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Spherical Retinal Flow for a Fixating Observer

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Abstract: When a human observer moves, the eye continually fixates on targets in the world. Although fixation is a common process in human vision, its role has not yet been established for computational purposes. The main contribution of this paper is to formalize the retinal flow for a fixating observer. A further contribution – a potentially more practical one – is to explore the role of the periphery in predicting collision. Utilizing fixation is expected to turn out to be especially fruitful in light of recent advances in computer vision for constructing active head/eye systems [3].

In this work we make the following assumptions: (i) the observer moves with respect to the world and fixates on a target; (ii) the world is rigid, with no independently moving elements; and (iii) the possible rotation axes of the eye lie on a plane (comparable to Listing's Plane). Assumptions (ii) and (iii) make the problem of determining retinal flow tractable.

We first define retinal flow for a 2D universe and then extend it to the full 3D case; the flow in 2D turns out to form a component of the flow in 3D. The retinal flow in 3D will be decomposed into longitudinal and latitudinal flow; the behavior of longitudinal flow along the retinal periphery will be further analyzed for interesting properties. Finally the results of a simulated experiment on retinal flow at the periphery will be presented.

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1 Introduction

When a human observer moves, the eye continually fixates on targets in the world. Although fixation is a common process in human vision, its role has not yet been established for computational purposes. The main contribution of this paper is to formalize the retinal flow for a fixating observer. A further contribution – a potentially more practical one – is to explore the role of the periphery in predicting collision. Utilizing fixation is expected to turn out to be especially fruitful in light of recent advances in computer vision for constructing active head/eye systems [3].

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2 Retinal Flow in a Rigid 2D Universe

For ease of exposition, we first consider a reduced case of a 2D universe in which we define the flow on the retina for any given point in the universe; as the observer moves the flow determines how each point projected on the retina moves. The crucial difference between the present work and traditional optical flow ([2]; cf. [5] for review) is the introduction of a fixating observer.

2.1 Calculating Retinal Flow

In a 2D universe consisting of a 2D plane, the eye of the observer corresponds to a circle (E in Fig. 1), and the retina corresponds to a semicircle (R).[1] As the observer moves, the center of the eye (O in Fig. 1) translates on the 2D plane. In addition, the eye may also rotate about its center (O). A combination of these two types of motion is sufficient to capture all possible movements of the eye in this 2D universe.

When the observer fixates on a target point (such as a corner of an object) this point – by definition – remains projected at the center of the retina, i.e., on the fovea (F in Fig. 1). In order to maintain fixation while moving, the observer has to rotate the eye about its center (O). Although the target point that the observer fixates on (T in Fig. 1) is stationary at the fovea, the retinal image of the rest of the points (e.g. P in Fig. 1) in the world can be expected to change; this change will be precisely defined below.

The instantaneous change in the retinal image will be referred to as *retinal flow*, and it will be defined in the present formalization in terms of angular coordinates. The retinal flow of a point P is an angular velocity, i.e., the change over time of an angle formed by the following two rays: (i) the direction of gaze (ray OT in Fig. 1) and (ii) the ray from the point in the world to the center of the eye (OP in Fig. 1).

The retinal flow may be decomposed into two components: one due to observer translation and the other due to the fixating rotation. The first component, due to observer translation, is:

$$\omega_1 = \frac{|\vec{\mathbf{v}} \times \vec{\mathbf{p}}|}{|\vec{\mathbf{p}}|^2} \tag{1}$$

where $\vec{\mathbf{v}}$ is the translational velocity of the center of the eye, and $\vec{\mathbf{p}}$ is the vector from the center of the eye to the point P.

The second component, arising due to fixation and corresponding to the rotation

[1]The eye and the retina considered in this paper will only correspond to the human eye in terms of optics and not in terms of the actual physical structure.

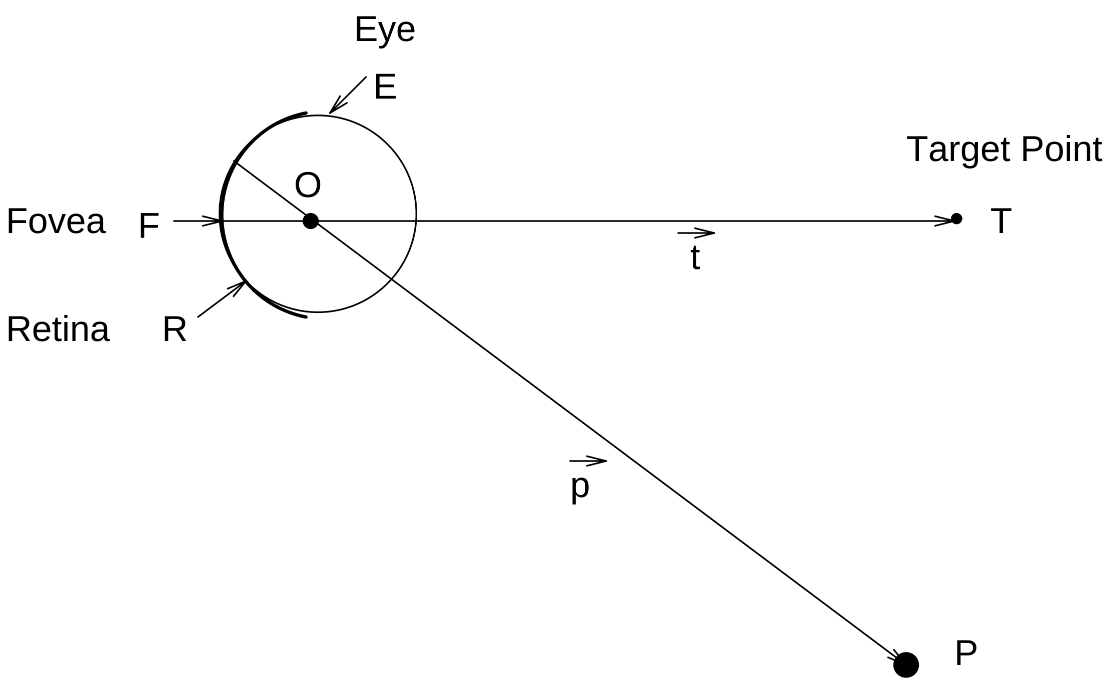


Figure 1: Model of the eye in a 2D universe.

