Localised States and Pattern Formation in a Neural Field Model of the Primary Visual Cortex

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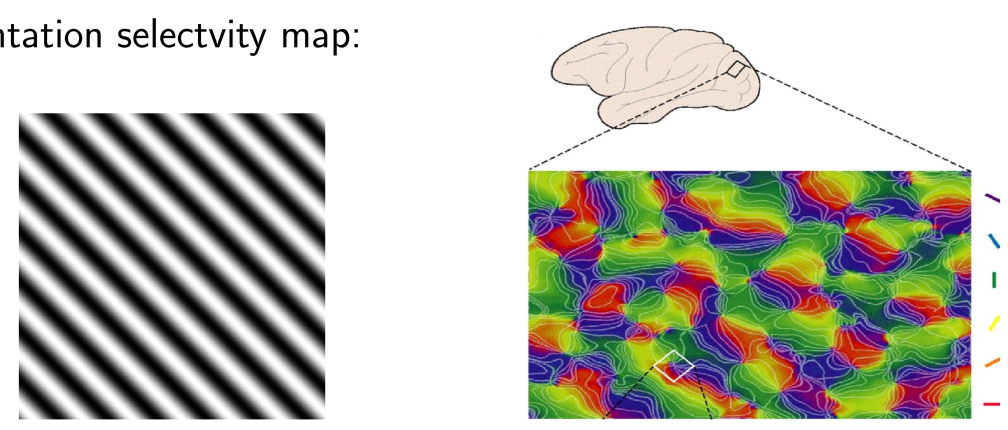
Summary: The primary visual cortex has been shown to maintain localised patterns of activity when local oriented stimuli are presented in the visual field [1,2]. We developed a computational framework to perform numerical continuation directly on the integral form of the planar neural field equation in [3]. Working with a biologically relevant connectivity function, we apply these methods to study localised patterns of activity with inhomogeneous firing rate function and input. The model captures the spatial and dynamic features of the experimentally observed patterns.

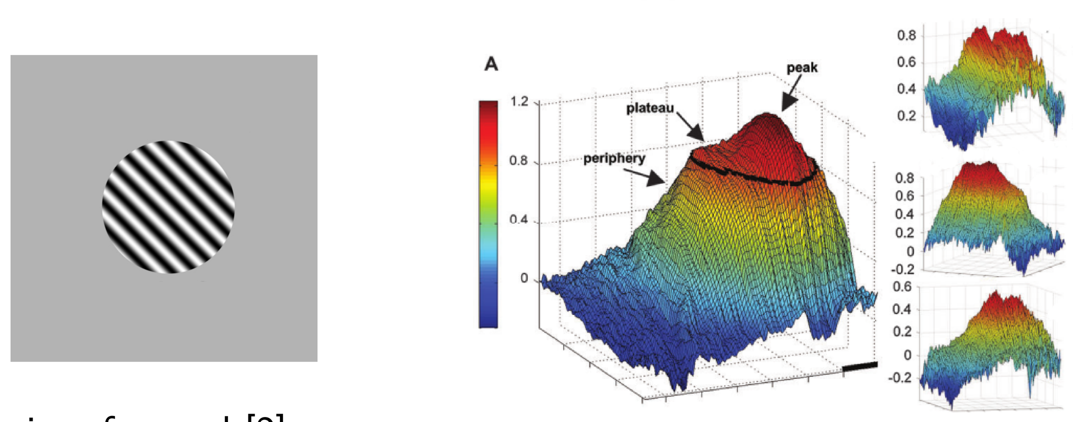
Motivation: localised states in the primary visual cortex (V1)

