

The logistic function

The logistic function is often used to fit a measured psychometric function. This is because it has the right general properties. It starts at 0 and increases to 1 in the sigmoidal manner characteristic of measured psychometric functions.

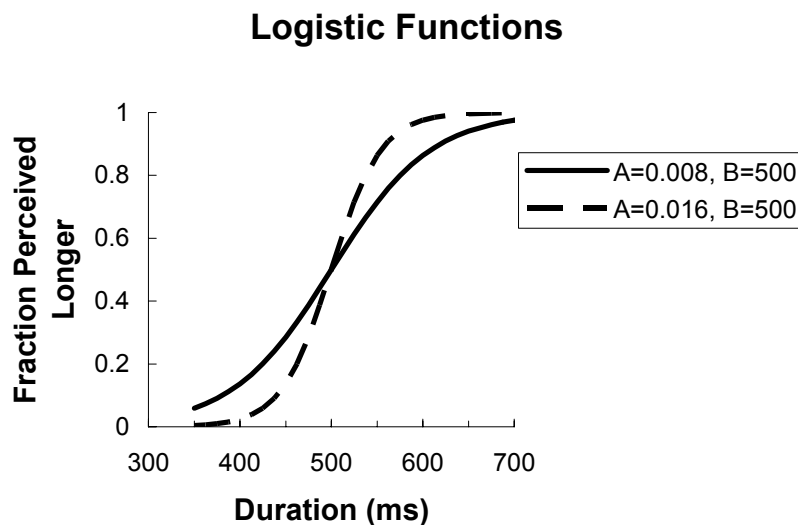
This handout describes the logistic function in the context of a duration discrimination experiment where a percent longer judgment is made as a function of stimulus duration. But the ideas are more general. For example, one could measure percent rightward judgments as a function of dot correlation in a motion discrimination experiment. The important thing to keep in mind is that the variable names may differ, but the ideas are the same.

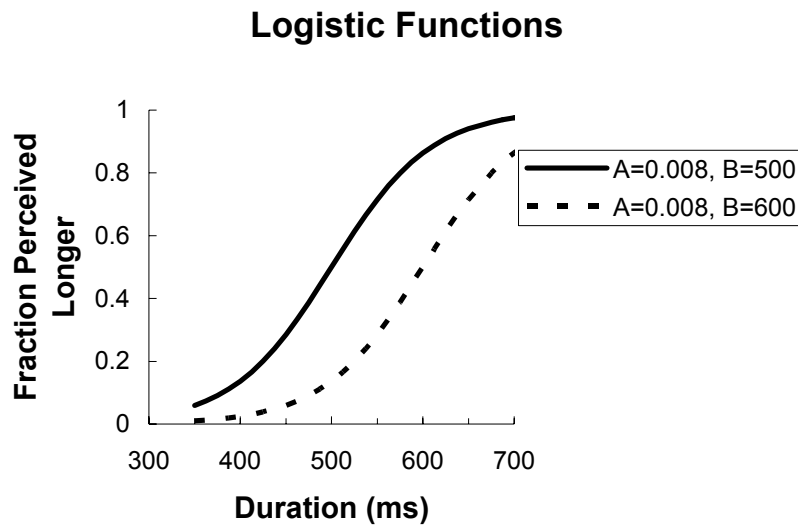
The formula for the logistic function is:

$$p = \frac{1}{1 + 10^{-A(d-B)}}$$

where p is the probability that duration d will be judged as longer than the standard duration. As you can see, the formula has two parameters, A and B . Changing these parameters affects the exact shape of the logistic function. The parameter A affects how steeply the function rises as it passes through its midpoint ($p = 0.5$), while the parameter B determines at which duration the midpoint occurs.

The graphs below illustrate the effects of changing each of the parameters while holding the other one constant.





Fitting the logistic function.

The reason for fitting a logistic function to your measured psychometric functions is to get a more accurate estimate of the true threshold. Each individual fraction longer judgment will have some error. By fitting the logistic, we can smooth the error to get a better estimate of what is going on.