about the center of the eye, is:

$$\omega_2 = -\frac{|\vec{\mathbf{v}} \times \vec{\mathbf{t}}|}{|\vec{\mathbf{t}}|^2} \tag{2}$$

where $\vec{\mathbf{t}}$ is the vector from the center of the eye to the target point (T in Fig. 1); the direction of $\vec{\mathbf{t}}$ is also called the direction of gaze, or optical axis. The negative sign signifies the fact that when the eye rotates – in order to fixate – the points on the retina move in the opposite direction.

Finally, the resultant retinal angular velocity of a point is the sum of the two angular velocities, i.e.

$$\begin{aligned} \omega &= \omega_1 + \omega_2 \\ &= \frac{|\vec{\mathbf{v}} \times \vec{\mathbf{p}}|}{|\vec{\mathbf{p}}|^2} - \frac{|\vec{\mathbf{v}} \times \vec{\mathbf{t}}|}{|\vec{\mathbf{t}}|^2} \end{aligned} \tag{3}$$

Given the observer velocity $\vec{\mathbf{v}}$, the vector representing gaze $\vec{\mathbf{t}}$, and the vector from the eye to any point in the world $\vec{\mathbf{p}}$, Equation 3 defines the retinal flow of that point in a 2D universe.

2.2 Level Sets of Retinal Flow

In this section we will consider the points in the 2D universe that give rise to the same value of retinal flow. Let us first isolate those points in the 2D universe that correspond to zero retinal flow. The projections of such points on the retina come to rest (for an instant) while the observer moves and fixates. This is true when ω in Equation 3 equals zero:

$$\frac{|\vec{\mathbf{v}} \times \vec{\mathbf{p}}|}{|\vec{\mathbf{p}}|^2} - \frac{|\vec{\mathbf{v}} \times \vec{\mathbf{t}}|}{|\vec{\mathbf{t}}|^2} = 0 \tag{4}$$

Factoring the magnitude of $\vec{\mathbf{v}}$ from both terms results in:

$$\frac{|\hat{\mathbf{v}} \times \vec{\mathbf{p}}|}{|\vec{\mathbf{p}}|^2} - \frac{|\hat{\mathbf{v}} \times \vec{\mathbf{t}}|}{|\vec{\mathbf{t}}|^2} = 0 \tag{5}$$

where $\hat{\mathbf{v}}$ is the unit vector in the direction of $\vec{\mathbf{v}}$.

Although the points in the 2D universe satisfying the above equation lie on a simple curve, reducing the solution to a recognizable form requires further vector algebraic manipulations. To this effect, let us introduce a unit vector $\hat{\mathbf{u}}$ perpendicular to $\hat{\mathbf{v}}$; then $|\hat{\mathbf{v}} \times \vec{\mathbf{t}}| = \hat{\mathbf{u}} \cdot \vec{\mathbf{t}}$. We can now rewrite equation 5 in terms of $\hat{\mathbf{u}}$, removing $\hat{\mathbf{v}}$.

$$\frac{\hat{\mathbf{u}} \cdot \vec{\mathbf{p}}}{\vec{\mathbf{p}} \cdot \vec{\mathbf{p}}} - \frac{\hat{\mathbf{u}} \cdot \vec{\mathbf{t}}}{\vec{\mathbf{t}} \cdot \vec{\mathbf{t}}} = 0 \tag{6}$$

This can be rewritten as:

$$\vec{\mathbf{p}} \cdot \vec{\mathbf{p}} - \hat{\mathbf{u}} \cdot \vec{\mathbf{p}} \left(\frac{\vec{\mathbf{t}} \cdot \vec{\mathbf{t}}}{\hat{\mathbf{u}} \cdot \vec{\mathbf{t}}} \right) = 0 \tag{7}$$

and further (by completing the square) as:

$$\left[\vec{\mathbf{p}} - \frac{1}{2} \left(\frac{\vec{\mathbf{t}} \cdot \vec{\mathbf{t}}}{\hat{\mathbf{u}} \cdot \vec{\mathbf{t}}} \right) \hat{\mathbf{u}} \right]^2 = \left[\frac{1}{2} \left(\frac{\vec{\mathbf{t}} \cdot \vec{\mathbf{t}}}{\hat{\mathbf{u}} \cdot \vec{\mathbf{t}}} \right) \right]^2 \tag{8}$$

For $r = \frac{1}{2} \left(\frac{\vec{\mathbf{t}} \cdot \vec{\mathbf{t}}}{\hat{\mathbf{u}} \cdot \vec{\mathbf{t}}} \right)$ and $\vec{\mathbf{c}} = r\ \hat{\mathbf{u}}$, we obtain the familiar equation of a circle:

$$(\vec{\mathbf{p}} - \vec{\mathbf{c}})^2 = r^2 \tag{9}$$

The above equation defines a set of points P in the 2D universe that lie on a circle. $\vec{\mathbf{c}}$ defines the center of this circle with respect to the center of the eye. Recall that $\hat{\mathbf{u}}$ is a vector perpendicular to the velocity of the observer $\vec{\mathbf{v}}$; this means that the center of the circle lies in a direction perpendicular to the direction of movement. This circle, corresponding to zero flow in the retina, is depicted by the solid line in Fig. 2. The circle passes through the target point of fixation and through the center of the eye. All points on the circle, including these two points, behave in the same way: momentarily, they are stationary. Furthermore, the points *within* this circle all move in the same

