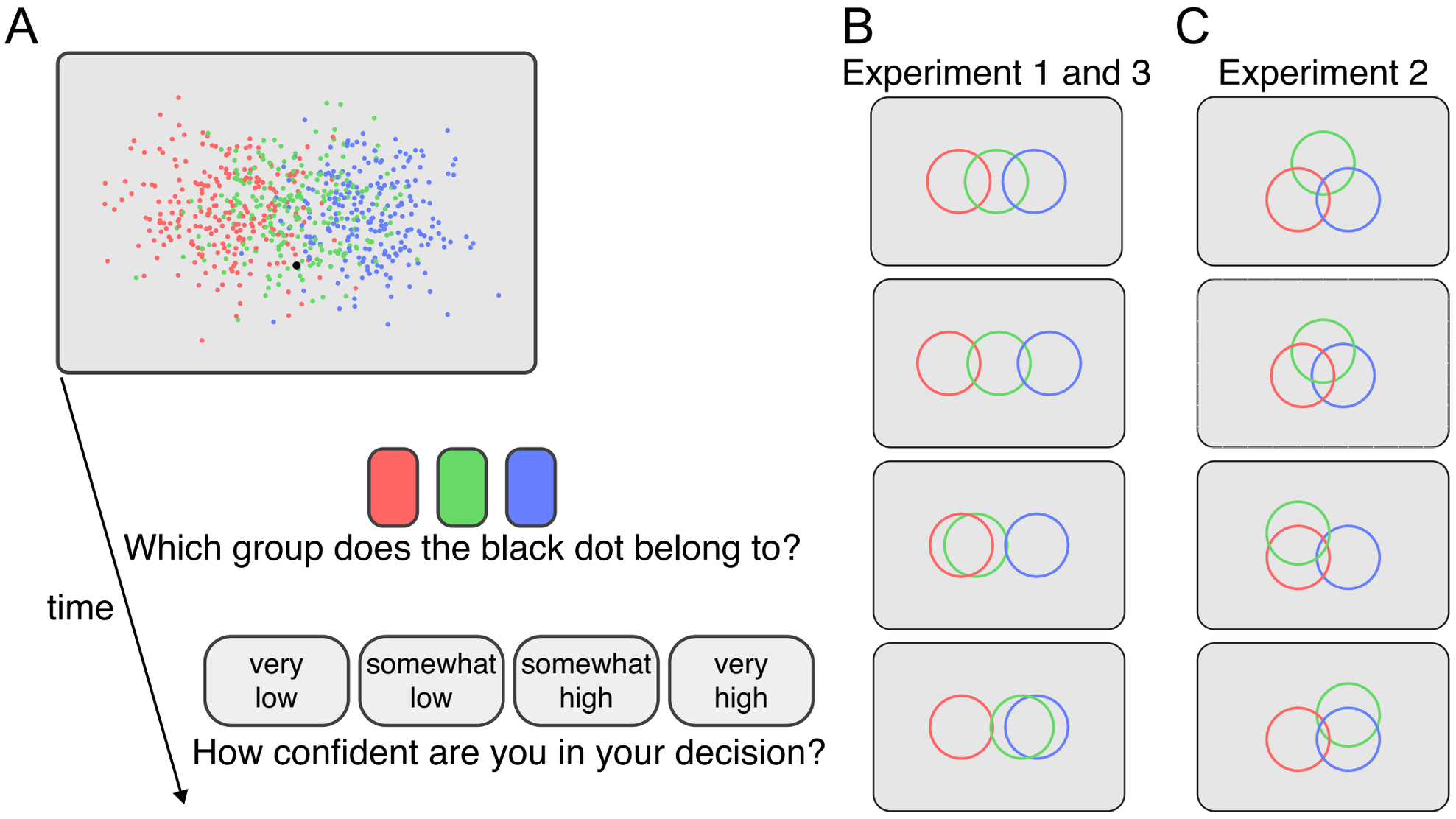


Fig. 1 Experimental procedure and stimuli.