Orientation selectivity and lateral spread of activity in V1

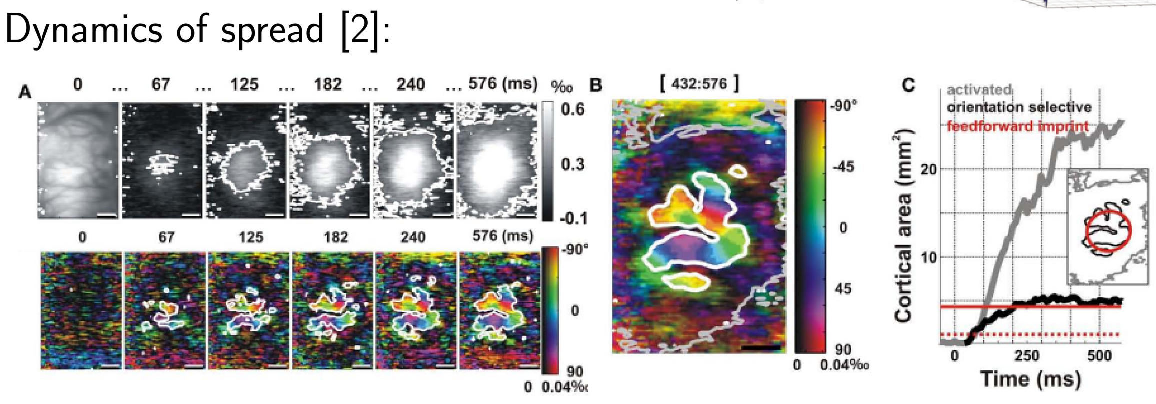
Orientation selectvity map:



Localised activity for a local, oriented input [1]:



Dynamics of spread [2]:



Local activation is selective (i.e. patchy), lateral spread is non-selective (i.e. non patchy).

Snaking and persistent localised states in a neural field

In [3] we studied pattern formation in the neural field equation, posed on the Euclidean plane, and given by

$$\frac{\partial}{\partial t}u(x,y,t) = -u(x,y,t) + \int_{\mathbb{R}^2} w(x-x',y-y')\, S(u(x',y',t))\, dx'dy',$$

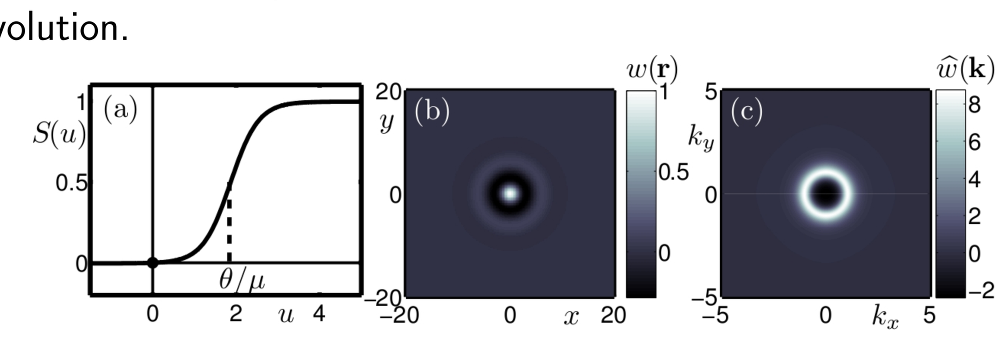
where S is the firing rate function with threshold θ and slope μ

$$S(u) = \frac{1}{1+e^{-\mu u+\theta}} - \frac{1}{1+e^{\theta}}, \qquad \mu,\theta > 0,$$