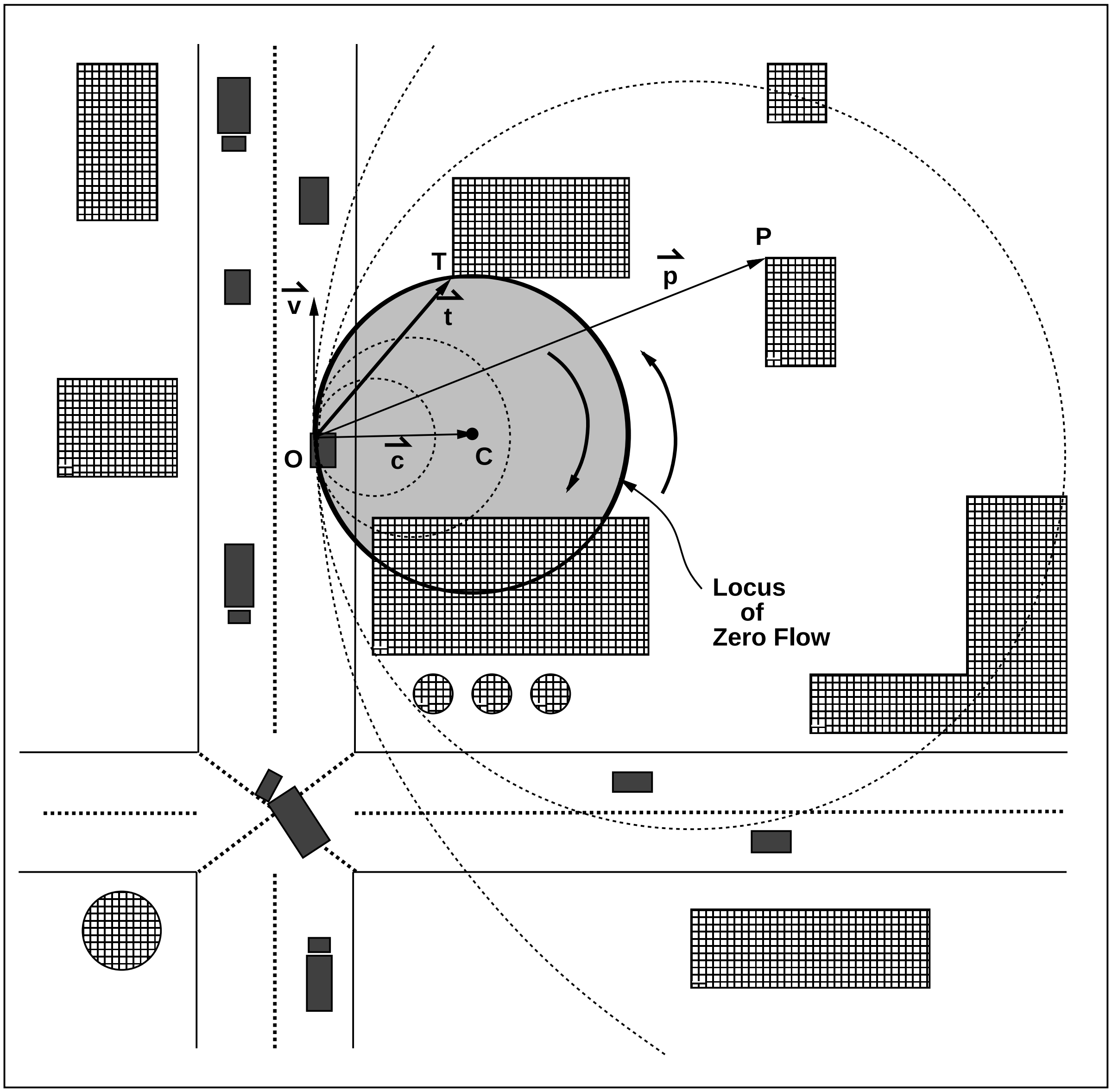


Figure 2: The level sets of retinal flow in a 2D universe. This scene depicts a traffic intersection, where the observer is moving along the road with velocity $\vec{\mathbf{v}}$, and fixates on a corner of a building T. O is the center of the eye of the observer, and P is an example point in the universe, the retinal velocity of which is being calculated. The points with zero retinal flow lie on the solid circle with center C.

direction on the retina, whereas the points *outside* of this circle move in the opposite direction.[2]

A similar analysis can be performed for any other value of retinal flow besides the zero flow. For any such value, the result corresponds to a circle of points in the 2D universe; the radius of the circle varies depending on the particular value chosen. Sample circles which correspond to points with the equal retinal flow are shown as dotted curves in Fig. 2. Note that the centers of all such circles lie on a straight line.

An interesting boundary case involves fixating straight ahead. In this case the direction of fixation coincides with the direction of observer movement. The resulting circle of zero flow has a radius tending to infinity, as depicted in Fig. 3. The circle of points corresponding to zero retinal flow opens up to a half plane; points within this half plane move in one direction, while points on the remaining half plane move in the opposite direction. This result fits intuition in the sense that when looking and moving straight ahead, points on the left half of the visual field move left, and points on the right half move right.

3 Retinal Flow in a Rigid 3D Universe

The case of moving and fixating in a 3D universe is clearly more complicated than the 2D case. However, the 3D case can be elegantly decomposed into two modules: one involving the retinal flow just as in the 2D case, and the other involving a new component.