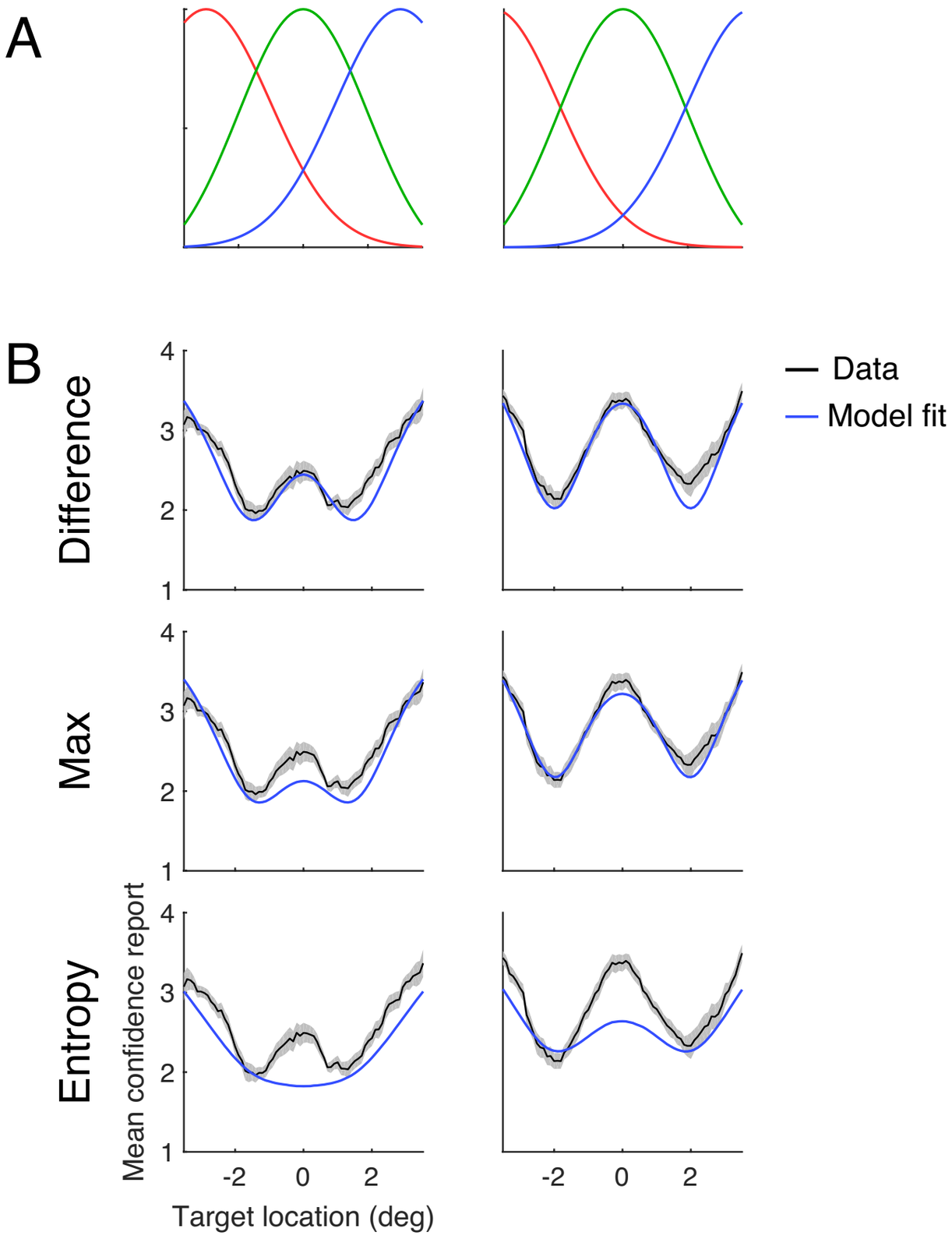
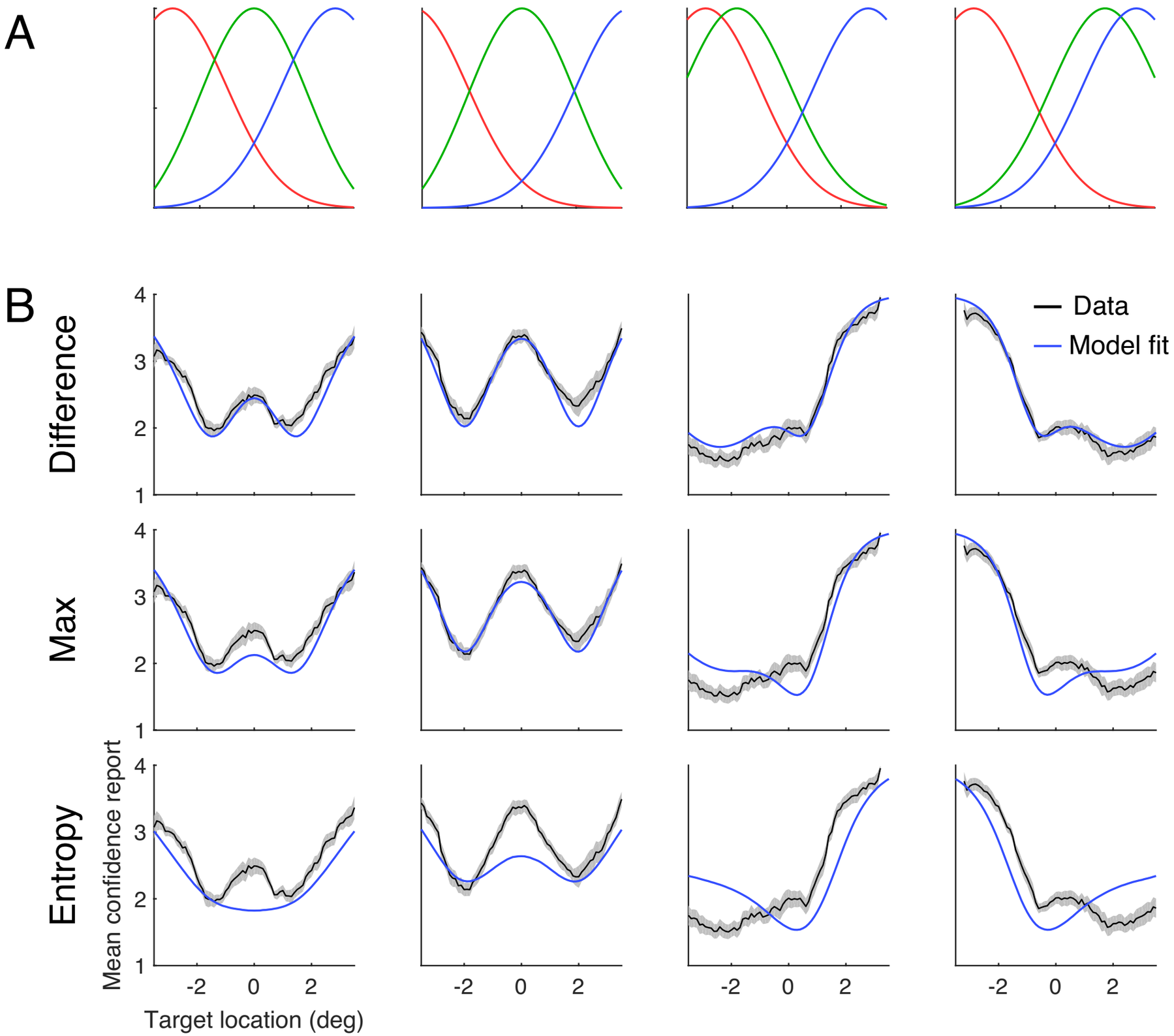


Fig. 3. Experiment 1.



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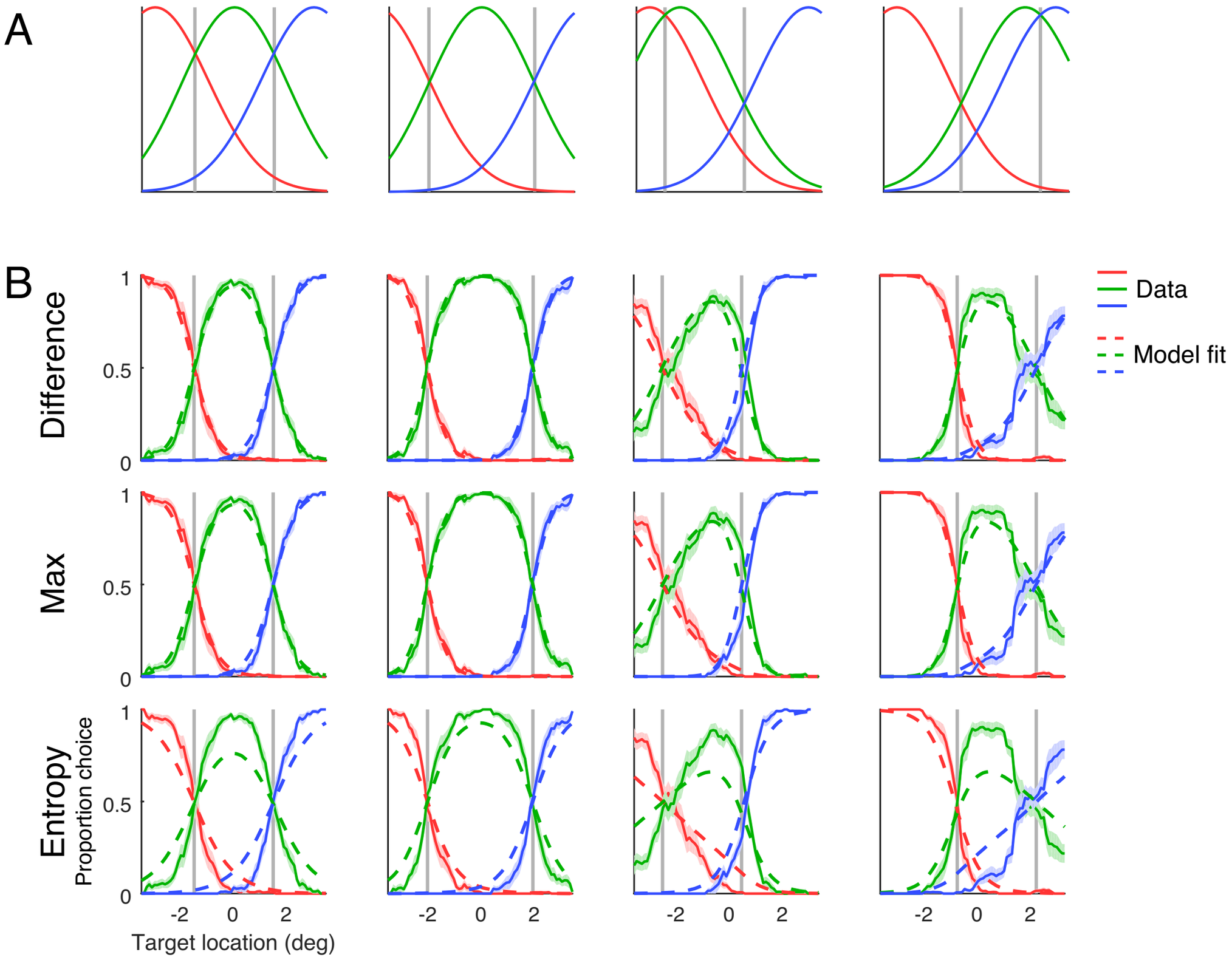


Supplementary Figure 2.

