Choose 2 out of the following 4 exercises. See following pages for the model’s eqns and description of Euler’s method for numerical integration.

1. Consider HH without $I_K$ (ie, $g_K=0$). Show that with adjustment in $g_{Na}$ (and maybe $g_{leak}$) the HH model is still excitable and generates an action potential. (Do it with $m=m_\infty(V)$.) Study this 2 variable (V-h) model in the phase plane: nullclines, stability of rest state, trajectories, etc. Then consider a range of $I_{app}$ to see if you get repetitive firing. Compute the freq vs $I_{app}$ relation; study in the phase plane. Do analysis to see that the rest point must be on the middle branch to get a limit cycle.

2. Convert the HH model into “phasic mode”. By “phasic” I mean that the neuron does not fire repetitively for any $I_{app}$ values – only 1 to a few spikes and then it returns to rest. Many neurons in the auditory system behave phasically. Do this by, say, sliding some channel-gating dynamics along the V-axis (probably just for $I_K$). [If you slide $x_\infty(V)$, you must also slide $\tau(x)(V)$.] If it can be done using $h=1-n$ and $m=m_\infty(V)$ then do the phase plane analysis.

3. Consider the FitzHugh-Nagumo model and describe its repetitive firing properties in terms of Hopf bifurcation theory:

\[
\begin{align*}
\frac{dv}{dt} &= -f(v) - w + I_{app} \\
\frac{dw}{dt} &= \varepsilon (v - \gamma w)
\end{align*}
\]

where \(f(v)=v(v-a)(v-1)\); \(0 \leq a \leq 1\); \(\varepsilon, \gamma >0\).

a. Show that the rest state \((v_R, w_R)\) is unique if \(\gamma\) is small enough

\[
\gamma < 3/(a^2-a+1).
\]

b. Find analytically the parameter conditions such that Hopf bifurcations occur for some critical current values $I_{app}=I_1, I_2$.

Answer: \(3\varepsilon \gamma < a^2-a+1\).

Find expressions for $I_1, I_2$ in terms of $\varepsilon, \gamma, a$. (Hint: first use $v_R$ as your control parameter and then later compute $I_1, I_2$.) Plot $I_{1,2}$ versus $\varepsilon$ (same axes). Interpret the results in terms of the repetitive firing regime, $\varepsilon$ as “temperature”, and the Hopf-predicted frequency.

c. With numerical simulations (or AUTO in XPP) compare the repetitive firing properties for $\varepsilon=0.07$ and $\varepsilon=0.02$ with $a=0.1$ and $\gamma=1$ – compute frequency vs. $I_{app}$, and amplitude ($v_{max}, v_{min}$ vs. $I_{app}$).

4. Explore the Morris-Lecar model. For parameter values in the Handout, obtain, plot and discuss the time courses for $I=20, 40, 60, 120$. Use numerical integration for $0< t < 900$, starting from the rest state (for $I=0$). (I used Runge-Kutta with $\Delta t=0.2$, but you could use Euler – maybe with a smaller step size.) Construct and describe features of the phase plane, $w$ vs $v$, for each of these cases: nullclines, singular points, stability, sample trajectories. For which values of $I$ is the system...
excitable, oscillatory, in nerve block, etc. Consult the Borisyuk/Rinzel Chapt, Fig 14, if you wish.

**HH and WC models.**

**Euler method for numerical integration.**

**HH Equations.**

XPP code.

```plaintext
#hh4jr.ode
p VNA=115, VK=-12, VL=10.5989, GNABAR=120, GKBAR=36, GL=.3
p I0=0, IP=0, PON=0, POFF=0, TEMP=6.3
AM(v)= PHI*0.1*(25.-v)/(EXP(0.1*(25.-v))-1.)
BM(v)=PHI*4.0*EXP(-v/18.)
AH(v)=PHI*0.07*EXP(-v/20.)
BH(v)=PHI*1.0/(EXP(0.1*(30.-v))+1.0)
BN(v)=PHI*.125*EXP(-v/80.)
AN(v)=PHI*0.01*(10.-v)/(EXP(0.1*(10.-v)))-1.0)
IAPP(t)=I0+heav(POFF-t)*heav(t-PON)*ip
PHI= 3**((TEMP-6.3)/10.)
#minf(v)=am(v)/(am(v)+bm(v))
#iionminf(v,h,n)=GNABAR*MINF(V)**3*H*(V-VNA)+GKBAR*N**4*(V-VK)+GL*(V-VL)
dv/dt= IAPP(T)-GNABAR*M**3*H*(V-VNA)-GKBAR*N**4*(V-VK)-GL*(V-VL)
dm/dt= AM(V)-(AM(V)+BM(V))*M
dh/dt= AH(V)-(AH(V)+BH(V))*H
dn/dt= AN(V)-(AN(V)+BN(V))*N
init v=20.001 m=.05932 h=.59612 n=.31768
@ METH=qualrk, TOL=.001, BOUND=100000, xlo=0, xhi=20, ylo=-20, yhi=140
done
```

**WC Equations.**

XPP code.

```plaintext
init u=.0426, v=.0834
u'=-u+f(aee*u-aie*v+Je+stim(t))
v'=(v+f(aei*u-aii*v+Ji))/tau
par aee=10, aie=9, Je=-3
par aei=20, aii=3, Ji=-3, tau=5
stim(t)=s0+s1*if(t<tdone)then(t/tdone)else(0)
par s0=0, tdone=1, s1=0
f(u)=1/(1+exp(-u))
@ xp=u, yp=v, xlo=-.1, ylo=-.1, xhi=1.1, yhi=1.1, total=50
done
```
ML Equations

XPP code.

# Morris-Lecar model
dv/dt=(i+gl*(vl-v)+gk*w*(vk-v)+gca*minf(v)*(vca-v))/c

dw/dt=lamw(v)*(winf(v)-w)

minf(v)=.5*(1+tanh((v-v1)/v2))

winf(v)=.5*(1+tanh((v-v3)/v4))

lamw(v)=phi*cosh((v-v3)/(2*v4))

param vk=-84,vl=-60,vca=120
param i=0,gk=8,gl=2, gca=4, c=20
param v1=-1.2,v2=18,v3=12,v4=17,phi=.06666667

# for type II dynamics, use v3=2,v4=30,phi=.04
# for type I dynamics, use v3=12,v4=17,phi=.06666667

v(0)=-60.899
w(0)=.0148

# track some currents
aux Ica=gca*minf(V)*(V-Vca)
aux Ik=gk*w*(V-Vk)
done

Euler’s Method.

For numerical simulation you could implement the forward Euler scheme. Suppose that \( t_n = n \Delta t \) where \( \Delta t \) is a small time step, say \( \Delta t=0.02 \) ms for HH (or smaller if necessary).

Then if \( x_n \) is the approximation to \( x(t_n) \) the Euler recipe for “integrating “ (getting the approximate time course, recursively) the ode:

\[
\frac{dx}{dt} = f(x), \quad x_0=x(0)
\]

is:

\[
x_{n+1} = x_n + \Delta t \ f(x_n)
\]

where \( x_0 \) is the starting (initial) value of \( x \) at \( t=0 \). There you go, just march forward in time (ie, \( n \)) repeating this recipe successively. Save \((t_n, x_n)\) as an array and plot it, \( x_n \) vs \( t_n \), to see the time course. Do at least one run where you reduce the \( \Delta t \) by \( \frac{1}{2} \) to see that you get good agreement in the time courses. This is an accuracy test.