3.1 Calculating Retinal Flow

In a 3D universe, the eye corresponds to a sphere (rather than a circle) with a center O (in Fig. 4) and the retina involves a hemisphere (rather than a semicircle). Recall that

[2] The retinal flow associated with each point in the world forms a vector field or dynamical system [1] with a *separatice* corresponding to the circle of zero flow which separates the two regions of opposite flow. This points to a possible interesting connection between the present analysis and the dynamics of well known systems.

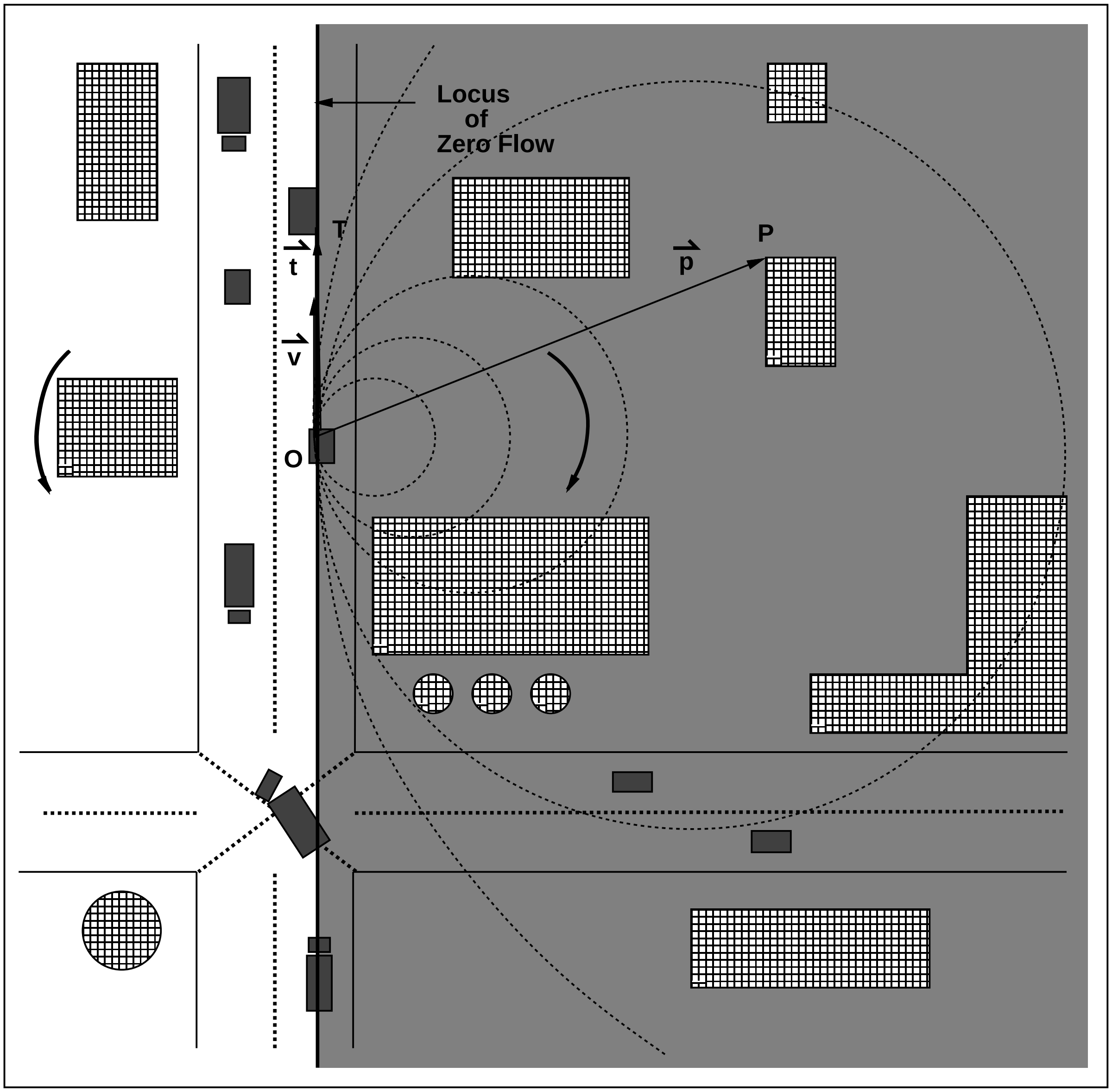


Figure 3: The special case of moving and fixating in the same direction; cf. Fig. 2 for explanation of symbols.

the center of the eye translates on a plane in the 2D universe, whereas in a 3D universe the center of the eye translates in any direction. As in the 2D case, the eye may also rotate in order to fixate on a target. However, in 3D the rotation that accomplishes fixation is not unique, since the eye can be rotated about various axes to correct for retinal movement of the target.[3]

In order to make the problem manageable, we constrain the way in which the eye can rotate in order to fixate. The constraint we impose is that the axis about which the eye rotates is always perpendicular to the direction of gaze; i.e., the possible rotation axes lie on a plane. Although this is an arbitrary constraint, the physiology of the eye suggests that a similar constraint operates in humans (involving the so–called Listing's Plane [6]). Furthermore, this particular formulation of the constraint allows us to decompose the retinal flow into two components.

In the 2D case the retinal flow consisted of a single measure, which expressed how fast the projections of points moved along the semicircular retina. In the 3D case, a new dimension is added, which gives rise to an additional measure. The two components of the 3D case can be thought of as encoding the velocity of points along the x and y axes of a coordinate system imposed on a flattened retina. In the 2D case the velocity along the equivalent of just the x–axis was defined.