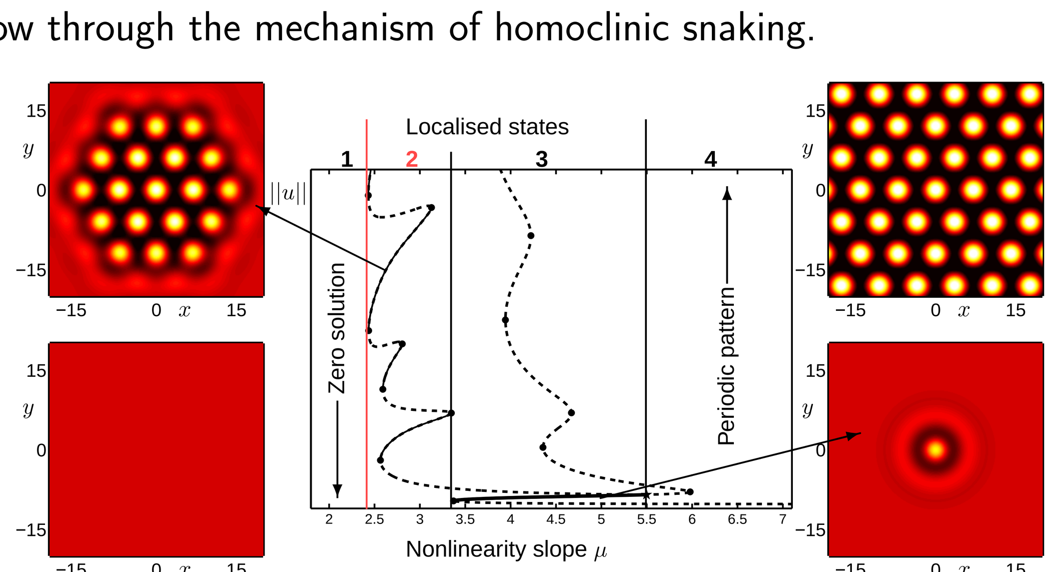
and w is the radial connectivity function with shape parameter b

$$w(r) = e^{-br}(b\sin r + \cos r), \qquad r = \sqrt{x^2+y^2}, \quad b > 0.$$

We employ matrix-free Newton-Krylov solvers and perform numerical continuation of localised patterns directly on the integral form of the equation. The scheme requires only that S be smooth (not necessarily spatial homogeneous) and that the integral term be expressible as a convolution.



We found there to be localised patterns, of varying spatial extent, that grow through the mechanism of homoclinic snaking.



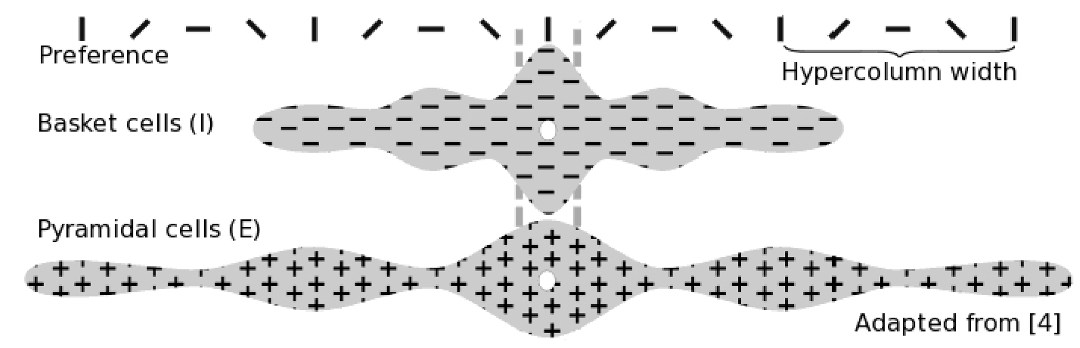
Questions

1) With input do the patterns persist and how are they modified?
2) How does the phase of the input with respect to a regular underlying cortical structure affect the patterns observed?
3) Can the neural field model capture the spatial features and temporal evolution of patterns observed experimentally?

