Glimcher
Decision Making
Signal Detection Theory

• With Gaussian Assumption
• Without Gaussian Assumption
• Equivalent to Maximum Likelihood without Cost Function
Distribution of internal responses when no tumor is present.

Distribution when tumor is present.
$d' = 1$

Hits = 97.5%
False alarms = 84%

Hits = 84%
False alarms = 50%

Hits = 50%
False alarms = 16%
Newsome Dot Task

A

- "Preferred" target

- Fixation point

B

- "Null" target

- Dot stimulus

- Receptive field

- Fixation point

- Stimulus

- Fixation point

- Targets

- 2 seconds

- Choice
QuickTime™ and a Video decompressor are needed to see this picture.
The diagram shows the number of trials (No. of trials) plotted against the number of spikes per trial (Spikes per trial) for three different correlation percentages:

1. Correlation = 12.8%
2. Correlation = 3.2%
3. Correlation = 0.8%
• Max Likelihood for an Individual Neuron
  (w/o Gaussian Assumption)
• Monkey’s Choice Behavior
Neurons vs Behavior

![Histogram showing the distribution of neurons vs behavior with threshold ratio (neuron/behavior).](image)
So How Should You Set Criterion? (What is the Loss Function)
More complicated theories of “Bias”

• Expected Value
• Expected Utility
• Prospect Theory

• Game Theory
The rise (and fall?) of expected utility

• Why EU?
  – EV-maximization $\Sigma x \cdot x \cdot g(x)$ easily disproved
    • St. Petersburg paradox
    …but EU is a simple generalization of EV
  – Axioms:
    • Completeness of preference (including transitivity)
    • Continuity (“solvability”)
      – If $f \succ g \succ h$ then there exists $p$ such that $pf + (1-p)h \sim g$
    • Cancellation (independence)
      – If $f \succ g$ then $pf + (1-p)z \succ pg + (1-p)z$ for all $z$, $p>0$

Paul: Explain EV and EU Here
Cancellation is logically sensible but perceptually unnatural

• Allais common consequence problem:
  – A: (.33, $2.5 M; 0.1, $0; .67, $2.4M) or
    (.33, $2.4 M; 0.1 $2.4M; .67, $2.4M)
  – B: (.33, $2.5 M; 0.1, $0; .67, $0) or
    (.33, $2.4 M; 0.1 $2.4M; .67, 0)

  – Majority choose $2.4 M in A
    and $2.5 M in B

Violates Cancellation because .01 payoff is identical in both experiments so it can be cancelled, this reveals that preferences have reversed
Many variants of expected utility

| Theory (figure no.) | Functional form for $U^*(F) = U(qX + (1-q)Y | X < Y)$ | Properties of curves | Miscellaneous |
|---------------------|-----------------------------------------------------|----------------------|---------------|
| Expected utility (1) | $qU(X) + (1-q)U(Y)$ | Straight lines? | Fanning out? | Fanning in? | Curves parallel |
| Weighted utility (2) | $qW(X)U(X) + (1-q)W(Y)U(Y)$ if $W(X) < 1$ | Yes | Yes | No | Curves meet in a point |
| Implicit expected utility (3) | $qU(X,U^*) + (1-q)U(Y,U^*)$ | Yes | Maybe | Maybe | Only testable property is between |
| The fanning-out hypothesis (4) | $-U''(x;F)/U'(x;F)$ if $F(x) < G(x)$ for all $x$ | Maybe | Yes | No | Movements to north cause steeper slope |
| Lottery-dependent utility (5) | $qU(X,C_F) + (1-q)U(Y,C_F)$ concave $C_F = \int h(X)dF(X)$ | No | Yes | No | Curves concave |
| Prospect theory (6) | $u(X) + \pi(q)U(Y - X)$ | No | Lower edge | Left edge, hypotenuse | Parallel along $P_{H} = (1 - P_{L})/2$ |
| Rank-dependent utility (7) | $g(q)U(X) + (1-g(q))U(Y)$ if $g$ concave | No | Lower edge | Left edge, hypotenuse | Parallel along hypotenuse |

Source: Camerer JRiskUnc 88, Edwards (Ed) Utility Theories, 92
Useful tool: Marschak-Machina triangle for representing 3-outcome gambles

- Outcomes
  $x_3 > x_2 > x_1$
- Each point is a gamble
- Theories dictate properties of indifference curves

*Figure 1a. The Allais Paradox Problems in a Probability Triangle*
• EU is equivalent to
  – Linear indiff curves
  – Parallel
• Allais common consequence effect → curves get steeper toward upper left
Prospect theory, I

- Key features:
  - Reference-dependence, nonlinear w(p)
- Nonlinear weighting of probability
  - Zeckhauser bullet example, 6→ 5 and 1→ 0
  - Inflection (sensitivity), elevation (risk attitude)
  - KT (’92 JRU) \( w(p)=p^\gamma/(p^\gamma+(1-p)^\gamma)^{1/\gamma} \)
  - Prelec (98 E’metrica) \( w(p)=1/\exp((\ln(1/p)^\gamma) \)
    - Large overweighting on low probabilities
    - \( \gamma=.7 \) \( w(1/10)=.165, w(1/100)=.05, w(1/1,000,000)=.002 \)
- Morgenstern 79 J Ec Lit:
  - [Like Newtonian mechanics] The domain of our axioms on utility theory is also restricted…For example, the probabilities used must be within certain plausible ranges and not go to .01 or even less to .001, then be compared with other equally tiny numbers such as .02 etc.”
Empirical weighting functions

