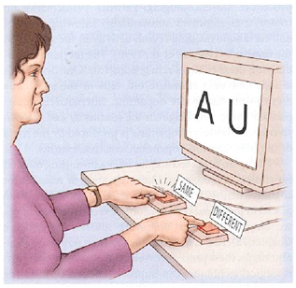
Three methods for measuring perception

1. Magnitude estimation
2. Matching
3. Detection/discrimination



Concerns of the Psychophysicist

- Bias/Criterion
- Attentiveness
- Strategy/Artifactual Cues
- History of stimulation
- Who controls stimulation

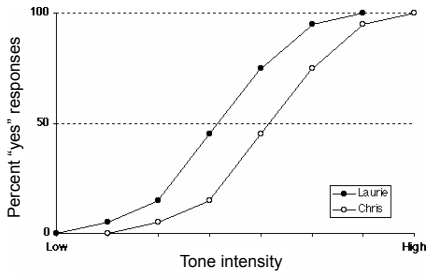
Detection / discrimination

In a detection experiment, the subject's task is to detect small differences in the stimuli.

Procedures for detection/discrimination experiments

- Method of adjustment
- Method of limits
- Yes-No/method of constant stimuli
- Forced choice

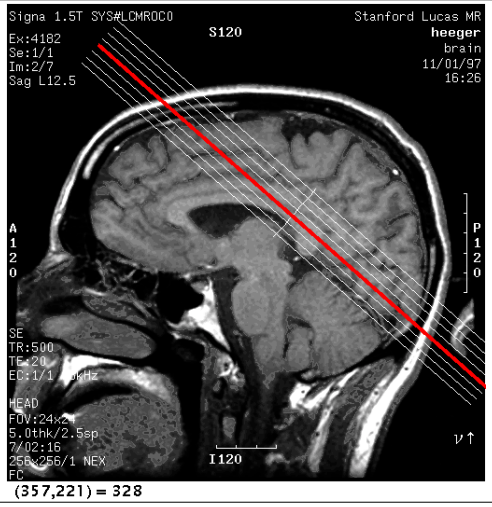
Yes/no method of constant stimuli



Do these data indicate that Laurie's threshold is lower than Chris's threshold?

Forced Choice

- Present signal on some trials, no signal on other trials (catch trials).
- Subject is forced to respond on every trial either "Yes" the thing was presented" or "No it wasn't". If they're not sure then they must guess.
- Advantage: With the forced-choice method, we have both types of trials so we can count both the number of hits and the number of false alarms to get an estimate of discriminability independent on the criterion.
- Versions: Yes-no, 2AFC, 2IFC



Forced choice: four possible outcomes

	Doctor responds "yes"	Doctor responds "no"
Tumor present	Hit	Miss
Tumor absent	False alarm	Correct reject

Information acquisition

	Doctor responds "yes"	Doctor responds "no"
Tumor present	Hit	Miss
Tumor absent	False alarm	Correct reject

Criterion shift

	Doctor responds "yes"	Doctor responds "no"
Tumor present	Hit	Miss
Tumor absent	False alarm	Correct reject

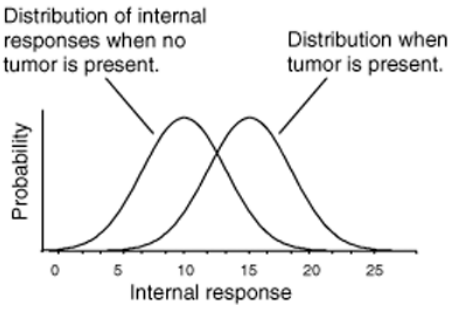
Information and criterion

Two components to the decision-making: signal strength and criterion.

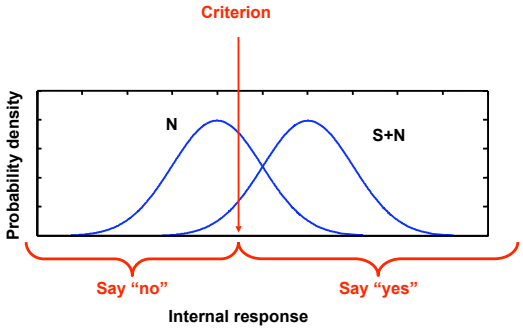
- **Signal strength:** Acquiring more information is good. The effect of information is to increase the likelihood of getting either a hit or a correct rejection, while reducing the likelihood of an outcome in the two error boxes.

- **Criterion**: Different people may have different bias/ criterion. Some may may choose to err toward "yes" decisions. Others may choose to be more conservative and say "no" more often.