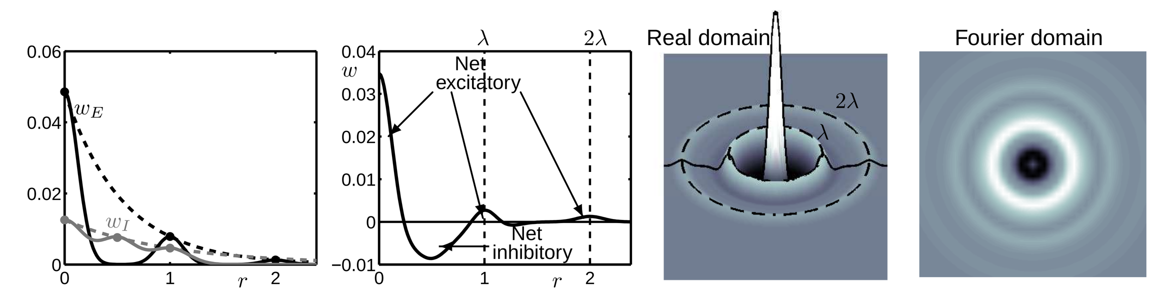
Mathematical model and parameter study

Neural field model of an iso-orientation subpopulation

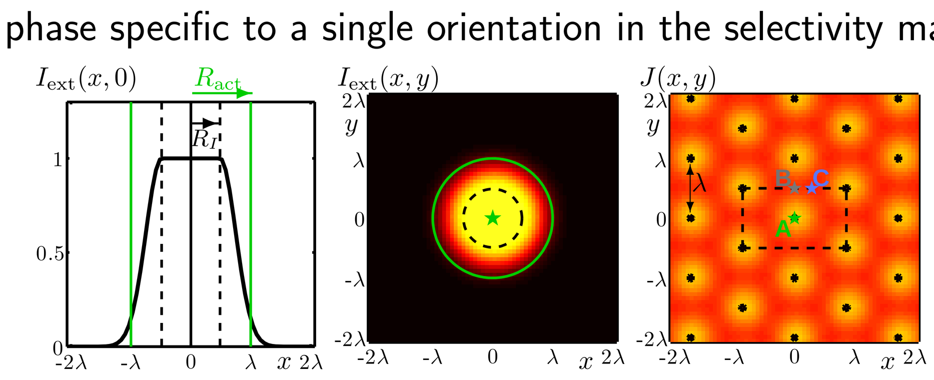
We introduce a connectivity more closely motivated from biology [4] and separable into excitatory and inhibitory contributions $w = w_E - w_I$. This allows for conversion of output into VSD-like signal and for the model to be extended to a two-population EI network in the future.



- w_E: peaks in excitation each hyper-column width λ up to $r = 2\lambda$.
- w_I: peaks in inhibition each half-hyper-column width $\frac{\lambda}{2}$ up to $r = \lambda$.



The input with radius R_I activates a region with radius R_{act}. Firing rates are modulated by a weak inhomogeneity J with spatial scale λ and a phase specific to a single orientation in the selectivity map.



Note that the patterns lie on a regular hexagonal lattice even with $\beta = 0$. Setting $\beta > 0$ fixes the spatial phase.

$$\tau\frac{\partial}{\partial t}u(x,y,t) = -u(x,y,t) + kI_{\text{ext}}(x-x_0,y-y_0) + \int_{\mathbb{R}^2} w(x-x',y-y')\,(1+\beta J(x',y'))\, S(u(x',y',t))\, dx'dy'$$

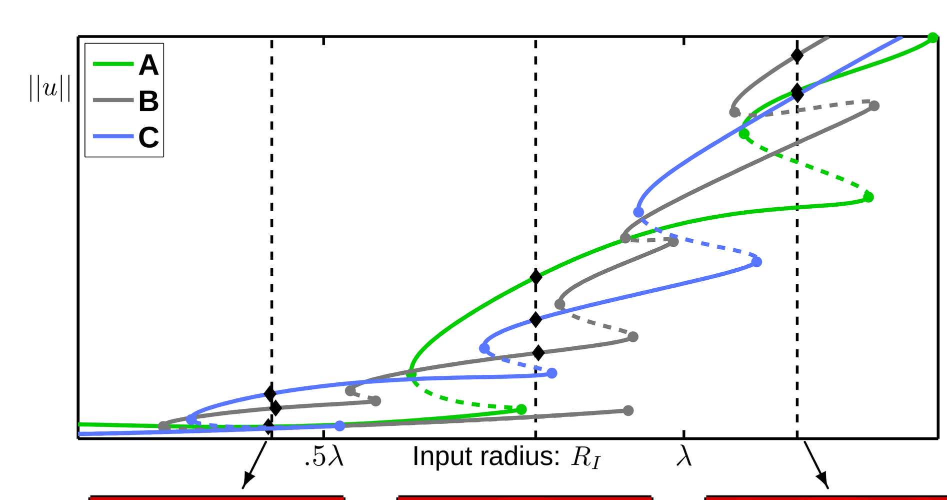
Parameter values: $\tau = 10$ms, $\beta = 0.2$, $\mu = 2.3$, $\theta = 5.6$ and $k = 1.8$.