In order to represent the two components of retinal flow in a 3D universe we impose a grid of longitudes and latitudes on the hemispherical retina. These longitudes and latitudes are comparable to the standard grid used to specify coordinates on the earth. In the present analysis, we wish to fix this grid on the retina in a such a way that the equatorial latitude the center of the eye, the target point (that is being fixated on), and the direction of movement all fall on the plane of the equator. This plane will be referred to as the *critical plane* (cf. Fig. 4). The other latitudes are semicircles on the

[3]For any fixating rotation an additional rotation about the optical axis can be added without loss of fixation. The family of such fixation rotation axes are obtained by varying the amount of this additional rotation.

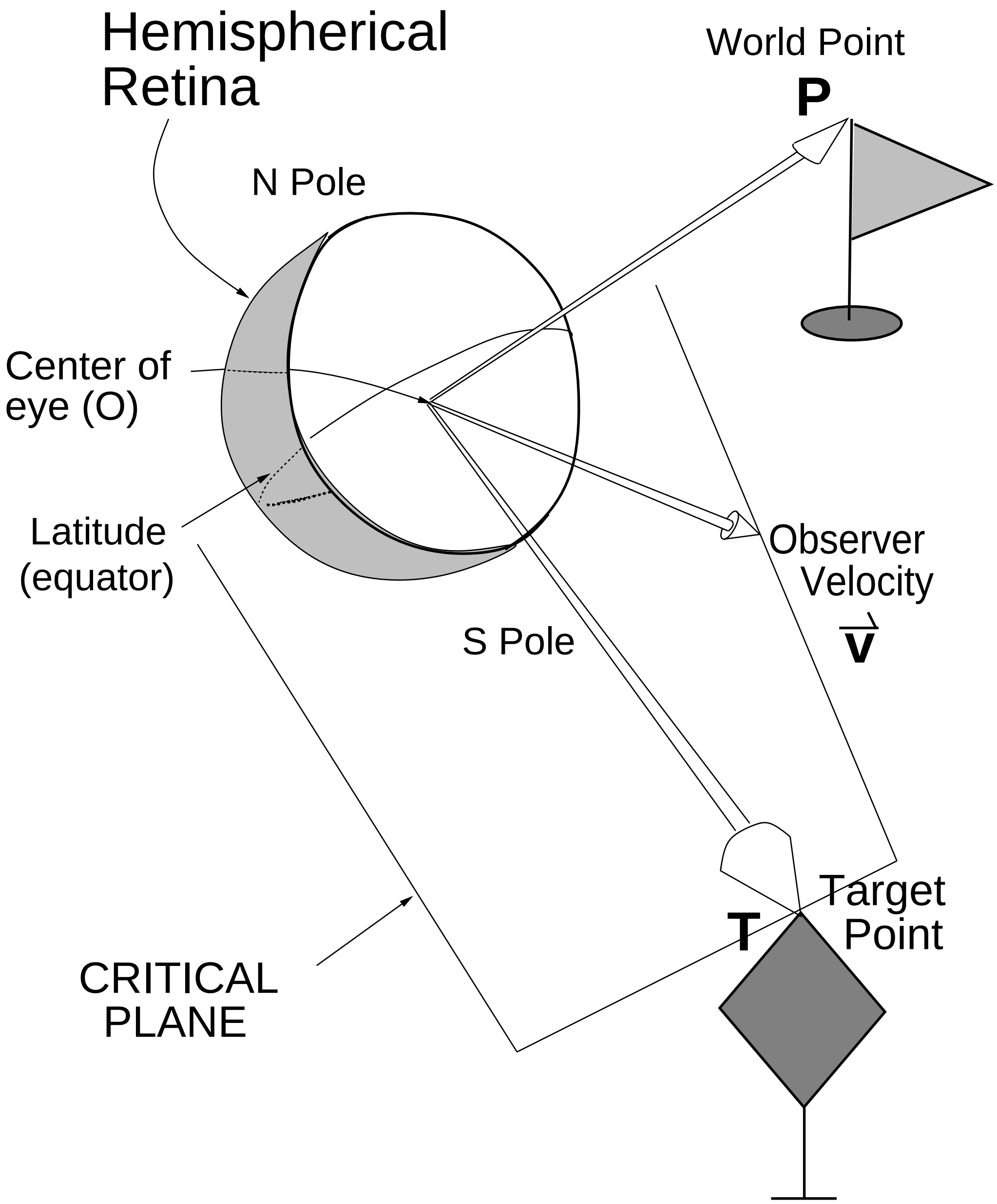


Figure 4: Model of the eye in a 3D universe. The critical plane contains the center of the eye, the target point and the velocity direction. The eye rotates about an axis passing through the North and South Poles in order fixate.

retina lying on planes parallel to the critical plane. The longitudes are semicircles[4] on the retina starting at the North Pole and ending at the South Pole. As is standard, the North Pole, center of the globe (eye) and the South Pole lie on a straight line perpendicular to the equatorial (critical) plane. Note that this line is the axis about which the eye rotates in order to fixate.

The retinal flow of points lying on the critical plane is identical to the retinal flow of points in the 2D universe, and was given by Equation 3. The points lying on the critical plane are special in the sense that their retinal flow consists of *latitudinal flow* (i.e. retinal flow along the latitude). In general, the retinal flow of points has two components: latitudinal flow and *longitudinal flow* (flow along the longitude).