Internal response: probability of occurrence curves

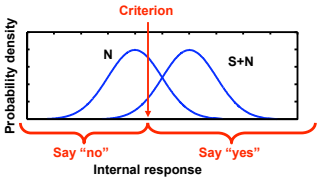


The criterion

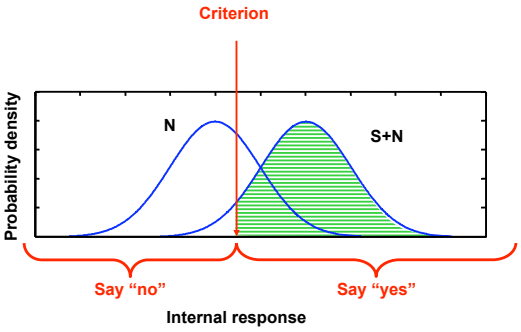


Applications of SDT: Examples

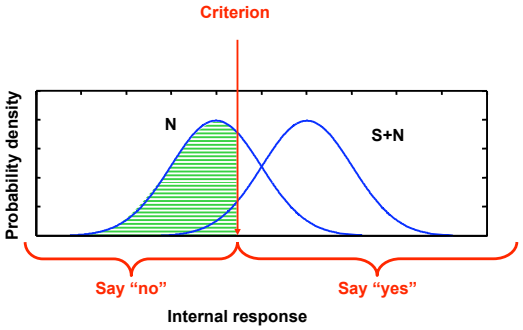
- Vision
  - Detection (something vs. nothing)
  - Discrimination (lower vs greater level of: intensity, contrast, depth, slant, size, frequency, loudness, ...
- Memory (internal response = trace strength = familiarity)
- Neurometric function/discrimination by neurons (internal response = spike count)



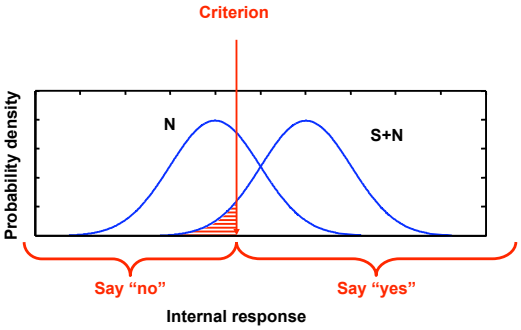
Hits
(response "yes" on signal trial)



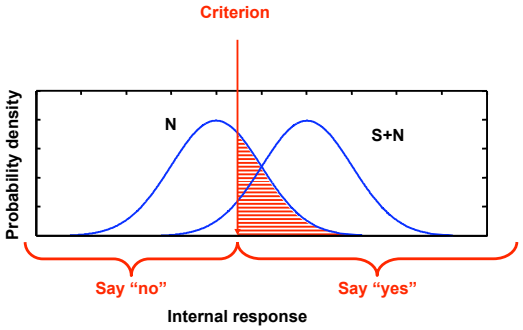
Correct rejects
(response "no" on no-signal trial)



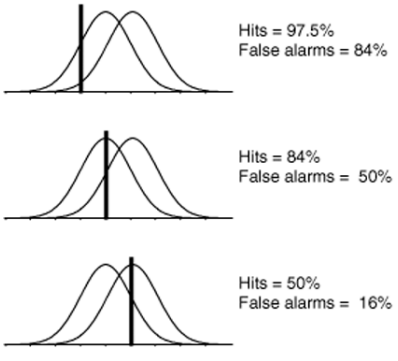
Misses
(response "no" on signal trial)



False Alarms
(response "yes" on no-signal trial)

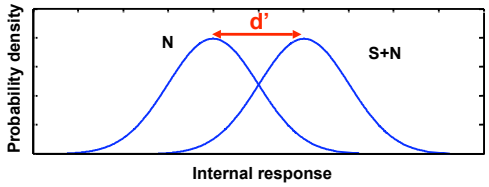


Criterion shift

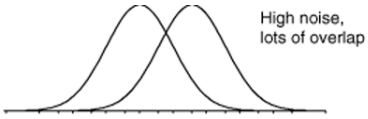


Discriminability (d-prime, d')

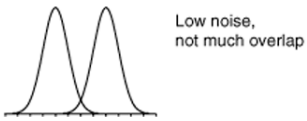
d-prime is the distance between the N and S+N curves



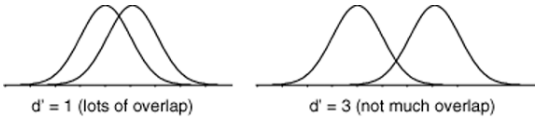
Discriminability (d-prime, d')