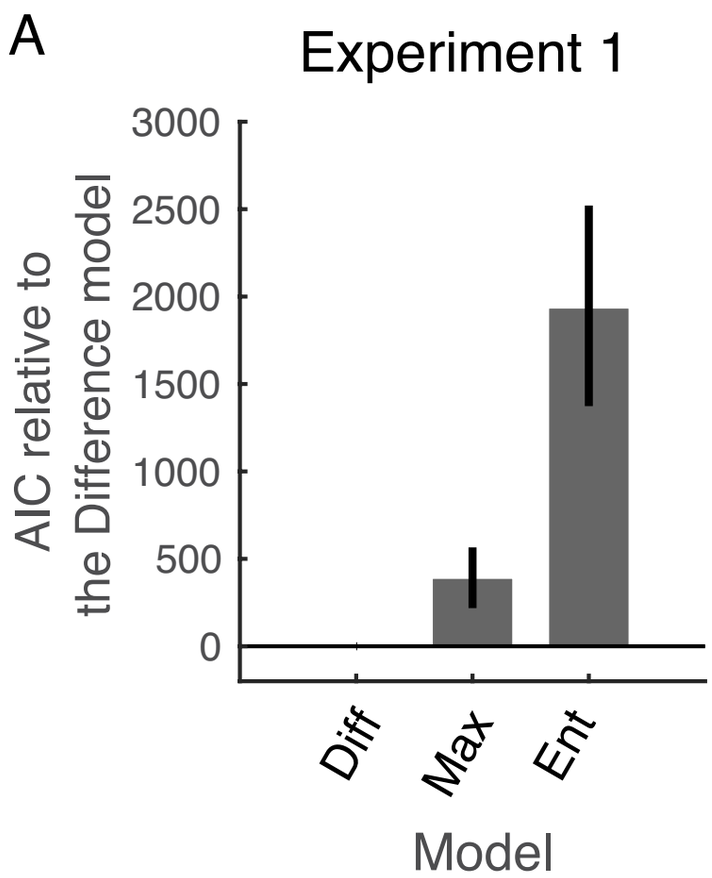
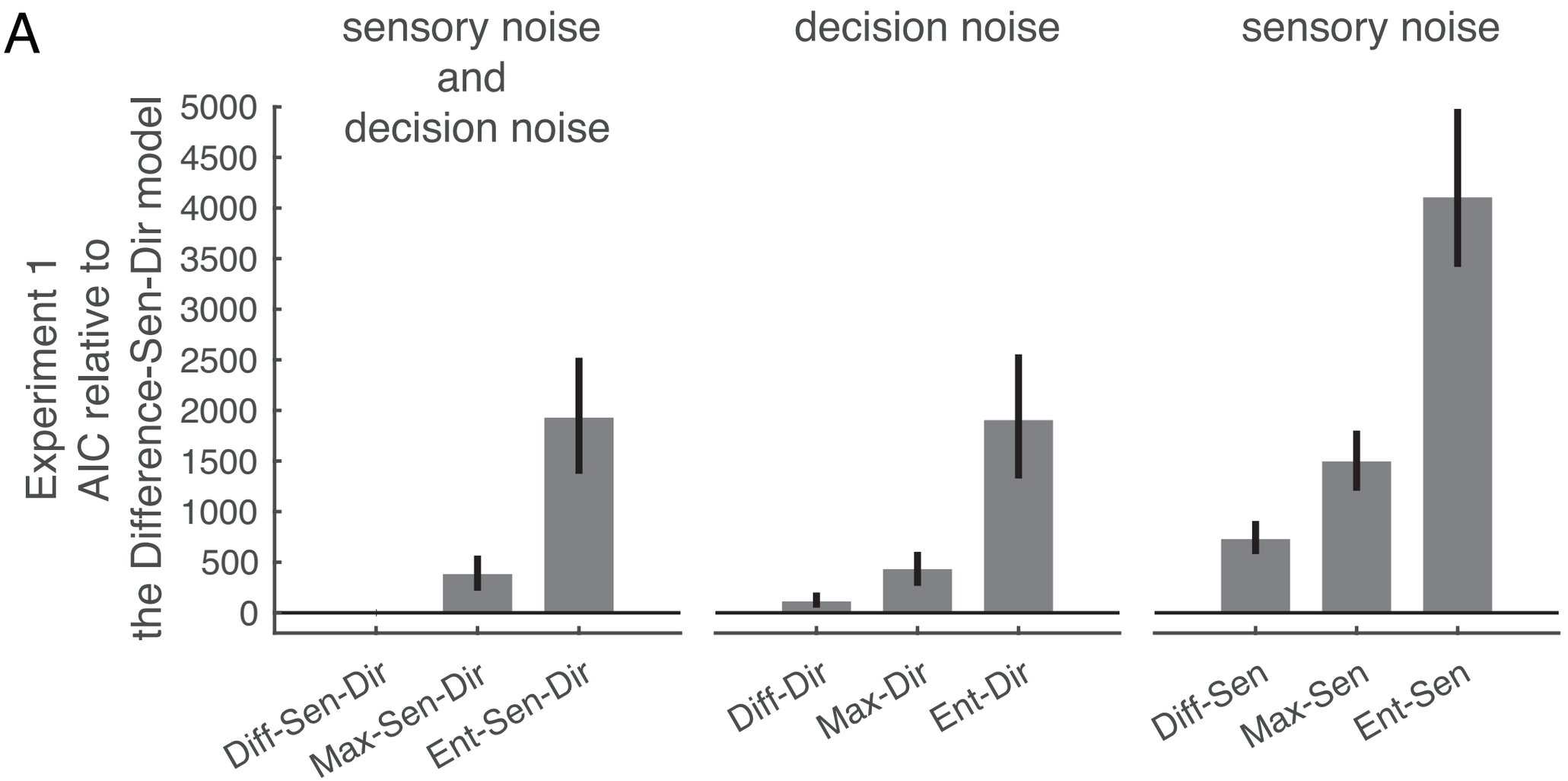


Fig. 4. Experiment 1. Model comparisons using ΔAIC: AIC of each model compared with the Difference model

