HW1 Exercises from Rinzel lecture #1, Sept 6, 2007.
Due Oct 4, 2007.

Choose 2 out of the following 4 exercises. See following pages for the model's eqns and description of Euler's method for numerical integration.

1. Consider HH without I_K (ie, $g_K = 0$). Show that with adjustment in g_{Na} (and maybe g_{leak}) the HH model is still excitable and generates an action potential. (Do it with $m = m_\infty(V)$.) Study this 2 variable (V-h) model in the phase plane: nullclines, stability of rest state, trajectories, etc. Then consider a range of I_{app} to see if you get repetitive firing. Compute the freq vs I_{app} relation; study in the phase plane. Do analysis to see that the rest point must be on the middle branch to get a limit cycle.

2. Convert the HH model into "phasic mode". By "phasic" I mean that the neuron does not fire repetitively for any I_{app} values – only 1 to a few spikes and then it returns to rest. Many neurons in the auditory system behave phasically. Do this by, say, sliding some channel-gating dynamics along the V-axis (probably just for I_K) . [If you slide $x_\infty(V)$, you must also slide $\tau_x(V)$.] If it can be done using h=1-n and $m = m_\infty(V)$ then do the phase plane analysis.

3. Consider the FitzHugh-Nagumo model and describe its repetitive firing properties in terms of Hopf bifurcation theory:
$$v' = -f(v) - w + I_{app}$$
$$w' = \varepsilon\,(v - \gamma\, w)$$
where $f(v) = v(v-a)(v-1)$; $0 \le a \le 1$; $\varepsilon, \gamma > 0$.
   a. Show that the rest state (v_R, w_R) is unique if γ is small enough
   $$\gamma < 3/(a^2 - a + 1).$$
   b. Find analytically the parameter conditions such that Hopf bifurcations occur for some critical current values $I_{app} = I_1, I_2$.
   Answer: $3\varepsilon\gamma < a^2 - a + 1$.
   Find expressions for I_1, I_2 in terms of ε, γ, a. (Hint: first use v_R as your control parameter and then later compute I_1, I_2 .) Plot $I_{1,2}$ versus ε (same axes). Interpret the results in terms of the repetitive firing regime, ε as "temperature", and the Hopf-predicted frequency.
   c. With numerical simulations (or AUTO in XPP) compare the repetitive firing properties for ε=0.07 and ε=0.02 with a=0.1 and γ=1 – compute frequency vs. I_{app}, and amplitude (v_{max}, v_{min} vs. I_{app}).

4. Explore the Morris-Lecar model. For parameter values in the Handout, obtain, plot and discuss the time courses for I=20, 40, 60, 120. Use numerical integration for $0 < t < 900$, starting from the rest state (for I=0). (I used Runge-Kutta with Δt=0.2, but you could use Euler – maybe with a smaller step size.) Construct and describe features of the phase plane, w vs v, for each of these cases: nullclines, singular points, stability, sample trajectories. For which values of I is the system

excitable, oscillatory, in nerve block, etc. Consult the Borisyuk/Rinzel Chapt, Fig 14, if you wish.