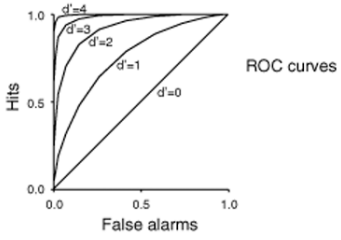


$$d' = \frac{\text{separation}}{\text{spread}} = \frac{\text{signal}}{\text{noise}}$$

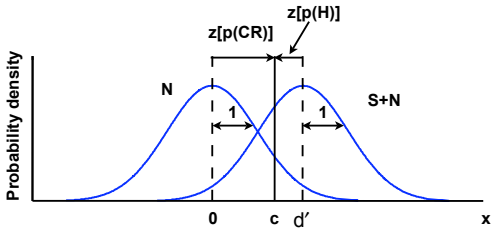


Receiver Operating Characteristic (ROC)





SDT: Gaussian Case



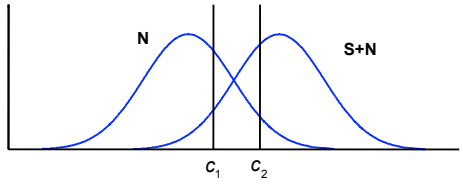
$$d' = z[p(H)] + z[p(CR)] = z[p(H)] - z[p(FA)]$$

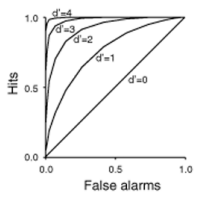
$$c = z[p(CR)]$$

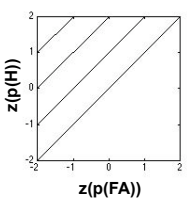
$$G(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(x-\mu)^2/2\sigma^2}$$

$$\beta = \frac{p(x = c \mid S+N)}{p(x = c \mid N)} = \frac{e^{-(c-d')^2/2}}{e^{-c^2/2}}$$

ROC: Gaussian Case







Optimal Criterion

Payoff Matrix

Stimulus	Response: Yes	Response: No
S+N	V^{Yes}_{S+N}	V^{No}_{S+N}
N	V^{Yes}_{N}	V^{No}_{N}

Optimal Criterion

Stimulus	Response: Yes	Response: No
S+N	V_{S+N}^{Yes}	V_{S+N}^{No}
N	V_{N}^{Yes}	V_{N}^{No}

$$E(Yes \mid x) = V_{S+N}^{Yes} p(S+N \mid x) + V_{N}^{Yes} p(N \mid x)$$
$$E(No \mid x) = V_{S+N}^{No} p(S+N \mid x) + V_{N}^{No} p(N \mid x)$$

Say yes if $E(Yes \mid x) \geq E(No \mid x)$

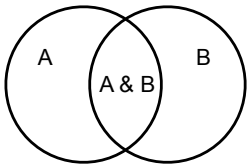
Optimal Criterion

$$E(Yes \mid x) = V_{S+N}^{Yes} p(S+N \mid x) + V_{N}^{Yes} p(N \mid x)$$
$$E(No \mid x) = V_{S+N}^{No} p(S+N \mid x) + V_{N}^{No} p(N \mid x)$$

Say yes if $E(Yes \mid x) \geq E(No \mid x)$

Say yes if $\dfrac{p(S+N \mid x)}{p(N \mid x)} \geq \dfrac{V_{N}^{No} - V_{N}^{Yes}}{V_{S+N}^{Yes} - V_{S+N}^{No}} = \dfrac{V(\text{Correct} \mid N)}{V(\text{Correct} \mid S+N)}$

Aside: Bayes' Rule



$p(A \mid B)$ = probability of A given that B is asserted to be true = $\dfrac{p(A \& B)}{p(B)}$

$$p(A \& B) = p(B)p(A \mid B)$$
$$= p(A)p(B \mid A)$$
$$\Rightarrow p(A \mid B) = \frac{p(B \mid A)p(A)}{p(B)}$$

Apply Bayes' Rule

Posterior — Likelihood — Prior

$$p(S+N \mid x) = \frac{p(x \mid S+N)p(S+N)}{p(x)}$$

Nuisance normalizing term

$$p(N \mid x) = \frac{p(x \mid N)p(N)}{p(x)}, \text{ hence}$$

$$\frac{p(S+N \mid x)}{p(N \mid x)} = \left(\frac{p(x \mid S+N)}{p(x \mid N)}\right)\left(\frac{p(S+N)}{p(N)}\right)$$

