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  • eg: tuning curves, receptive field, LNP

• Mechanistic (how?)
  • eg: compartmental models, Hodgkin-Huxley

• Interpretive/Explanatory (why?)
  • eg: efficient coding, optimal estimation/decision, wiring length, metabolic cost, etc

- Dayan & Abbott
Interaction with Experiments
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  - to refine model
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• Develop new experiments...
  - to refine model
  - to differentiate models
  - with optimized stimuli, to characterize cells
Descriptive Response Models (outline)

- Receptive fields and tuning curves
- Linear models
- Rate models
- Wiener/Volterra (polynomial) models
- LN models
- Poisson spiking
- Fitting/validating LNP models
- Dayan & Abbott
Rate coding

Some of Adrian’s first recordings from very small numbers of individual nerve fibres. Each spiky deflection is a single nerve impulse. These records were taken from the sensory nerves of a cat’s toe. The toe was flexed slowly, more quickly and very rapidly to produce these three traces. The frequency of firing depends on the strength of the stimulus – Adrian’s law.
Receptive Fields

- Classical: A region of the retina (visual field) that must be stimulated directly in order to obtain a response in a neuron
  - Sherrington (1906), Hartline (1938), Kuffler (1953)
Receptive Fields

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• Modern generalization: Kernel that captures those attributes of the stimulus that generate/modulate responses. Often assumed linear.
- Dayan & Abbott, after Hubel & Wiesel ‘62
Estimating firing rates

- Dayan & Abbott
Polynomial Model
(Volterra/Weiner Kernels)

\[ r(\vec{x}) = k_0 + \vec{k}_1 \cdot \vec{x} + \vec{x}^T K_2 \vec{x} + \ldots \]
Polynomial Model
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const \hspace{1cm} vector \hspace{1cm} matrix \hspace{1cm} 3-tensor
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\# pars:
\begin{align*}
\text{const} & \quad \boxed{1} \\
\text{vector} & \quad n \\
& \quad (20) \\
\text{matrix} & \quad n^2 \\
& \quad (400) \\
\text{3-tensor} & \quad n^3 \\
& \quad (8000) \\
\end{align*}
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- estimate kernels using moments of spike-triggered stimuli
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- estimate kernels using moments of spike-triggered stimuli
- in practice, insufficient data to go beyond 2\textsuperscript{nd} order
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const \hspace{1cm} vector
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# pars:
- const: 1
- vector: \(n\) (20)
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- 3-tensor: \(n^3\) (8000)
Polynomial Model
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- **const**: 1
- **vector**: \( n \)
  - \((20)\)
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Low-order polynomials do a poor job of representing the nonlinearities found in neurons
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Low-order polynomials do a poor job of representing the nonlinearities found in neurons.
• Threshold-like nonlinearity $\Rightarrow$ linear classifier
• Classic model for Artificial Neural Networks
  - McCullough & Pitts (1943), Rosenblatt (1957), etc
• No spikes (output is firing rate)
• Simplest successful descriptive spiking model
• Easily fit to (extracellular) data
• Descriptive, and interpretable (although not mechanistic)
Geometric view of Poisson models

1D stimulus over time
(e.g., flickering bars)

- 8 x 6 stimulus block
  = 48-dimensional vector
A geometric picture illustrating the relationship between stimuli and responses over time. The diagram shows raw stimuli on the left and spiking stimuli on the right, with time indicated by the horizontal axis.
Neural response is captured by relationship between the distribution of red points (spiking stim) and blue points (raw stim)

Expressed in terms of Bayes’ rule:

\[
P(\text{spike}|\text{stim}) = \frac{P(\text{spike, stim})}{P(\text{stim})}
\]

Cannot be estimated directly
ML estimation of LNP

[on board]
ML estimation of LNP

If $f_\theta(\vec{k} \cdot \vec{x})$ is convex (in argument and theta), and $\log f_\theta(\vec{k} \cdot \vec{x})$ is concave, the likelihood of the LNP model is convex (for all observed data, $\{n(t), \vec{x}(t)\}$).