A



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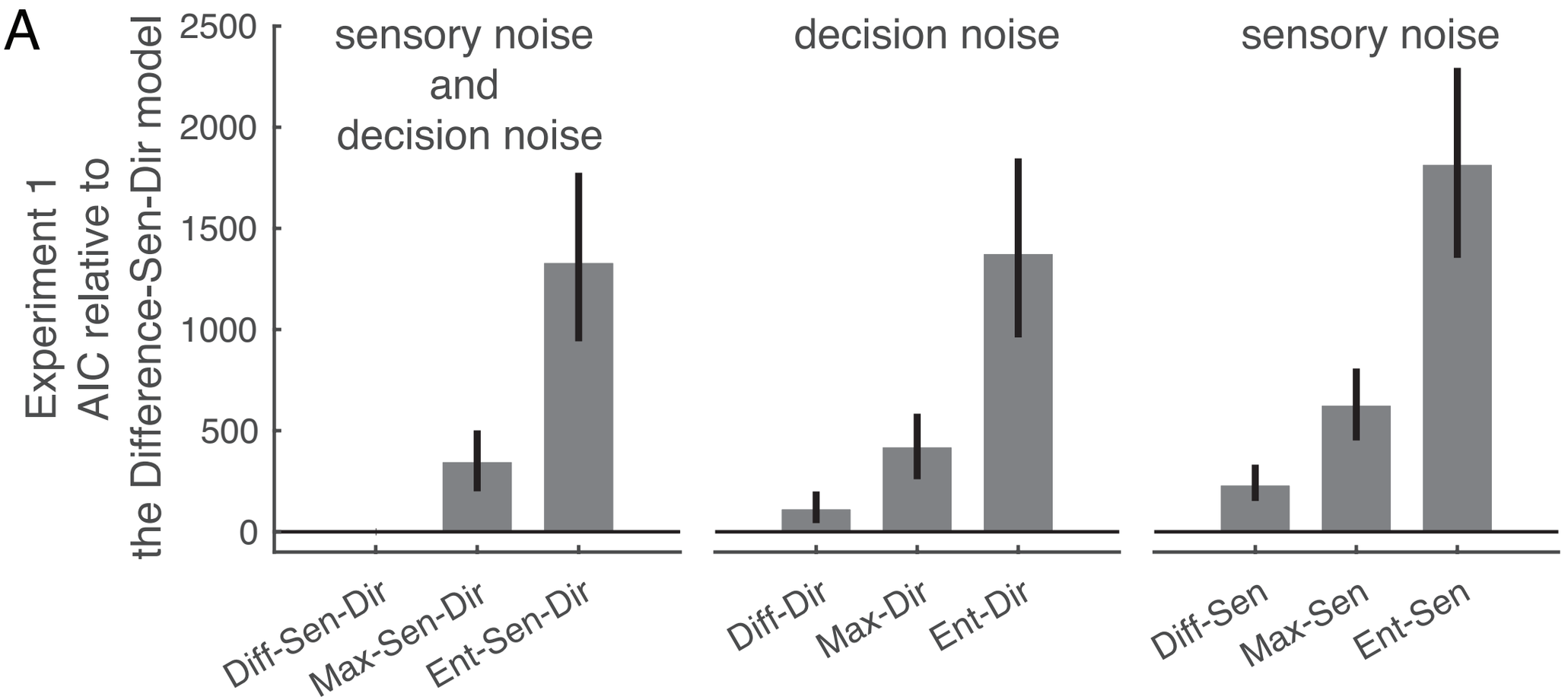
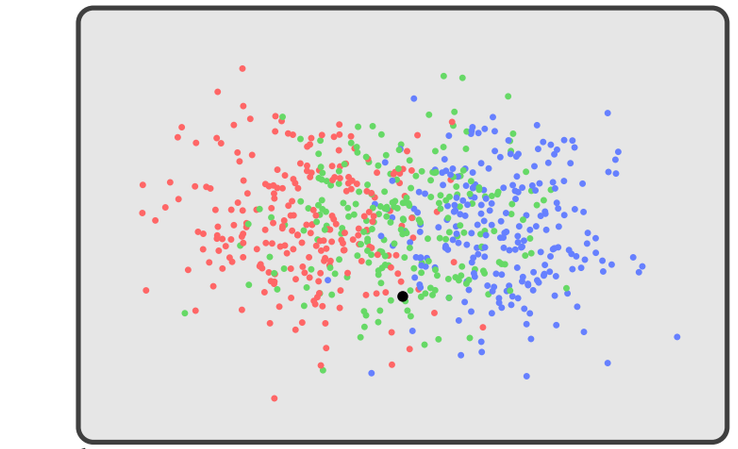


Supplementary Figure 6. Fitting confidence reports only

A



Which group does the black dot belong to?

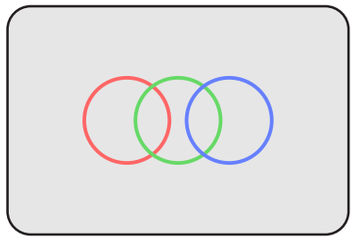
time

very low | somewhat low | somewhat high | very high

How confident are you in your decision?