Equation 10 defines the complete retinal flow of any point P in a 3D universe when the observer moves with velocity $\vec{\mathbf{v}}$ and fixates on a target T:[5]

$$\vec{\omega} \quad = \quad \frac{\vec{\mathbf{v}} \times \vec{\mathbf{p}}}{|\vec{\mathbf{p}}|^2} - \frac{\vec{\mathbf{v}} \times \vec{\mathbf{t}}}{|\vec{\mathbf{t}}|^2} \tag{10}$$

In order to obtain the latitudinal and longitudinal flows from Equation 10, let us decompose the observer's velocity into two components: (i) $\vec{\mathbf{v}}'$, which is the projection of $\vec{\mathbf{v}}$ onto a plane containing P, the North Pole and the center of the eye (this plane will be referred to the *longitudinal plane*) and (ii) $\vec{\mathbf{v}}''$ perpendicular to the longitudinal plane, such that:

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}' + \vec{\mathbf{v}}'' \tag{11}$$

Substituting for $\vec{\mathbf{v}}$ in Equation 10 gives rise to three terms, two of which correspond to vectors that lie on the longitudinal plane and which give rise to only latitudinal flow. The third term is a vector that is perpendicular to the longitudinal plane and thus gives rise to purely longitudinal flow. After substituting for $\vec{\mathbf{v}}$ (Equation 11) in

[4] These semicircles are half of the so–called great circles.

[5] Equation 3 was a special case of Equation 10 in that **om$\vec{\text{e}}$ga** always pointed in one direction.

Equation 10 the *latitudinal flow of a point P* is given by:

$$\vec{\omega}_x = \frac{\vec{\mathbf{v}}'' \times \vec{\mathbf{p}}}{|\vec{\mathbf{p}}|^2} - \frac{\vec{\mathbf{v}} \times \vec{\mathbf{t}}}{|\vec{\mathbf{t}}|^2} \tag{12}$$

The *longitudinal flow of a point P* consisting of the remaining term in Equation 10 is:

$$\vec{\omega}_y = \frac{\vec{\mathbf{v}}' \times \vec{\mathbf{p}}}{|\vec{\mathbf{p}}|^2} \tag{13}$$

As Equation 13 shows, the amount of longitudinal flow of a point P depends only on the movement of the observer. The rotation of fixating eye does not produce any longitudinal flow; rather the rotation only affects the latitudinal flow. The direct relationship between the observer motion and longitudinal flow will be exploited below in Section 4; this relationship is a result of the particular choice of the latitudes and longitudes in this formalization.

3.2 Points with Zero Longitudinal Flow in the 3D Universe

The points with zero flow are interesting boundary cases that enhance an intuitive understanding of how flow is related to positions in the world. The points that are momentarily stationary on the retina are those that have neither latitudinal nor longitudinal flow. However, in general, the points of zero latitudinal flow are difficult to isolate. For points on the critical plane, the latitudinal flow is zero along a circle defined as in the 2D case (cf. Equation 9). For other points, zero latitudinal flow is a subject of future study.

In the case of longitudinal flow, all points with zero flow lie on either of two planes (cf. Figure 5). One such plane is the critical plane. All points on the critical plane project onto a single latitude, the equator. Any movement along or rotation perpendicular to the critical plane will not induce the points to change latitudes; i.e. the points remain on the equator regardless of observer motion or fixation. Since the points remain on the same latitude, they have no longitudinal flow.[6] The second plane with zero longitudinal flow is perpendicular to the direction of the observer's velocity $\vec{\mathbf{v}}$ and passes through the center of the eye. For points on this plane, the modified velocity ($\vec{\mathbf{v}}'$ in Equation 13) is zero, resulting in no longitudinal flow.

4 A Systematic Pattern at the Periphery

When a target is fixated on by rotating as we have defined earlier, the retinal *periphery* has a unique invariant property: it is the only longitude that is a constant across all possible rotations of the eye. This makes the retinal periphery an interesting location to look at for certain visual tasks.

Let us consider a situation where the moving observer has to decide whether he is heading towards the fixated target or not. In the former case, the observer will *hit* the target if he continues in the current direction of motion, whereas in the latter case the observer will *miss* the fixated target.[7] Such an ability to predict *hit* and *miss* situations (assuming that the current direction of movement is maintained, and assuming that the target does not move) should turn out to be useful in navigation.

An analysis of the retinal flow at the periphery of our model eye indicates that the characteristics of the longitudinal flow distinguish *hit* from *miss* situations (in the sense described above). The magnitude of the longitudinal flow on the periphery (from a point P in the world) depends on how far the point is from the eye as well as how fast the observer moves. On the other hand, the direction – or sign – of the longitudinal flow *within a quadrant of the retina* only depends on whether the observer is heading towards the target or not. The quadrants are defined by the location of the North and South Poles on the periphery.

[6]For points on the critical plane note that $\vec{\mathbf{v}}'$ and $\vec{\mathbf{p}}$ are in the same direction, leading to a zero cross–product term in equation 13.

[7]This holds for an idealized case where the observer is a point. For a practical situation where the observer has finite dimensions, determining a *hit* situation is more involved.