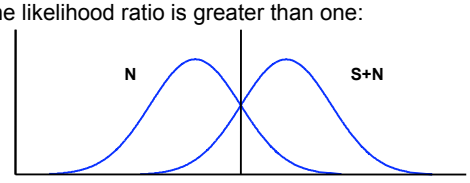
Posterior odds = Likelihood ratio × Prior odds

Optimal Criterion

Say yes if $\dfrac{p(S+N \mid x)}{p(N \mid x)} \geq \dfrac{V(\text{Correct} \mid N)}{V(\text{Correct} \mid S+N)}$

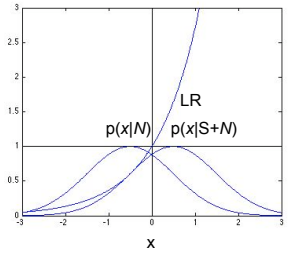
i.e., if $\dfrac{p(x \mid S+N)}{p(x \mid N)} \geq \dfrac{p(N)}{p(S+N)} \dfrac{V(\text{Correct} \mid N)}{V(\text{Correct} \mid S+N)} = \beta$

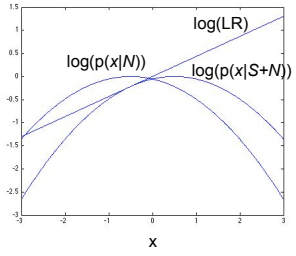
Example, if equal priors and equal payoffs, say yes if the likelihood ratio is greater than one:



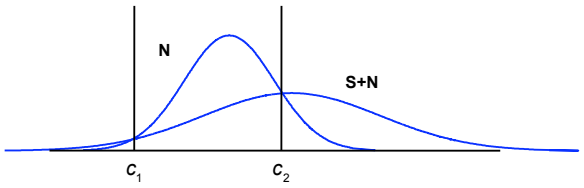
Likelihood Ratio

Optimal binary decisions are a function of the likelihood ratio (it is a sufficient statistic, i.e., no more information is needed or useful).





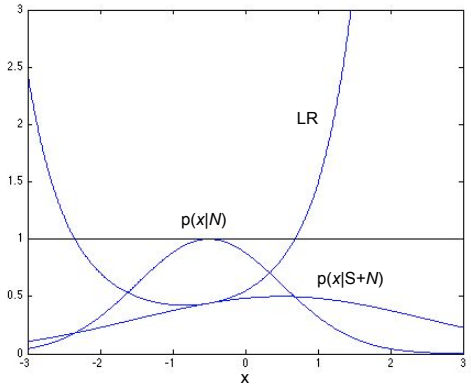
Gaussian Unequal Variance



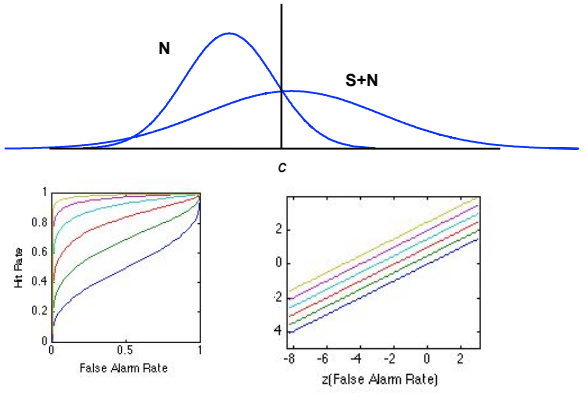
SDT (suboptimal): Say yes if $x \geq c_2$

Optimal strategy: Say yes if $x \geq c_2$ or $x \leq c_1$

Gaussian Unequal Variance



Gaussian Unequal Variance



Poisson Distribution

Counting distribution for processes consisting of "events" that occur randomly at a fixed rate **λ** events/sec.

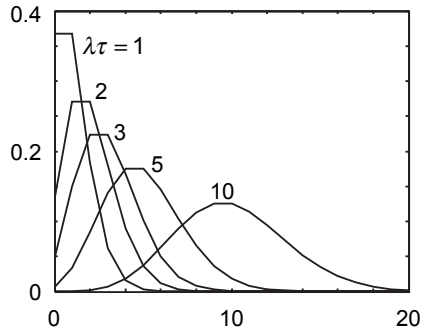
The time until the next event occurs follows an exponential distribution: $p(t) = \lambda e^{-\lambda t}$