The idea in fitting is as follows. We use simple algebra to transform both sides of the logistic equation to express A and B as the parameters of a line. This algebra goes as follows:

$$1) p = \frac{1}{1 + 10^{-A(d-B)}} \quad \text{Logistic equation}$$

$$2) p(1 + 10^{-A(d-B)}) = 1 \quad \text{Multiply both sides by } 1 + 10^{-A(d-B)}$$

$$3) p + p10^{-A(d-B)} = 1 \quad \text{Expand left side by multiplying}$$

$$4) p10^{-A(d-B)} = 1 - p \quad \text{Subtract } p \text{ from both sides}$$

$$5) 10^{-A(d-B)} = \frac{1-p}{p} \quad \text{Divide both sides by } p$$

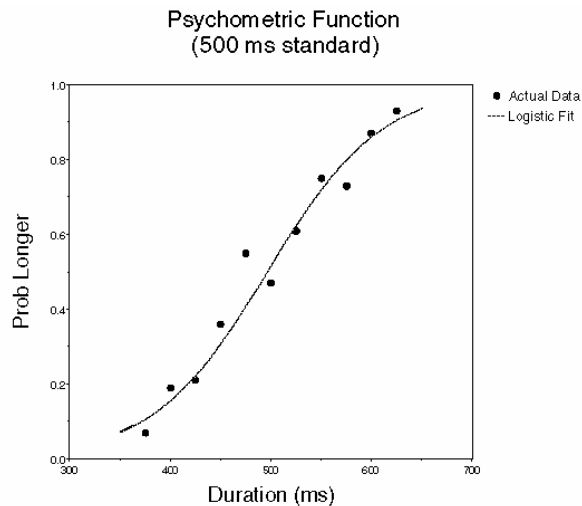
$$6) \log(10^{-A(d-B)}) = \log\left(\frac{1-p}{p}\right) \quad \text{Take the log of both sides}$$

$$7) -A(d-B) = \log\left(\frac{1-p}{p}\right) \quad \text{Simplify left side using fact that } \log(10^x) = x$$

$$8) -\log\left(\frac{1-p}{p}\right) = A(d-B) \quad \text{Multiply both sides by -1 and switch left and right}$$

Notice that the expression we arrive at is exactly the equation for a straight line. To see this, let $y = -\log((1-p)/p)$ and let $x = d$. Then Equation 8 is exactly equivalent to $y = ax + b$, where $a=A$ and $b=-AB$.

Recall that the data we measure is durations (d) and corresponding fractions longer (p). To find A and B , then, we use the quantities $y = -\log((1-p)/p)$ and $x = d$. This may be done in Excel, but sometimes we do it for you in MATLAB. We then find A and B by fitting the best line between x and y . Once we have A and B , we can plug them into Equation 1 above for the best fitting logistic function. A logistic function fit to data in this way is shown below.



Finding the threshold

The idea behind finding a threshold is that we find d_{25} and d_{75} , the durations that produce 25% and 75% longer judgments. It is pretty easy to do this graphically, and the following graph illustrates what is meant by d_{25} and d_{75} . But once we know A and B , it is possible to find these two durations directly from the logistic formula. We know that d_{25} must satisfy the equation

$$0.25 = \frac{1}{1 + 10^{-A(d_{25} - B)}}.$$

If we follow the same steps as in 1-9 above, we get

$$-\log\left(\frac{1-0.25}{0.25}\right) = A(d_{25} - B).$$

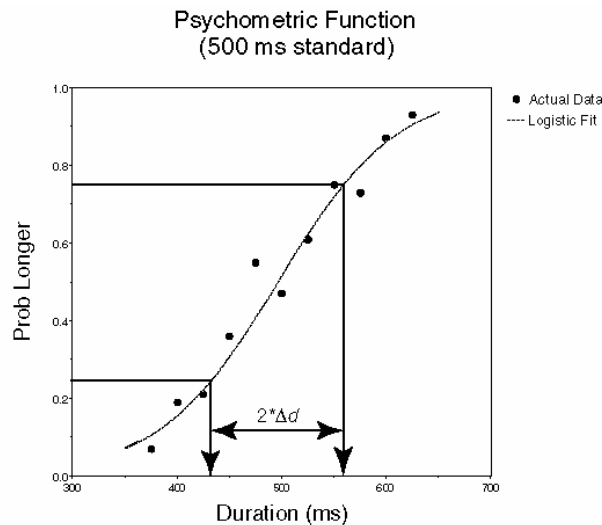
After a little more algebra, this leads to

$$d_{25} = B - \frac{1}{A} \log\left(\frac{1-0.25}{0.25}\right).$$

Once we have fit the logistic function, we know A and B , so it is simple to plug in and find d_{25} . A similar expression holds for d_{75} . It is common to take as the overall threshold

$$\Delta d = \frac{d_{75} - d_{25}}{2}.$$

This is known as the 75% correct threshold.



Finding the PSE

The logic for finding the point of subjective equality is just the same as for finding either d_{25} or d_{75} above. The only difference is that for the PSE, we want to find d_{50} , the point corresponding to 50-50 judgments. We just substitute .50 for .25 in the formula for d_{25} above.