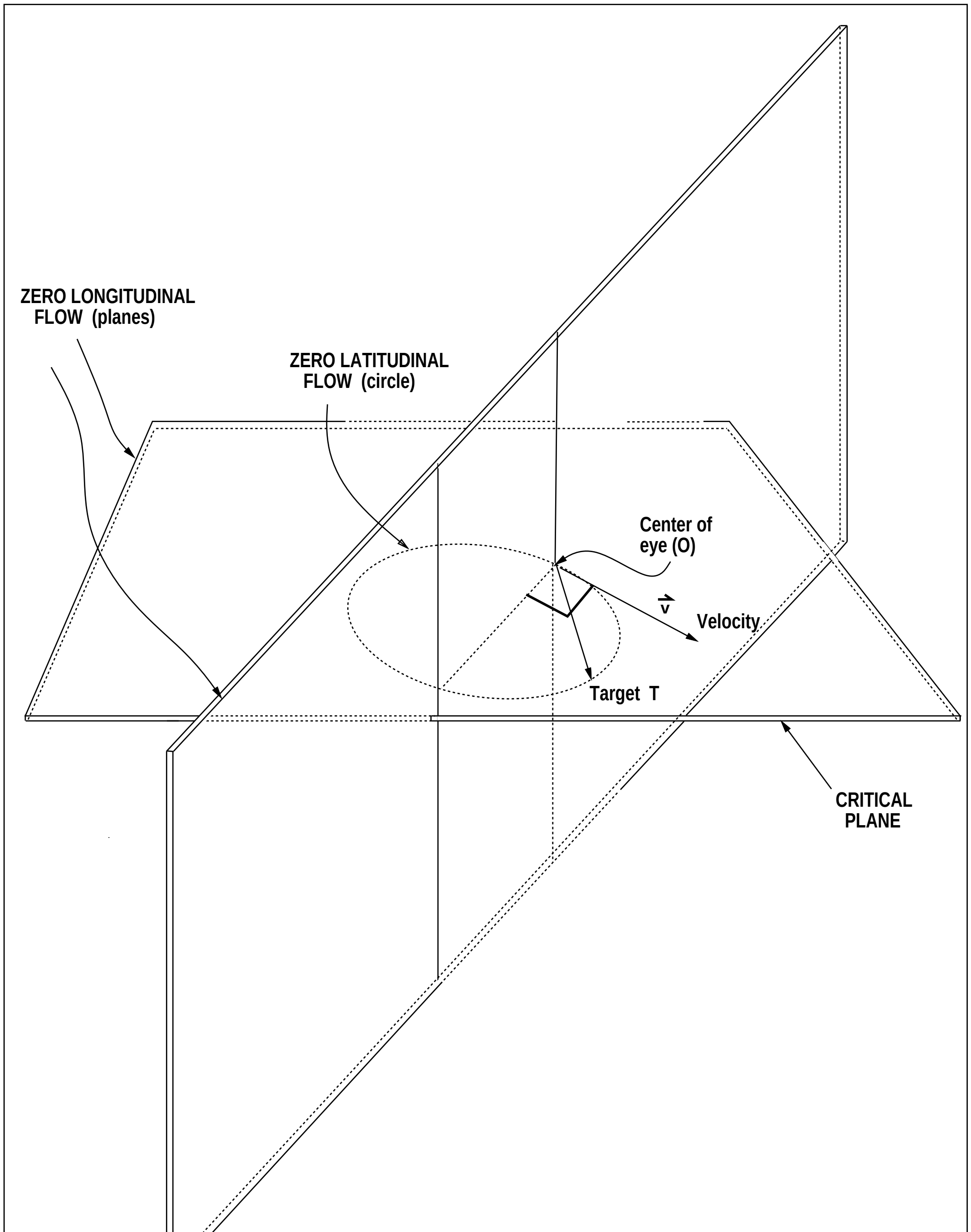


Figure 5: Points with zero longitudinal flow lie on two planes.

In the *miss* situation, the sign of the longitudinal flow (sign of $\vec{\omega}_y$ in Equation 13) switches exactly four times as one traces along the periphery (i.e., once for each quadrant). Figure 6(a) indicates the sign of the longitudinal flow along the periphery as well as across the entire retina for the most extreme *miss* situation. Even in less extreme cases the sign changes four times.

As we continuously move from a *miss* situation towards a *hit* situation, the velocity directions gets closer to the direction of gaze. When the velocity direction and the direction of gaze coincide, one of the planes with zero longitudinal flow (in Fig. 5) ends up containing the entire periphery. Thus the longitudinal flow of all the points on the periphery is zero in the *hit* situation. However, the zero flow holds only for an infinitely thin periphery. Immediately adjacent to this infinitely thin periphery, a different picture emerges. As one traces the longitudinal flow along the immediate neighbor of the periphery, the sign of the longitudinal flow changes *exactly twice*. Figure 6(b) shows the sign of the longitudinal flow across the entire retina in the *hit* situation.

Thus, in theory, the infinitely thin line of the periphery will contain the four–way change for the *miss* situation and zero flow for the *hit* situation. However, in practice, the observable difference at a periphery with finite thickness involves a four–way change in the *miss* situation and a two–way change in the *hit* situation. The practically observable situation is illustrated in Fig. 7.

5 A Simulated Experiment

In a simulated experiment we attempted to obtain the sign of the longitudinal flow on the retina as in Fig. 6. The number of times the sign changes along the periphery (two vs. four times) allows us to distinguish between the *hit* and the *miss* situations.

In the simulation we created a rigid world consisting of 1024 points. The points were uniformly and randomly distributed in all directions around the initial position of the eye. There were more points closer to the eye than farther away[8] which gave

[8]The distance (d) of the points from the eye varied randomly between $2ft$ and infinity, such that

