# 1. Psychophysical Methodology and Scaling

## I. Psychophysics

Definition/Goals
Type A vs. type B experiments, Sensitivity vs. appearence
Psychometric function $P = f(x)$
Ogive curve
- 50% point, Point of subjective equality (PSE), Threshold
- Slope

## II. Psychophysical Methodology

Concerns
- Bias
- Criterion
- Attentiveness
- Strategy
- Artifactual cues
- History of stimulation
- Who controls stimulation

Methods
- Method of adjustment
- Method of (ascending/descending) limits
- Method of constant stimuli (Yes-No)
- Forced choice (2I2AFC, etc.)
- Method of single stimuli
- Sequential testing (staircase methodologies)

Psychometric functions
- Range of dependent variable, log/linear scale, dB
- Models
  - Random threshold
  - Noise
    - Additive
    - Multiplicative (log law)
    - Multiple channels (Quick)
    - Uncertainty (Pelli)
- Parametric models

Probit/Cumulative normal: $\int_{-\infty}^{\beta(x-\alpha)} \frac{e^{-t^2/2}}{\sqrt{2\pi}}\,dt$

Logit/Logistic: $\frac{1}{1+e^{-\beta(x-\alpha)}}$

Weibull: $1-e^{-(x/\alpha)^{\beta}}$

Quick: $1-2^{-R(x)^{\beta}}$

Probability summation:

$P(\text{Detect}) = 1 - P(\text{Not detect})$

$$= 1 - \prod_i P(\text{Not detect in channel } i)$$

$$= 1 - \prod_i (1 - P(\text{detect in channel } i))$$

$$= 1 - \prod_i (1 - (1 - 2^{-R_i(x)^{\beta}}))$$

$$= 1 - \prod_i 2^{-R_i(x)^{\beta}}$$

$$= 1 - 2^{-\sum_i R_i(x)^{\beta}}$$

Thus, probability summation is like a response summed over multiple channels (e.g. a vector length)

Staircase procedures

- Concerns
  - Computation during trials
  - Efficiency/sweat factor/number of trials/trial placement
  - Subject fatigue (boredom if too easy, frustration if too hard)
  - Stationarity
  - Finger errors
  - Desired estimates: $L_{.5}$, $L_p$, slope
  - Sequential dependencies, interleaved staircases
  - Estimation bias
  - Correction for guessing and for finger errors
- Assumptions
  - Monotonic
  - Threshold approximately known
  - Slope β known or approximately known
  - Parametric form of $f$
  - Stationary
  - Independent trials (interleaving)
- Basics
  - How to place trials
  - When to stop
  - How to estimate parameters
- Procedures
  - Robbins/Munro

    $x_{n+1} = x_n - \frac{c}{n}(y_n(x_n) - p)$

    to estimate $L_p$, biased away from 50%
  - Up-Down (Dixon/Mood, Cornsweet)
  - Transformed Up-Down (Levitt/Weatherill)
    - 1-up-2-down, 2-up-1-down, 1-up-3-down, etc.
    - Halve stepsize every other turnaround and restart at current threshold estimate
    - Notion of a transformed response curve

PEST (Taylor & Creelman, Findlay, Pentland)
Wald test to change levels, changes in step size to deal with closeness and distance from correct spot, stop at minimum step size
APE (Watt & Andrews)
method of constant stimuli for blocks of trials, then fit previous 2 blocks and choose a new set of levels ranging over ±1.35 SD with momentum based on prior change
QUEST (Watson & Pelli), β as a constant for log scaled stimulus strength
sweat factor = $p(x)q(x)/(dP_T/dx)^2$
don't know $T$ so use maximum a posteriori
Maximize $P(T \mid D) = \frac{P(T)P(D \mid T)}{P(D)}$, by Bayes rule
So, maximize Quest function $Q(T) = \ln P(T) + \ln P(D \mid T)$
Assume independent trials, so

$$\ln P(D \mid T) = \ln \prod_i P(R_i \mid x_i, T)$$
$$= \sum_i \ln P(R_i \mid x_i, T)$$

Addend is either $\ln P_T(x) = \ln \Psi(x - T)$
or $\ln(1 - P_T(x)) = \ln(1 - \Psi(x - T))$
so precompute these and accumulate over trials
Log likelihood $L(T) = Q(T) - Q_0(T)$
where $Q_0(T) = \ln P(T)$, the *prior*
Stop based on a likelihood ratio test (a $\chi^2$ test)

When to stop
N trials
N turnarounds
Given standard error of the estimate

Estimation
Probit analysis
Midrun estimates
Maximum likelihood
Final values
Minimum $\chi^2$

Confidence intervals
Variance of mid-run estimates
Variance across blocks
Bootstrap of the same procedure
Closed form solution for ML (generalized variance)

**III. Scaling**

Internal representation
Weber's Law
Fechner/Log law
Stevens/Power law (magnitude estimation and production)
Cross-modal matching and log/power laws

**Exercise**: Design staircase procedures for making estimates of the .1, .2, and .75 points of a psychometric function as accurately as you can manage.