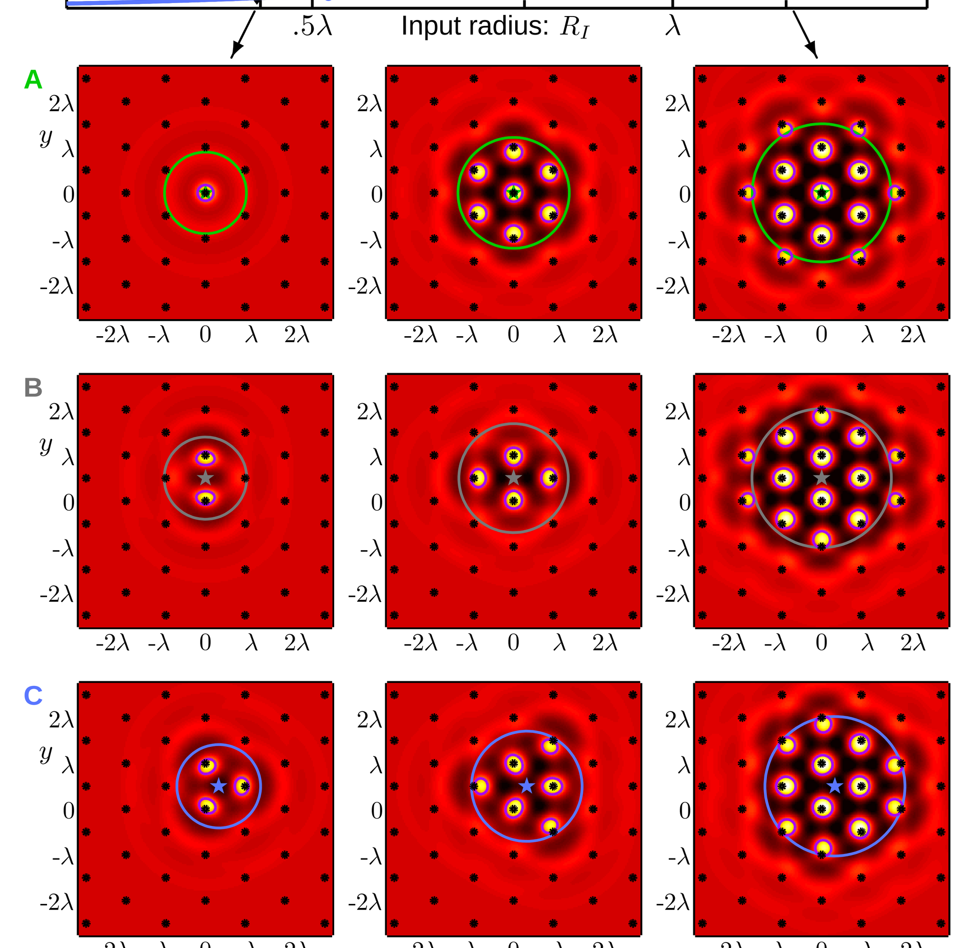
Bifurcation diagrams for stimulus-driven patterns

The numerical continuation scheme allows us to rapidly tune the model parameters θ, μ and set k so that we operate just above the input threshold. We consider three stimulus locations.