Since **λ** is the rate, then over a period of **τ** seconds, the expected count is **λ τ**. The distribution of event count is Poisson:

$$p(k) = \frac{e^{-\lambda\tau}(\lambda\tau)^k}{k!}$$

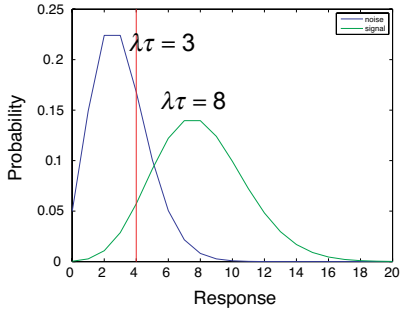
Poisson Distribution

$$p(k) = \frac{e^{-\lambda\tau}(\lambda\tau)^k}{k!}$$

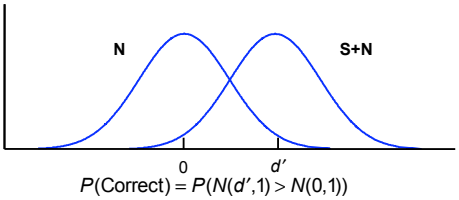


Poisson Distribution SDT

$$p(k) = \frac{e^{-\lambda\tau}(\lambda\tau)^k}{k!} \qquad LR(k) \propto \left(\frac{\lambda_{S+N}}{\lambda_N}\right)^k$$



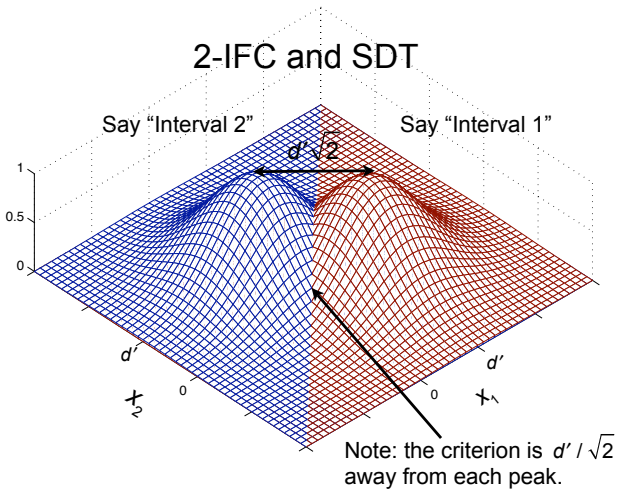
2-IFC and SDT



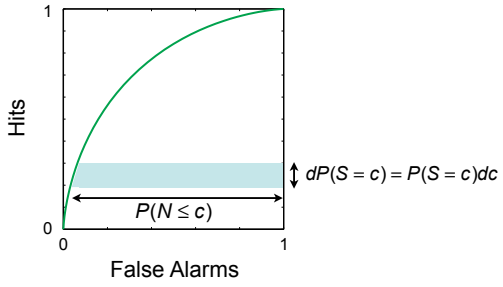
$$P(\text{Correct}) = P(N(d',1) > N(0,1))$$
$$= P(N(d',1) - N(0,1) > 0)$$
$$= P(N(d',\sqrt{2}) > 0)$$
$$= P(N(d'/\sqrt{2},1) > 0)$$
$$= P(N(0,1) < d'/\sqrt{2})$$

Where $N(\mu,\sigma)$ means a random variable with mean μ and SD σ.

2-IFC and SDT

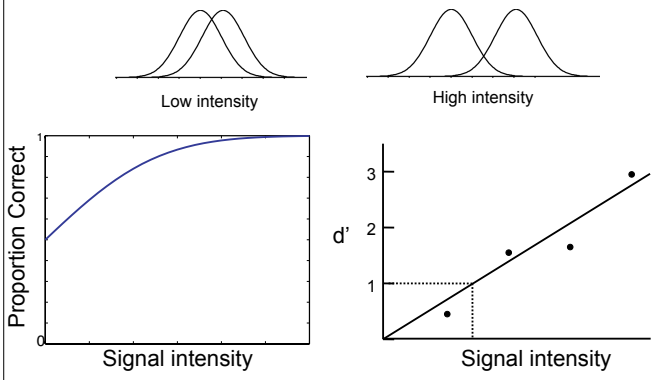


2-IFC and the Yes-No ROC



$$P(\text{Correct in 2-IFC}) = \int P(S=c)P(N \le c)dc$$
$$= \text{Area under the ROC}$$