Figure 1.—The compound invariant form (solid line) and several empirical probability weighting functions. Estimates of the one-parameter equation (3.5) are taken from Tversky and Kahneman (1992) and Wu and Gonzalez (1996a); estimates of the two-parameter equation (3.6) are taken from Tversky and Fox (1994).
Prospect theory, I

- **Key features:**
  - Reference-dependence, nonlinear \( w(p) \)
- **Reference-dependence**
  - Extension of psychophysics (e.g. hot-cold)
  - \( U(x-r) \)
    - Koszegi-Rabin \( u(x) + \omega v(x-r) \)
  - \( U(.) \) “reflects” (gamble over losses)
  - Loss-aversion

\[
\frac{\mu'_-(0)}{\mu'_+(0)} \equiv \lambda > 1
\]
Prospect theory value function:
Note kink at zero and diminishing marginal sensitivity
(concave for $x>0$, convex for $x<0$)
Do Classical Decision Variables Influence Brain Activity in LIP?
Varying Movement Value
Same Movement, Different Values

Firing Rate (Hz)

Time (ms)

Gain Ratios
- Red: 0.75
- Blue: 0.25

Large Gain Expected
Small Gain Expected
What Influences LIP?

Related to Movement Desirability

- Value/Utility of Reward
- Probability of Reward
- Overall Desirability (EU)

NB: Utility is used here without any intended reference to welfare
Varying Movement Probability

Block 1
- 50% green to 50% red

Block 2
- 20% green to 80% red

Block 3
- 80% green to 20% red
Same Movement, Different Likelihoods

Firing Rate (Hz)

Time (ms)

Movement Likely
Movement Unlikely

Reward Prob.
0.80
0.20

CH990215
What Influences LIP?

Related to Movement Desirability

- Value/Utility of Reward
- Probability of Reward
- Overall Desirability (EU)
What Influences LIP?

Related to Movement Desirability
- Value/Utility of Reward
- Probability of Reward
- Overall Desirability (EU)
Is LIP a Decision Making Map?

V1:
A Map of Orientation, Spatial Frequency, Phase, Ocular Dominance, Color...
Theory of Games:

By The Nash Equations:
At Behavioral Equilibrium

\[ EU_{\text{working}} = EU_{\text{shirking}} \]
Paul: Show Equilib Computation here

**Work or Shirk**

<table>
<thead>
<tr>
<th>Player A</th>
<th>Inspect</th>
<th>No-Inspect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work Player B</td>
<td>0.50</td>
<td>1.00</td>
</tr>
<tr>
<td>Work</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Shirk</td>
<td>-0.50</td>
<td>-1.00</td>
</tr>
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</tbody>
</table>

**Game Tree:**

- **Player A: Inspect or No-Inspect**
  - **No-Inspect**
    - **Player B: Work or Shirk**
      - **Work:** 0.50, 0.50
      - **Shirk:** 0.00, -0.50
  - **Inspect**
    - **Player B: Work or Shirk**
      - **Work:** 0.50, 1.00
      - **Shirk:** 1.00, -1.00
Behavior Fixed

EU Changes

Behavior Changes

EU Fixed
## Work or Shirk

<table>
<thead>
<tr>
<th>Player B</th>
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Along Indifference Lines
Both Actions Have Equal
Expected Utility
Adaptive Computer Algorithm

Inspect

No-Inspect

Player B

-or-

Player A

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Human versus Computer
Employee Behavior
20-Trial Running Average

$P_{\text{shirk}}$

Trial #
Monkey Work of Shirk
Monkey versus Computer

Employee Behavior

20-Trial Running Average
In Lottery Task:

Behavior Held Constant
50% into Response Field
Expected Utility Varies
High vs. Low
Instructed Movements

Firing Rate (sp/s)

Time from Target Presentation (ms)

Big Reward: black line
Small Reward: gray line

Insertions with green and red dots indicate movement directions and reward conditions.
In Strategic Game:

Behavior Varies
Nash Equilibrium
Expected Utility Fixed
On Average $EU_1 = EU_2$
LIP Tracks Average EU
Local Fluctuations in EU?
Perhaps

A

Pre-target  Visual *  Delay *

Cue *  Pre-motor  Post-motor

Firing Rate (sp/s)

Relative Expected Utility

B

Free Choice

Regression Slope

Pre-target  Visual  Delay  Cue  Pre-motor  Post-motor

Amplitude  Velocity  Latency  Nash  Expected Utility
LIP Codes Absolute or Relative Expected Utility?

Firing Rate (sp/s)

Time from Target Presentation (ms)

N=18

- Double Rewards
- Normal Rewards
Stochastic Decision Model
Stochastic Decision Model

Relative Expected Utility Map

Expected Utility Estimates

Frontal Eye Field Winner-Take-All

Superior Colliculus Biophysical Threshold

Normalization

\[ \sum \]
Stochastic Decision Model

Relative Expected Utility Map

Intrinsic Noise $N$

Expected Utility Estimates

$\sum$

Normalization

Frontal Eye Field Winner-Take-All

Superior Colliculus Biophysical Threshold