- **A**: centred at a peak.
- **B**: midpoint between two peaks.
- **C**: midpoint between three peaks.

Solid branch segments are stable:





Summary of bifurcation results:

- With increasing R_I a series of folds give rise to localised patterns with increasing spatial extent.
- The patterns have either D6 (A), D2 (B) or D3 (C) symmetry dependent on the spatial phase of the input with respect to J.
- For $R_I > 1.3\lambda$ activated spots start to form outside the stimulated region.

Comparison with experiments and predictions

Voltage Sensitive Dye (VSD) signal

Following the method presented in [4] we convert the model output in terms of a membrane potential u into a VSD signal OI:

$$OI(x,y,t) = g(x,y) \star [m(x,y) \star S(u(x,y,t))],$$

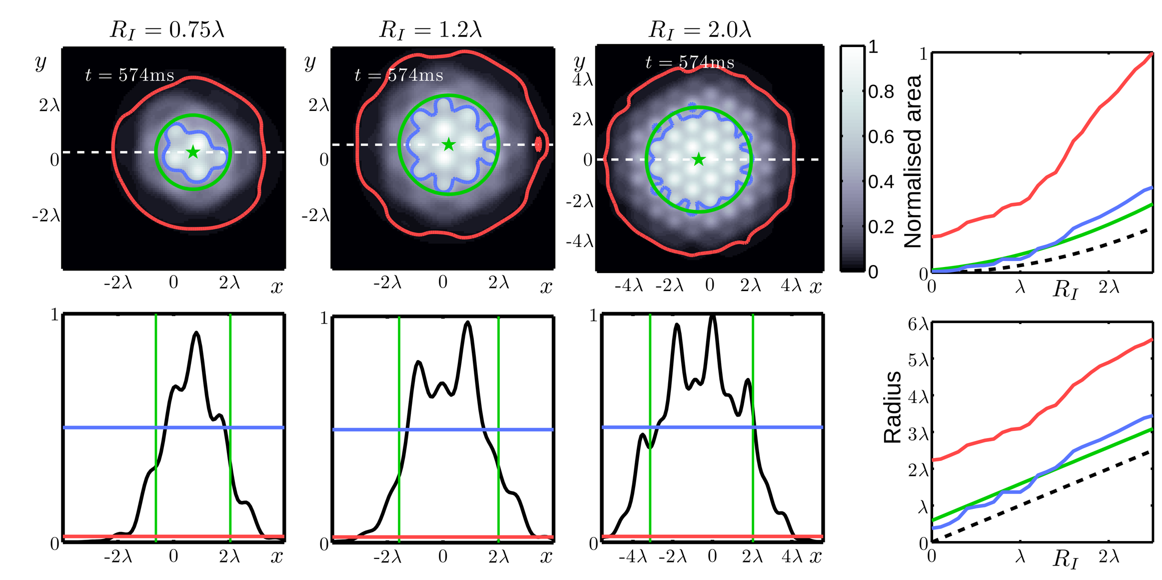
where

- $S(u(x,y,t))$ is the activity profile at time t,
- $m(x,y)$ is the connections-only kernel $m = w_E + w_I$,
- $g(x,y)$ is a Gaussian smoothing kernel.

Spatial profile of lateral spread

Characteristics of the spatial profile are determined by selected contours. A cross-section reveals:

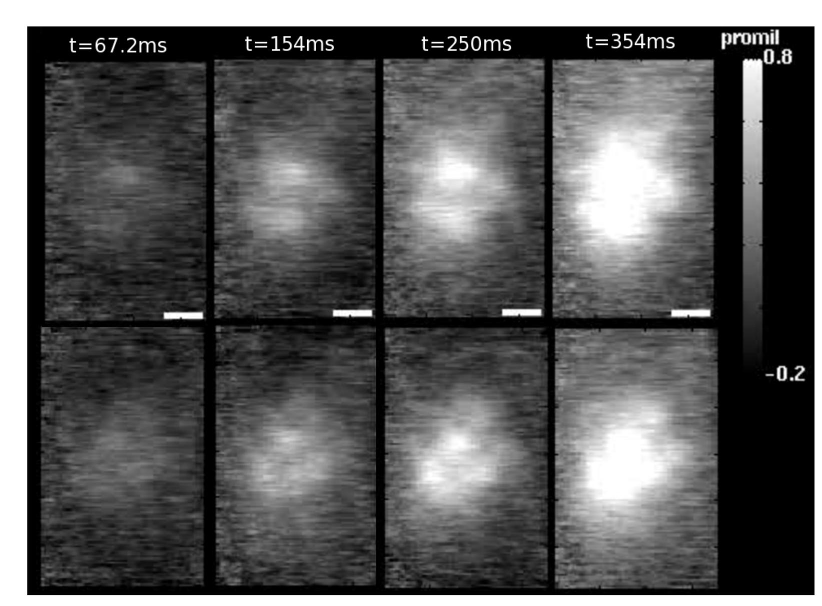
- A plateau with individual peaks.
- Longer range lateral spread through the excitatory connections.



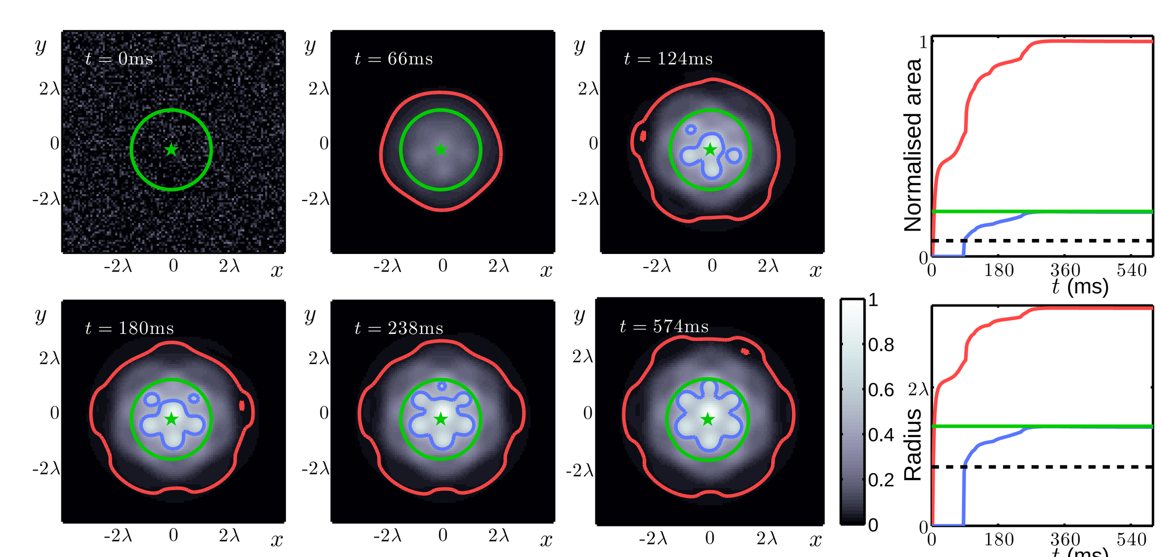
We find the relationship between R_I and the spread of activity with a brute force computation. At each R_I-value we average across 10 simulations with random phase for the input (← dashed-black box).

Dynamics of the lateral spread

Data extracted from [2]:



Patchy activity arises after around 100ms and the system converges to steady state after 250ms. The model accurately captures the dynamics of the spread, isolated peaks merge to form a coherent region.



Key results:

- The location of an input with respect to the underlying map effects the size, shape and symmetry properties of the observed patterns.
- The main spatial features of the localised activity are captured: a plateau, one or more peaks and non-patchy peripheral spread.
- The dynamic spread of local selective and longer-range non-selective activation is also captured.
- Prediction: for large inputs, patchy (selective) spread is observed outside the imprint of the stimulus.

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