Measuring thresholds

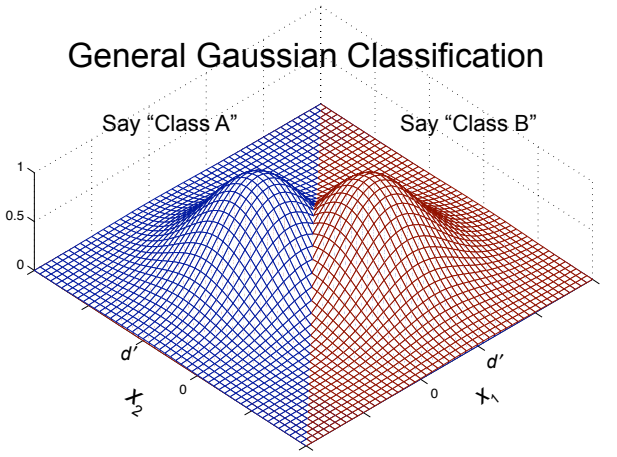


Assumptions: $x \propto$ signal strength, σ constant

Aside: 2-IFC and Estimation of Threshold

- Frequently one wishes to estimate the signal strength corresponding to a fixed, arbitrary value of d’, defined as threshold signal strength.
- For this, one can measure performance at multiple signal strengths, estimate d’ for each, fit a function (as in the previous slide) and interpolate to estimate threshold.
- Staircase methods are often used as a more time-efficient method. The signal strength tested on each trial is based on the data collected so far, trying to concentrate testing at levels that are most informative.
- Methods: 1-up/1-down (for PSE: point of subjective equality), 1-up/2-down, etc., QUEST, APE, PEST, ...

General Gaussian Classification



General Gaussian Classification

The *d*-dimensional Gaussian distribution:

$$p(\vec{x}) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(\vec{x}-\vec{\mu})^T \Sigma^{-1}(\vec{x}-\vec{\mu})\right]$$

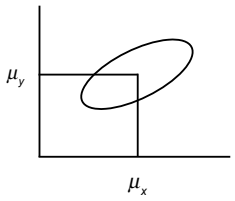
$\Sigma =$ the covariance matrix

General Gaussian Classification

In 2 dimensions: Mahalanobis distance

$$p(\vec{x}) = \frac{1}{2\pi|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(\vec{x}-\vec{\mu})^T \Sigma^{-1}(\vec{x}-\vec{\mu})\right]$$

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$$



General Gaussian Classification

Say "Category A" if

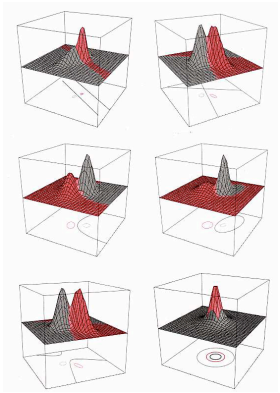
$$LR(\vec{x}) = \frac{\frac{1}{(2\pi)^{d/2}|\Sigma_A|^{1/2}} \exp\left[-\frac{1}{2}(\vec{x}-\vec{\mu}_A)^T \Sigma_A^{-1}(\vec{x}-\vec{\mu}_A)\right]}{\frac{1}{(2\pi)^{d/2}|\Sigma_B|^{1/2}} \exp\left[-\frac{1}{2}(\vec{x}-\vec{\mu}_B)^T \Sigma_B^{-1}(\vec{x}-\vec{\mu}_B)\right]} > 1$$

Take log and simplify. Say "Category A" if

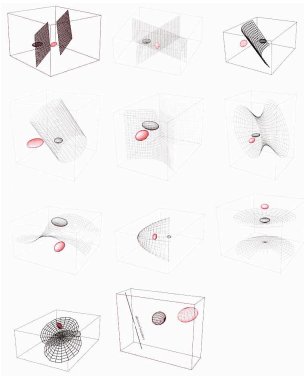
$$(\vec{x}-\vec{\mu}_B)^T \Sigma_B^{-1}(\vec{x}-\vec{\mu}_B) - (\vec{x}-\vec{\mu}_A)^T \Sigma_A^{-1}(\vec{x}-\vec{\mu}_A)$$
$$= MD(\vec{x},\vec{\mu}_B) - MD(\vec{x},\vec{\mu}_A) > C$$

which is a quadratic in $\vec{x}$.

General Gaussian Classification



General Gaussian Classification

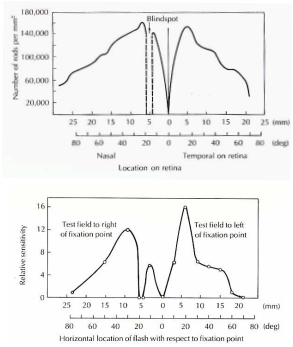


Application: Absolute Threshold

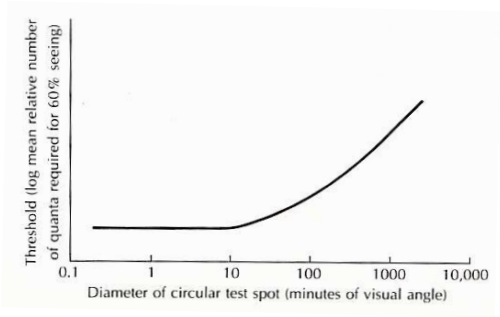
Question: What is the minimal number of photons required for a stimulus to be visible under the best of viewing conditions?

