Exploring the richness of center-surround dynamics: A bifurcation study

N V Kartheek Medathati1, James Rankin2, James Rankin2, James Rankin2, Guillaume S. Masson3, Pierre Komprobst1
1 Inria, Team NeuroMathComp, France, France; 2 Center for Neural Sciences, NYU; 3 Institut des Neurosciences de la Timone, Team InVibe

ABSTRACT
The balance of excitatory and inhibitory interactions between neurons is one of the characteristic aspects of neural computation. In both neural network and neural field models these interactions have been modeled using center-surround connectivity kernels. Depending on the relative strength of excitation and inhibition these networks have been found to exhibit rich and interesting dynamical behavior. Although many models have been reported in the literature using center-surround connectivity kernels and many experimental studies have shown evidence for changes in observed behavior from winner-take-all to gain control, a thorough bifurcation analysis of these networks in terms of sensitivity of the network to peak strength, discriminability of the peaks and speed of convergence has not been done. In our present work we visit this question in order to identify the parameter regimes where this important switch in the behavior of the network occurs and also establish the trade-offs that arise with the choice of a particular connectivity kernel.

Keywords: Center-surround interactions, lateral inhibition, winner take all, neural fields, bifurcation analysis

INTRODUCTION

Lateral inhibition leading to competition among neurons has been found to produce a number of different behaviors (e.g., winner take all, oscillations)

State of the Art
Recurrent neural networks with lateral inhibition have been studied in many ways.

• Short term memory in hippocampus[1].
• Velocity estimation in MT[3].
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• Synaptic weights of lateral inhibition.
• Dynamics of local inhibition (e.g. instantaneous, slow).

Future work
Bifurcation study (d and h)

Definition of
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We begin our analysis by considering a variant of
J1
and
J2
having different excitatory and inhibitory regions but matching first modes in the Fourier spectra.

Model Description
Ring model of orientation selection
The activity of a population of neurons is denoted by
u(x,t)
with a feature space of orientation
x ∈ [−π, π].
The neural field equation is given by:

\[ \partial_t u(x,t) = -u(x,t) + \int_{-\pi}^{\pi} J(x, y) S(\sin(y/\lambda)) dy + I_{ext}(x), \]

where,
• \( J \) is the connectivity kernel in direction space \( x \),
• \( S \) is a sigmoid function with slope \( \mu \),
• \( I_{ext} \) is the input.

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Definition of
J

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is defined as a weighted difference of gaussians:

\[ J(x) = g_1(x, \sigma_1) - g_2(x, \sigma_2), \]

where, \( G(x, \sigma) \) is a one dimensional Gaussian function \( G(x, \sigma) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \).

The four parameters of \( J \) \( (\sigma_1, \sigma_2, \alpha_0, \alpha_1) \) allow us to describe kernels introduced earlier:

• Connectivity A: \( \alpha_0 = \alpha_1 = \alpha_2 = > 0 \).
• Connectivity B: \( \alpha_0 = > 0, \alpha_1 = \alpha_2 = 0 \).
• Connectivity C: \( \alpha_0 = \alpha_1 = > 0, \alpha_2 = 0 \).

Considering the three mode kernel explored by [4],

\[ J_{3M}(x) = J_1 + 2J_2 \cos(x) + 2J_2 \cos(2x), \]

where \( J_0 = 1, J_1 = J_2 \),

best fit of \( J \) is obtained for \( \alpha_0 = 0.07, \sigma_0 = 2.65, \sigma_1 = 2.27, \alpha_1 = 4.83 \) denoted by \( J_1 \).

Equations

\[ \partial_t u(x,t) = -u(x,t) + \int_{-\pi}^{\pi} J(x, y) S(\sin(y/\lambda)) dy + I_{ext}(x), \]

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Bifurcation study using two bumps input with \( J_2 \) as connectivity kernel.