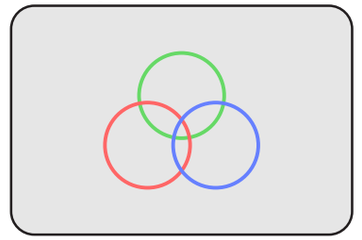
B

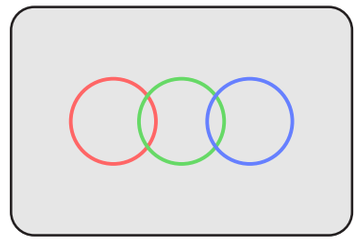
Experiment 1 and 3

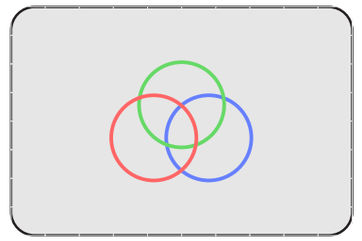


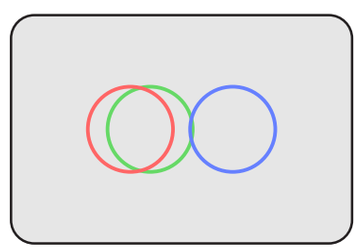
C

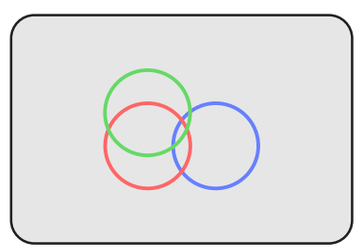
Experiment 2

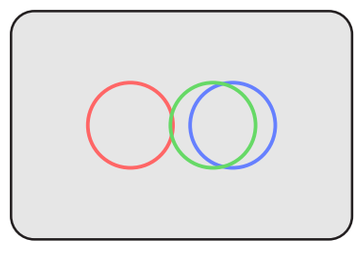


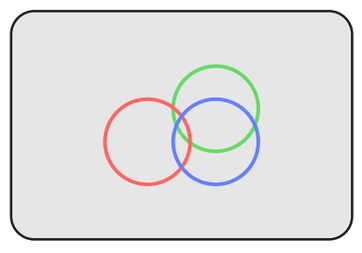


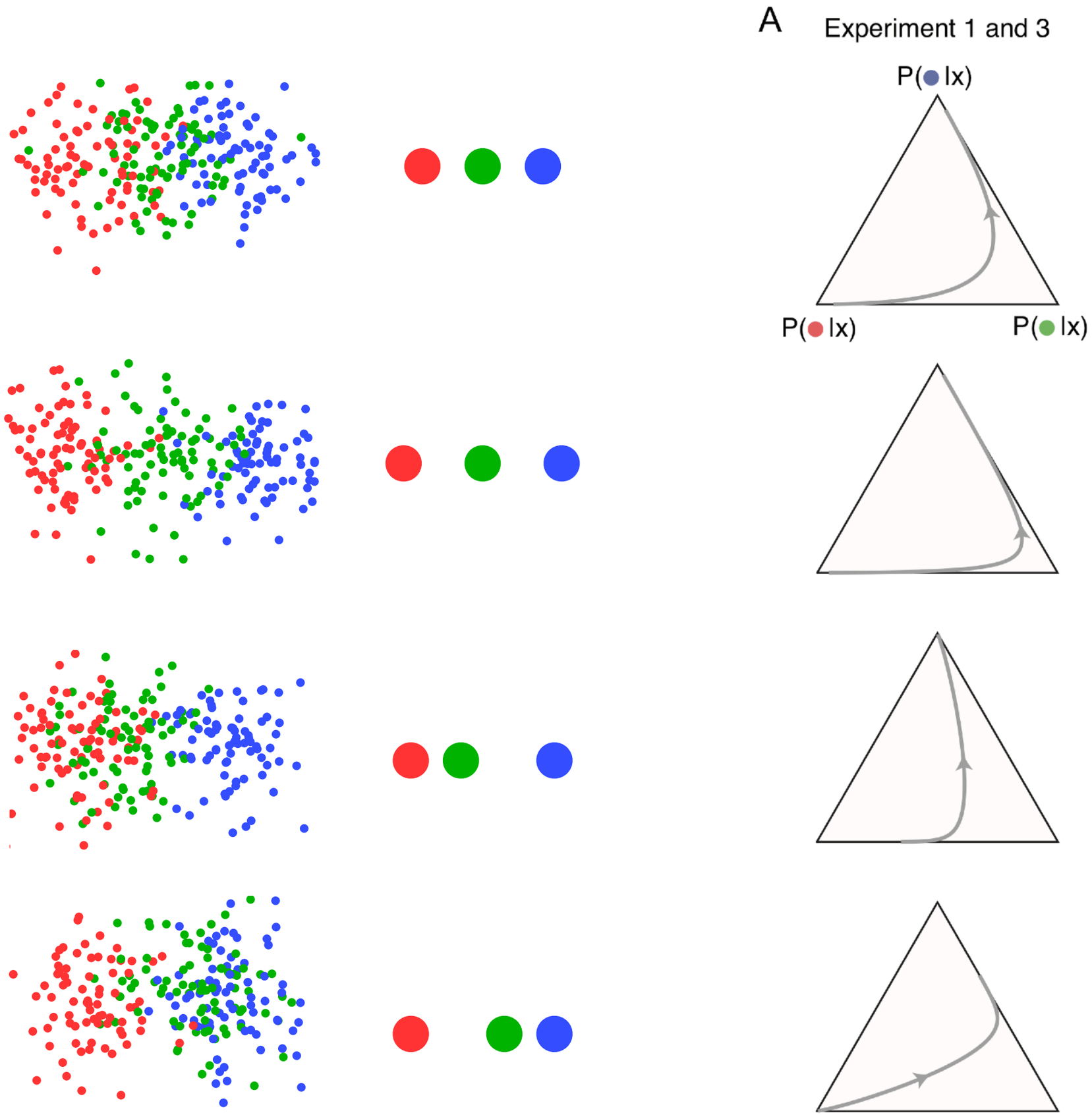




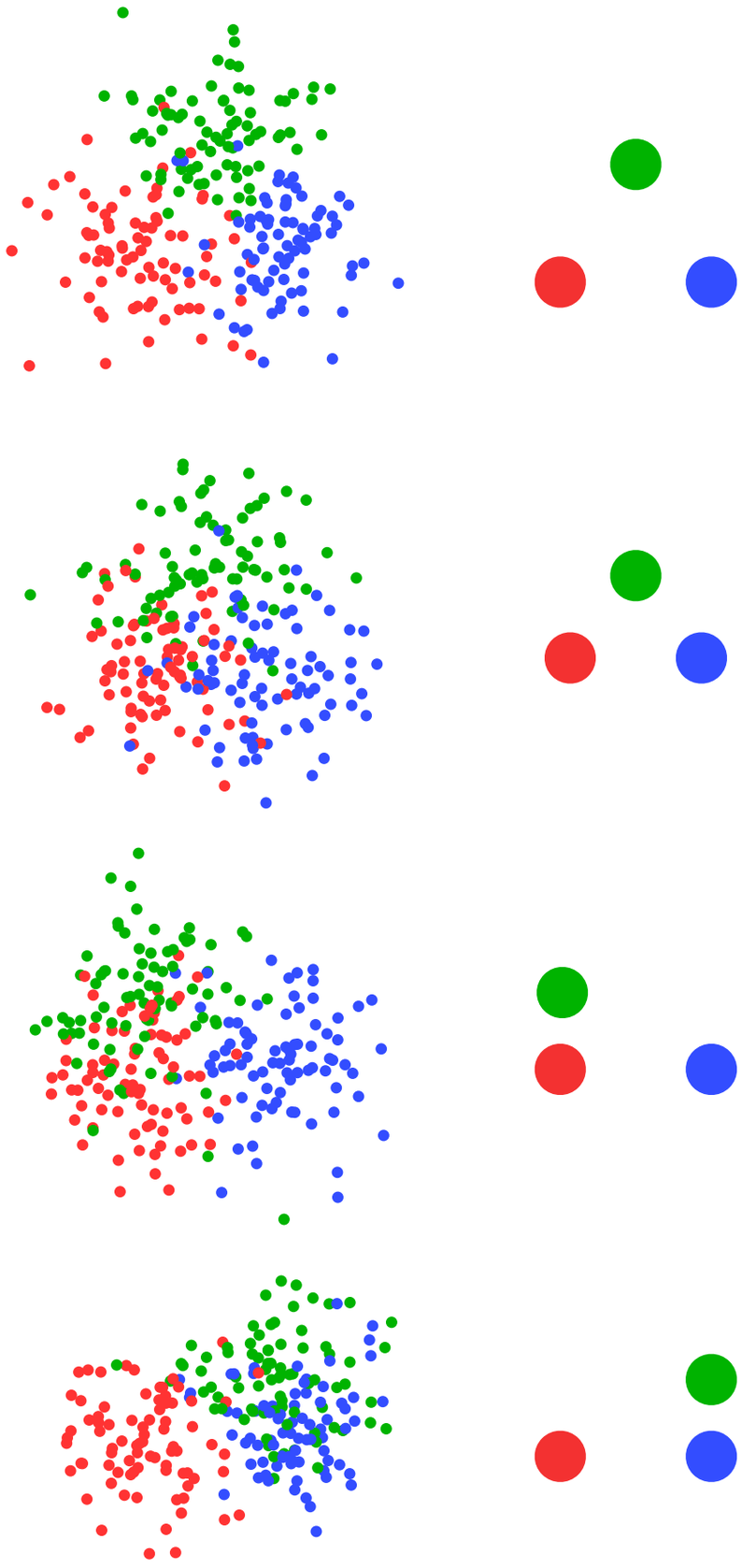






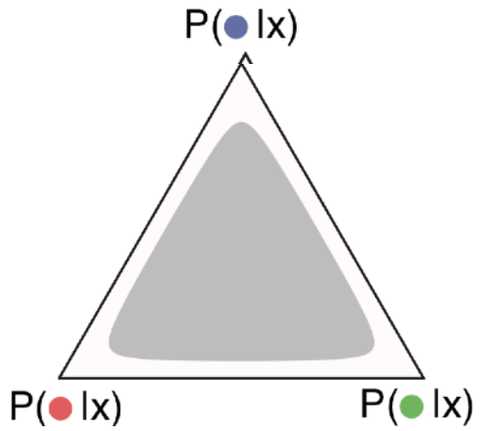


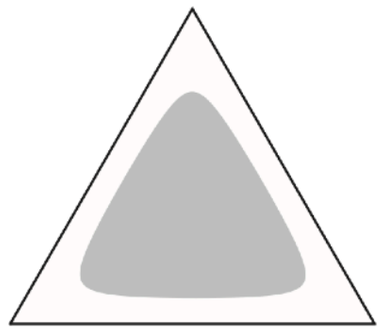
Supplementary Figure 1.