[Paninski, '04]
ML estimation of LNP

If \( f_\theta(\vec{k} \cdot \vec{x}) \) is convex (in argument and theta), and \( \log f_\theta(\vec{k} \cdot \vec{x}) \) is concave, the likelihood of the LNP model is convex (for all observed data, \( \{n(t), \vec{x}(t)\} \) )

Examples: \( e^{(\vec{k} \cdot \vec{x}(t))} \)

\( (\vec{k} \cdot \vec{x}(t))^\alpha, \quad 1 < \alpha < 2 \)

[Paninski, ’04]
Simple LNP fitting

• Assuming:
  – stochastic stimuli, spherically distributed
  – spike counts in small time bins (0,1)
  – neural response is such that mean of spike-triggered ensemble is shifted

• Reverse correlation gives an unbiased estimate of $k$ [on board]

• For exponential $f$, this is same as ML

- Bussgang 52; de Boer & Kuyper 68
Computing the STA

Stimulus

Response
time

STA

s

raw stimuli
spiking stimuli
STA corresponds to a “direction” in stimulus space
Projecting onto the STA

$$P \left( \text{spike}(t) \mid \vec{k} \cdot \vec{s}(t) \right) = P \left( \text{spike}(t) \land \vec{k} \cdot \vec{s}(t) \right) / P \left( \vec{s}(t) \right)$$
Projecting onto the STA
Projecting onto the STA
Projecting onto the STA
Projecting onto an axis orthogonal to the STA
Projecting onto an axis orthogonal to the STA
Projecting onto an axis orthogonal to the STA

[Diagram showing projections and stimulus/spike histograms]
Figure 3. Characterization of light response in one ON cell (A, B) and one OFF cell (C, D) simultaneously recorded in salamander retina. A, C,
RGC

LNP

- Pillow et al., 2004

74% of var
RGC

LNP

- Pillow et al., 2004

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Pillow et al., 2004

- 74% of var

- Pillow et al., 2004
V1 simple cell

- Ozhawa, et al.
LNP summary

• LNP is the defacto standard descriptive model, and is implicit in much of the experimental literature
• Accounts for basic RF properties
• Accounts for basic spiking properties (rate code)
• Easily fit to data
• Easily interpreted
• BUT, non-mechanistic, and exhibits striking failures (esp. beyond early sensory/motor) ...
LNP limitations

- Symmetric nonlinearities and/or multi-dimensional front-end (e.g., V1 complex cells)
LNP limitations

- Symmetric nonlinearities and/or multi-dimensional front-end (e.g., V1 complex cells)

→ Subspace LNP
Classic V1 models

Simple cell

Complex cell

computational complexity and provides an estimate of the network's functional connectivity [7] (see Methods).
V1 simple cell

[Rust, Schwartz, Movshon, Simoncelli, ‘05]
LNP limitations

- Symmetric nonlinearities and/or multi-dimensional front-end (e.g., V1 complex cells) → Subspace LNP
- Responses depend on spike history, other cells
LNP limitations

- Symmetric nonlinearities and/or multi-dimensional front-end (e.g., V1 complex cells)
  ➡  Subspace LNP
- Responses depend on spike history, other cells
  ➡  Recursive models (GLM) [paninski]
Linear-Nonlinear-Poisson (LNP)
Recursive LNP

[Truccolo et al ‘05; Pillow et al ‘05]
LNP limitations

• Symmetric nonlinearities and/or multi-dimensional front-end (e.g., V1 complex cells)
  ➡ Subspace LNP [movshon lecture?]

• Responses depend on spike history, other cells
  ➡ Recursive models (GLM) [paninski lecture]

• White noise doesn’t drive mid- to late-stage neurons well
LNP limitations

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- Responses depend on spike history, other cells
  ➡ Recursive models (GLM) [paninski lecture]

- White noise doesn’t drive mid- to late-stage neurons well
  ➡ Specialized “afferent” stimuli [movshon lecture]
Credits

• Spike-triggered covariance: Odelia Schwartz, Jonathan Pillow, Liam Paninski, Nicole Rust

• Stochastic integrate-and-fire & rLNP models: Jonathan Pillow, Liam Paninski

• V1/MT physiology/modeling: Nicole Rust, Tony Movshon (NYU)

• Retinal physiology/modeling: Jonathon Shlens, Valerie Uzzell, Divya Chander, EJ Chichilnisky (Salk Institute)