Hecht, Schlaer & Pirenne (1942)

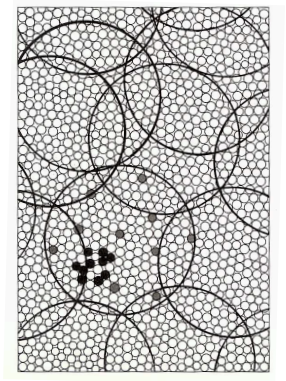
Sensitivity by Eccentricity



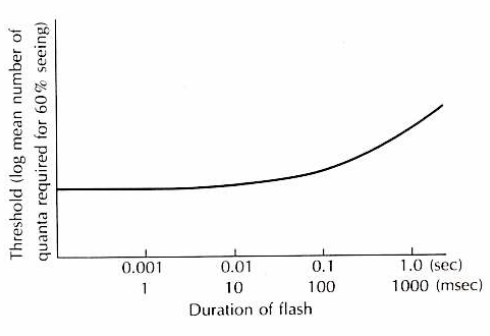
Spatial Summation



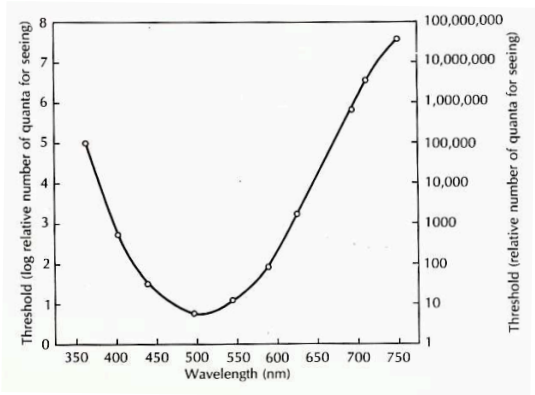
Spatial Summation



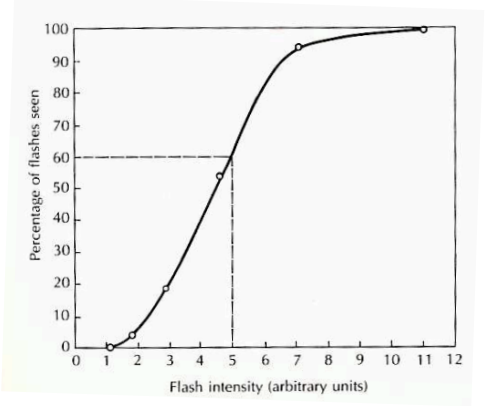
Temporal Summation



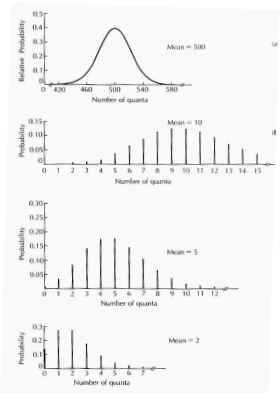
Choice of Wavelength



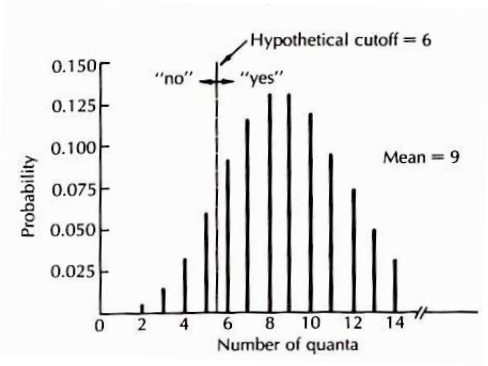
The Data



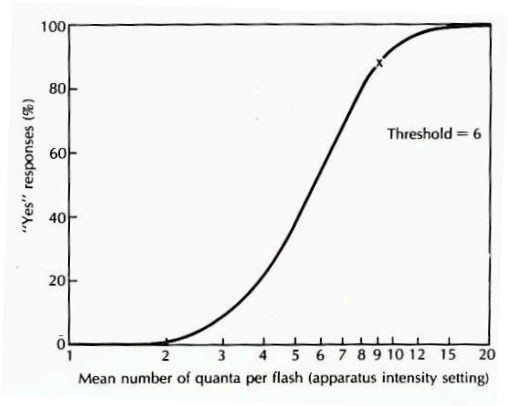
Stimulus Variability (Poisson)