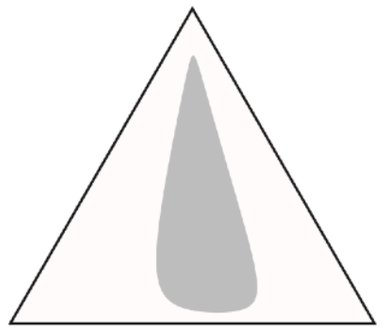


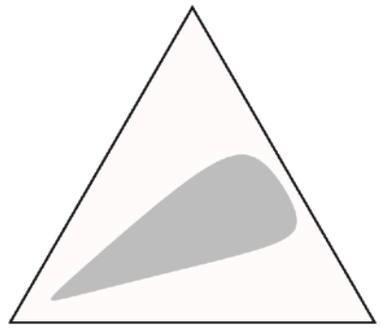
B

Experiment 2









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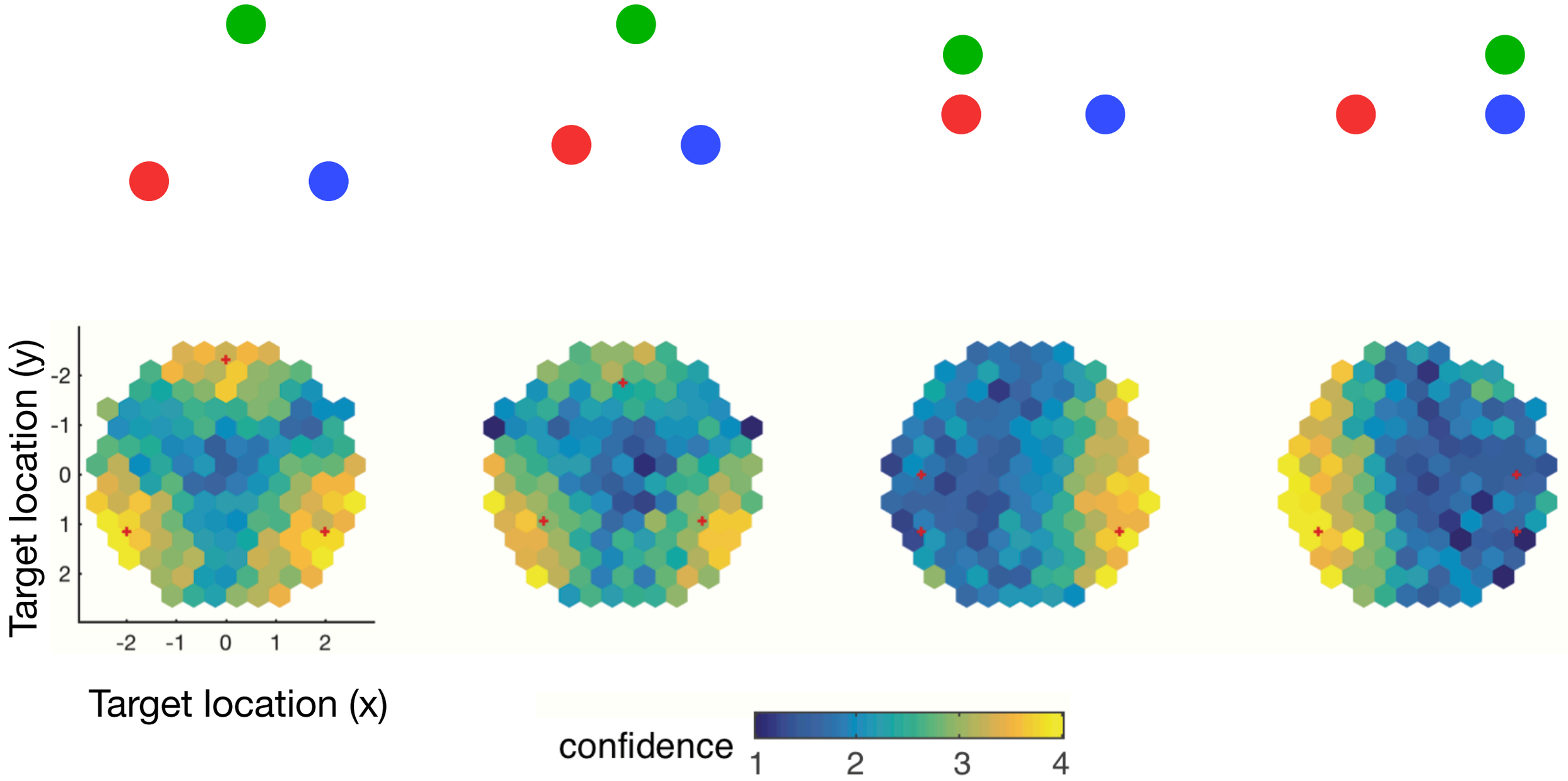
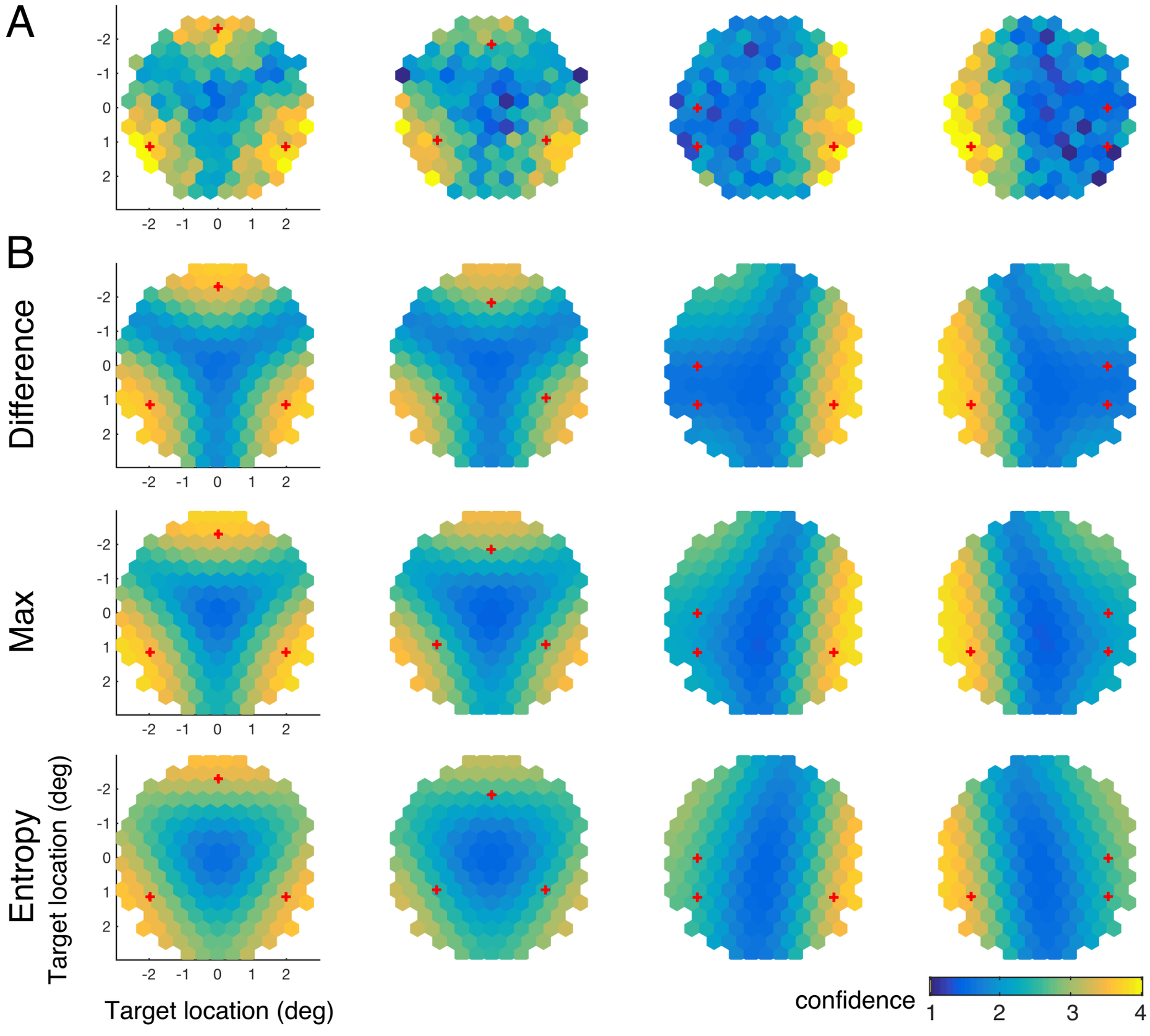


