Trichromacy

- Spectral nature of light (1650+)
- Human color matching + theory (1850+)
- Physiology of photoreceptors (1980+)

[See Wandell, Foundations of Vision]
Representing spectra with vectors

Intensity Variation

[c/o David Brainard]
Additive mixtures

\[ b = b_1 + b_2 \]

Color perception: what’s it for?

[c/o David Brainard]
Perceptual color matching experiment

Arbitrary test light

Mixture of three “primary” lights

[Young, Helmholtz, Grassman, etc, 1800’s]
Grassmann’s Laws (1853)

1) Any light can be matched with a mixture of 3 primaries
2) Adding two lights results in a sum of their mixtures
3) Rescaling the light results in a rescaled mixture

⇒ Color matching can be described by an Nx3 linear system!

* Fine print: i) Normal human observer; ii) photopic intensity levels, not too bright; iii) independent primaries; iv) negative primary amplitude = add to test light.
Implications

- If $P$ is an $N \times 3$ matrix containing the primary spectra, and $H$ is an $3 \times N$ “matching matrix” that captures human color matching responses (mapping a spectrum to 3 “knob settings”), then for any light (spectrum) vector $\vec{l}$:

$$\vec{l} \sim P H \vec{\tilde{l}}$$

where $\sim$ means “looks the same as”

- Two lights look the same if (and only if) they produce the same match settings:

$$\vec{l}_1 \sim \vec{l}_2 \iff H \vec{\tilde{l}}_1 = H \vec{\tilde{l}}_2$$
Characterization

400 nm

\[
\begin{array}{c}
\text{\vdots} \\
\text{\vdots} \\
\text{\vdots} \\
\end{array}
\xrightarrow{0.27}
\begin{array}{c}
9.78 \\
0.27 \\
46.4 \\
\end{array}
\]

410 nm

\[
\begin{array}{c}
\text{\vdots} \\
\text{\vdots} \\
\text{\vdots} \\
\end{array}
\xrightarrow{0.82}
\begin{array}{c}
29.7 \\
0.82 \\
142 \\
\end{array}
\]

420 nm

\[
\begin{array}{c}
\text{\vdots} \\
\text{\vdots} \\
\text{\vdots} \\
\end{array}
\xrightarrow{2.73}
\begin{array}{c}
91.8 \\
2.73 \\
440 \\
\end{array}
\]

\[\ldots\]

700 nm

\[
\begin{array}{c}
\text{\vdots} \\
\text{\vdots} \\
\text{\vdots} \\
\end{array}
\xrightarrow{2.80}
\begin{array}{c}
7.79 \\
2.80 \\
0.00 \\
\end{array}
\]

[c/o David Brainard]

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Stiles-Burch Data, Monochromatic RGB Primaries of 645, 526, and 444 nm.

[Stiles & Burch, 1959]
Useful calculations

- Scientific: given results of one matching experiment, predict the results of another one with different primaries.

- Practical: calibrate a display device, so as to generate mixtures of three colors that match the appearance of any desired real-world spectrum.

[derive on board]

In summary:
Theory: the visual system projects the wavelength spectra of light onto a 3-dimensional subspace

- Predicts/explains perceptual “metamers” - lights that appear identical, but have physically distinct wavelength spectra (1800’s).
- Codified in CIE standards for color representation in 1931
- Underlying mechanism (cones) verified in 1987!
The underlying mechanism...
Physiological basis for color “metamers”

Confirmation of prediction (100+ years later):

Perceptual trichromatic “knob settings” for monochromatic test lights

Primary intensities that produce cone responses equal to those of monochromatic test lights

[Baylor, Nunn & Schnapf, ’87]
Some dimensionality caveats

- Normal human vision, at high ("photopic") light levels is tri-chromatic
- At low ("scotopic") light levels, we are all monochromats
- At intermediate ("mesopic") light levels, we are quadrachromats!
- Common genetic forms of color blindness are due to lack of one or more cone types