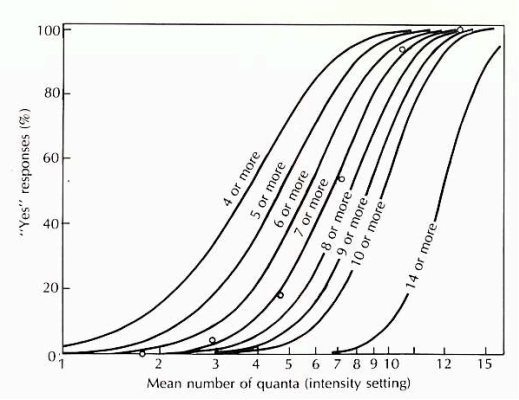
Model with no Subject Variability



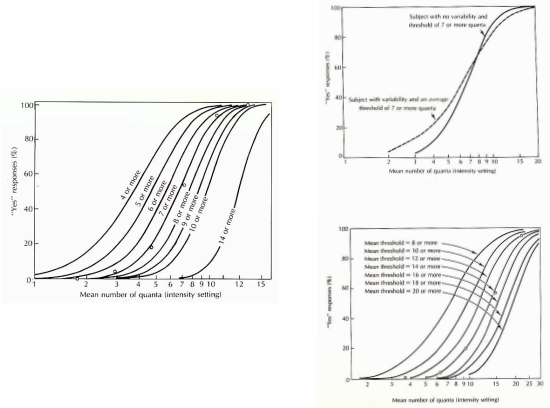
Model with no Subject Variability



Model with no Subject Variability



Model with Subject Variability

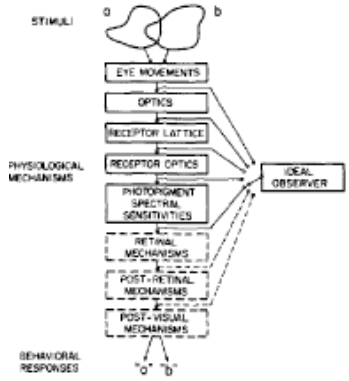


Application: Ideal Observer

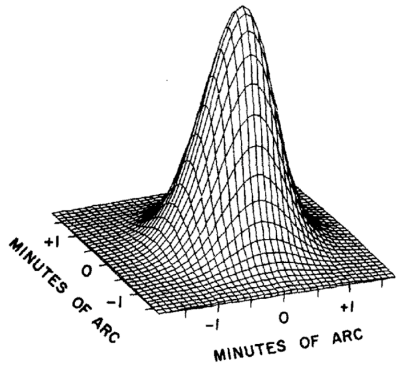
Question: What is the minimal contrast required to carry out various detection and discrimination tasks?

Geisler (1989)

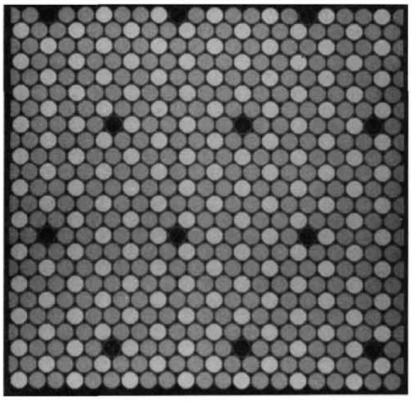
Sequential Ideal Observer



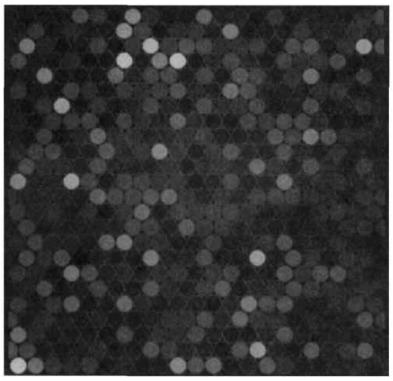
Point-Spread Function



Idealized Receptor Lattice



Poisson Statistics



Ideal Discriminator

Given two known possible targets A and B with expected photon absorbtions a_i and b_i and actual photon catches Z_i in receptor *i,* respectively, calculate the likelihood ratio $p(Z_i \mid a_i)/p(Z_i \mid b_i)$, take the log, do some algebra and discover the following quantity is monotonic in likelihood ratio:

$$Z = \sum_i Z_i \ln\left(a_i / b_i\right)$$

Decisions are thus based on an “ideal receptive field.”

Example: 2-Point Resolution