Fig. 5 Experiment 2.



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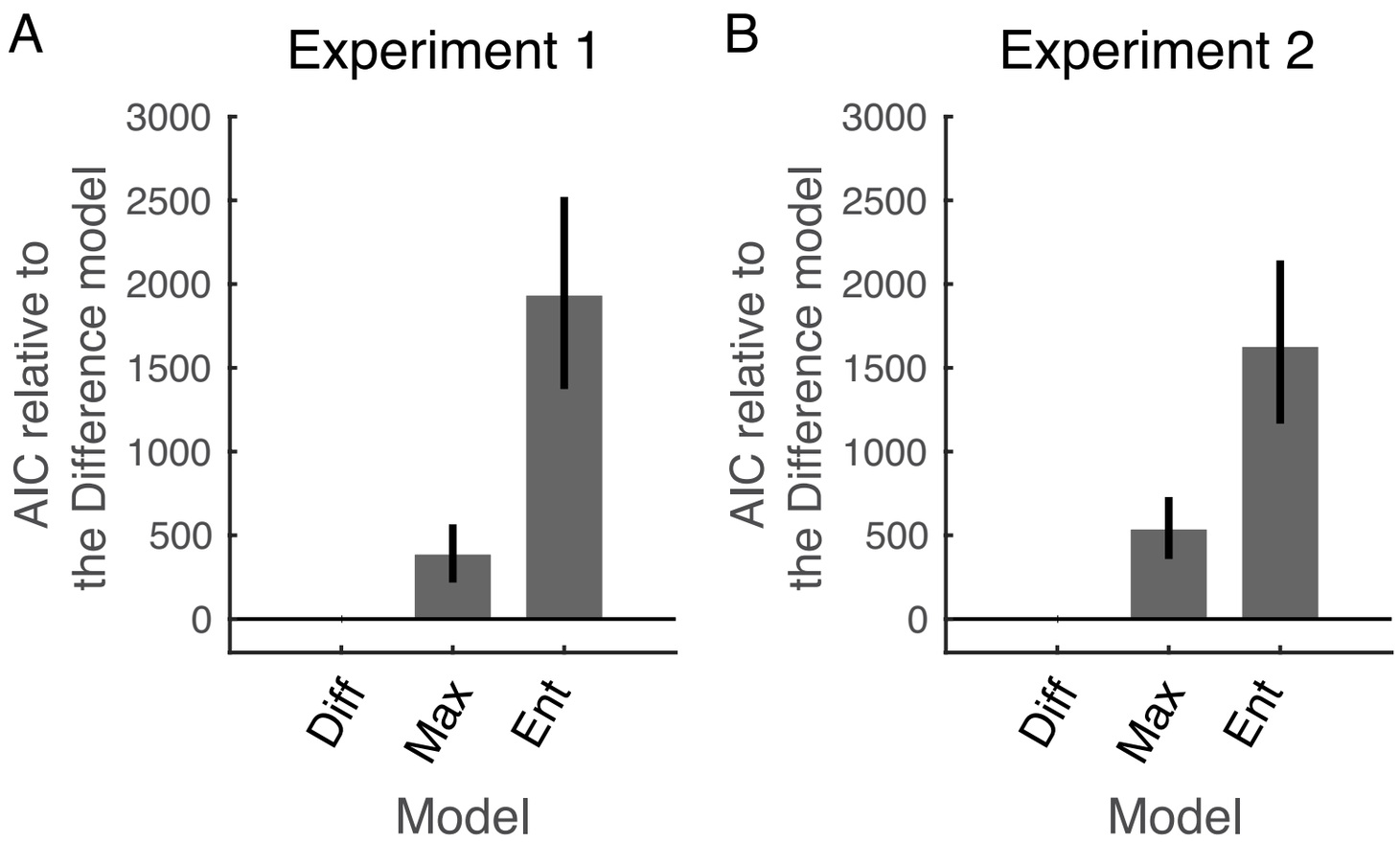

