Implement a simplified version of an Oscillatory Recurrent Gated Neural Integrator Circuit (ORGaNICs) using the equations below. This circuit has 3 neurons, a primary neuron and 2 modulatory neurons and an input neuron. In these equations, $y(t)$ is the membrane potential of the primary neuron and $y^+(t)$. The membrane potential responses of the modulator neurons are $a(t)$ and $u(t)$, and their firing rates are $a^+(t)$ and $u^+(t)$. The value of $x^+(t)$ is the firing rate of the input neuron. The value of $b_0$ is a constant that determines the input gain. The value of $\sigma$ is constant that determines the contrast gain.

\begin{align*}
\tau_y \frac{dy}{dt} &= -y + \frac{b_0}{1 + b_0} x^+ + \frac{1}{1 + a^+} \hat{y} & (1) \\
y^+ &= y^2 & (2) \\
\hat{y} &= \sqrt{y^+} & (3) \\
\tau_a \frac{da}{dt} &= -a + u^+ + au^+ & (4) \\
a^+ &= a & (5) \\
\tau_u \frac{du}{dt} &= -u + uy^+ + u_{\min} & (6) \\
u^+ &= \sqrt{u} & (7)
\end{align*}

Use the following values for the various constants and the time step:

\begin{align*}
u_{\min} &= \left( \frac{\sigma b_0}{1 + b_0} \right)^2 \\
b_0 &= 0.2 \\
\sigma &= 0.1 \\
\tau_y &= 1 \text{ ms} \\
\tau_a &= 2 \text{ ms} \\
\tau_u &= 10 \text{ ms} \\
\Delta t &= 1 \text{ ms}
\end{align*}

The firing rate of the input neuron $x^+(t) = x_{\text{Amp}}$ (a positive non-zero value) for $t = 0$ to $t = 500$ ms, and then $x^+(t) = 0$ for $t = 500$ to $t = 1000$ ms.
1) Graph the firing rate responses of the principal neuron over time from $t = 0$ to $t = 1000$ ms for each of several values of the input firing rate: $x_{\text{Amp}} = 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1.0$. You should get results that look like the blue curves in Fig. 3E of Heeger & Zemlinanova (2020).

2) Compute the mean response of the principal neuron from $t = 250$ to $t = 500$ ms for each value of the input firing rate and make a graph of the mean responses versus the input rate ($x_{\text{Amp}}$). Plot this on a log axis for the input rate and you should get a result that looks like the blue curve in Fig. 3A of Heeger & Zemlinanova (2020).

3) Repeat parts 1 and 2 with $\tau_u = 1$ ms. You should get results that look like the blue curves in Figs. 3F and 3A of Heeger & Zemlinanova (2020).