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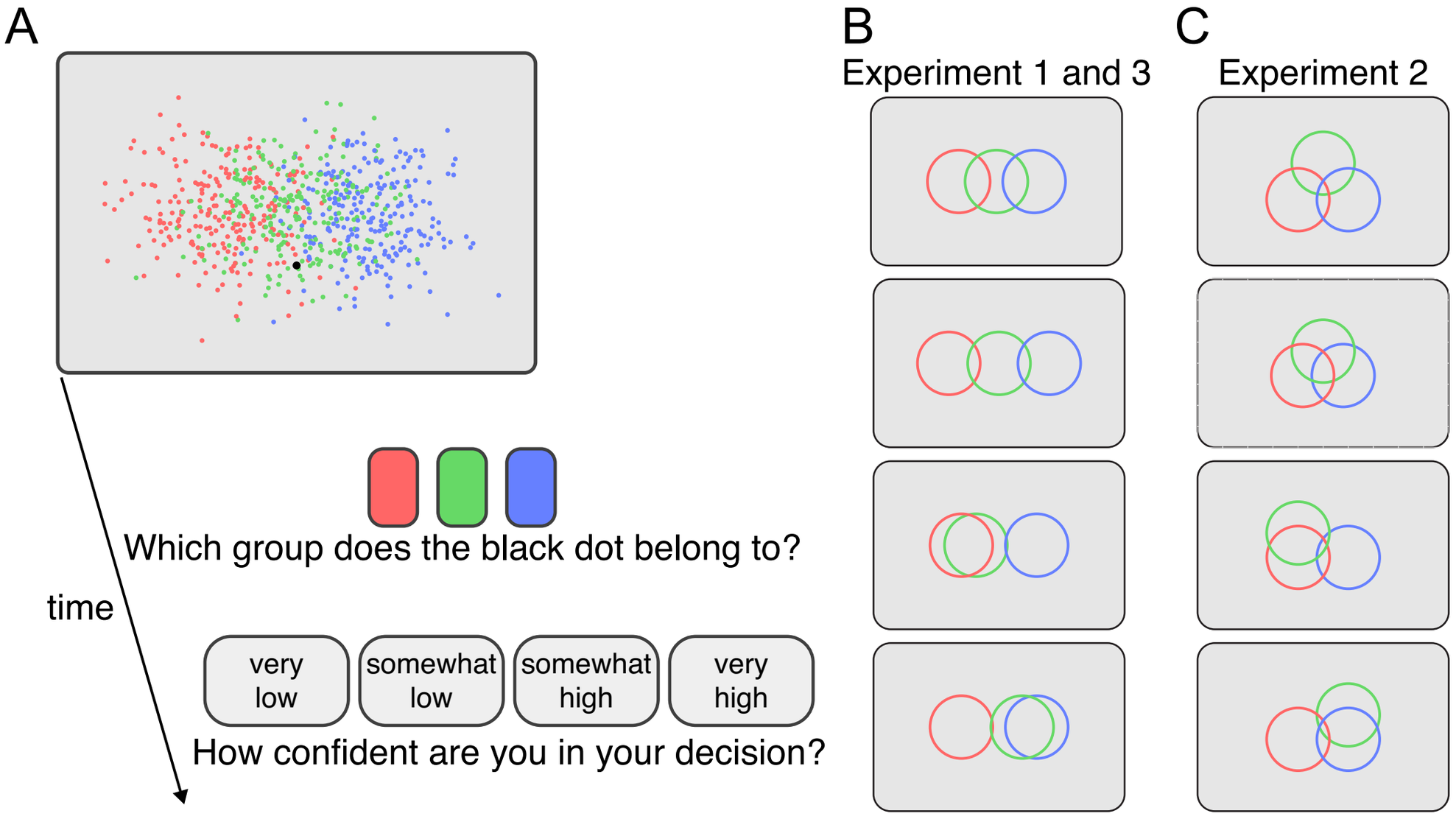
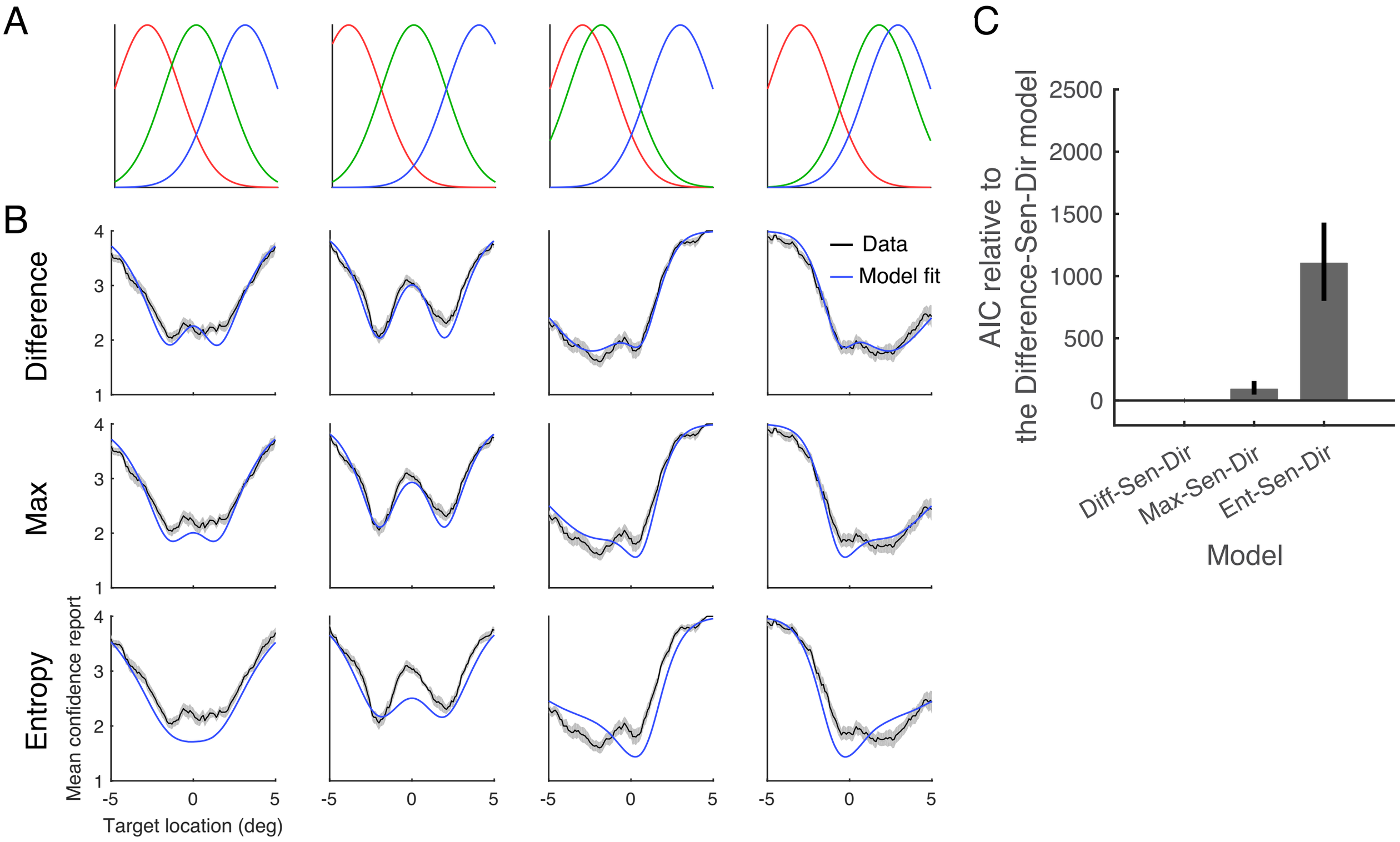


Experiment 3

1. Feedback
2. Sampling from the true stimulus distribution



Supplementary Figure 11.

If you are interested in:
-Model recovery analysis (Supplementary Figure 3)
-Model comparisons using BIC index (Supplementary Figure 5)
-Fitting confidence reports alone (Supplementary Figure 6 and 7)
-Fitting category decisions alone (Supplementary Figure 9)
-21 additional models (various implementations of variabilities; heuristic models; Supplementary Figure 10)
-Ratio model (Supplementary Figure 12)

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Difference model outperforms Max and Entropy model
Posteriors of unchosen options matter

Doby Rahnev @DobyRahnev · 5月2日

回覆給 @weijima01 和 @hsinhungli

Great paper, congrats to you both! A bit of tangential question: are you thinking of this task as a "perceptual choice" (as the tweet reads)? I always thought of it as a cognitive choice. This may be semantics but I also think that perception may actually work differently.

1 1 1

Hsin-Hung Li @hsinhungli · 5月2日

@DobyRahnev Thank you :-) This sounds like an interesting topic to discuss in our virtual journal club: May 4 at 12:30 EST
nyu.zoom.us/j/99274459619
This task could be on the borderline between perception and cognition. Essentially this is a location/position categorization task.

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THANK YOU!
Time to discuss