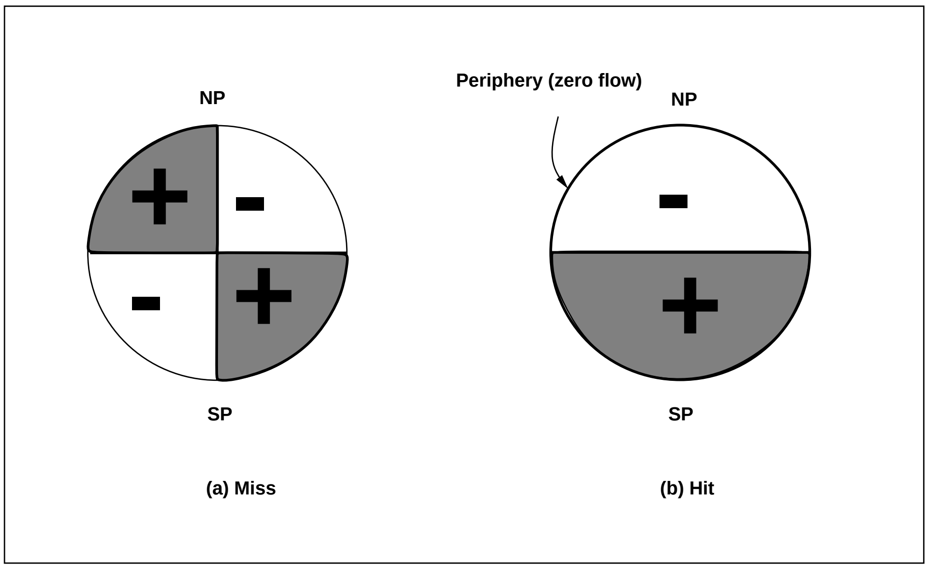


Figure 6: Sign of longitudinal flow on the hemispherical retina as viewed from the direction of the target T.

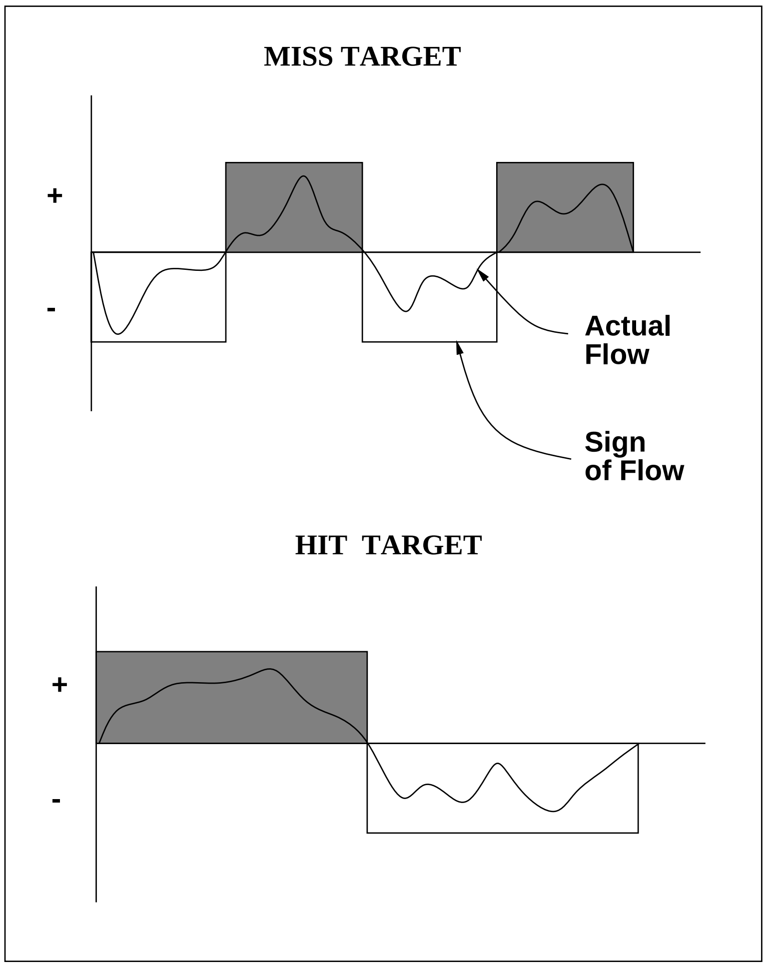


Figure 7: The sign of the retinal flow at the periphery. (See text for description of "at the periphery".)

rise to a natural situation where further away points tend to be occluded. Due to the hemispheric nature of the retina only those points in front of the eye were projected on the retina; the visual field was 180° along any diameter of the retina.

Three images of the retina were generated while viewing the retina from the direction of the target T. The images contained the projections of points in the world onto the retina. The target on which the eye fixated was located $3ft$ directly ahead of the initial position of the eye. The three images were: (i) an image from the initial position of the eye, (ii) an image after a sideways movement of $0.25ft$ (*miss* situation) and (iii) an image after a movement of $0.25ft$ towards the target (*hit* situation) from the initial position. The two image sequences (each consisting of the first image and either of the second or third image) are shown in Fig. 8. These image sequences were the result of the generation stage of the simulation.

In order to test the sign of the longitudinal flow on the retina, the optical flow in both image sequences was first obtained, using a standard algorithm of Simoncelli [4]. The longitudinal flow was then extracted from the total optical flow. [9]

Fig. 9 shows the sign of the longitudinal flow across the entire retina for the two sequences. The longitudinal flow in one direction is shown in white, and in the opposite direction in black. For the *hit* situation these regions approximately divide the retina into two halves – shown on the left in Fig. 9 – while in the *miss* situation the retina is divided into quarters, shown on the right. Each row corresponds to a different range of values of longitudinal flow that was ignored (shown grey in figure).

Although the overall pattern of the sign of the longitudinal flow on the simulated retina is in agreement with the theoretical prediction (cf. Fig. 6), there are certain regions on the retina that have an unexpected sign. Whether this is due to imperfect optical flow, low resolution of images or other factors remains to be determined.

$\frac{1}{d}$ was uniformly distributed in the range $[0, 0.5]$.

[9] The optical flow contained both the latitudinal and longitudinal flow. For the purposes of this demonstration only the longitudinal flow was needed. Simoncelli's optical flow algorithm could be modified to obtain just the longitudinal flow.

Figure 8: Image sequences depicting the retina for the *hit* situation (left column) and the *miss* situation (right column). The first image is common to both sequences.

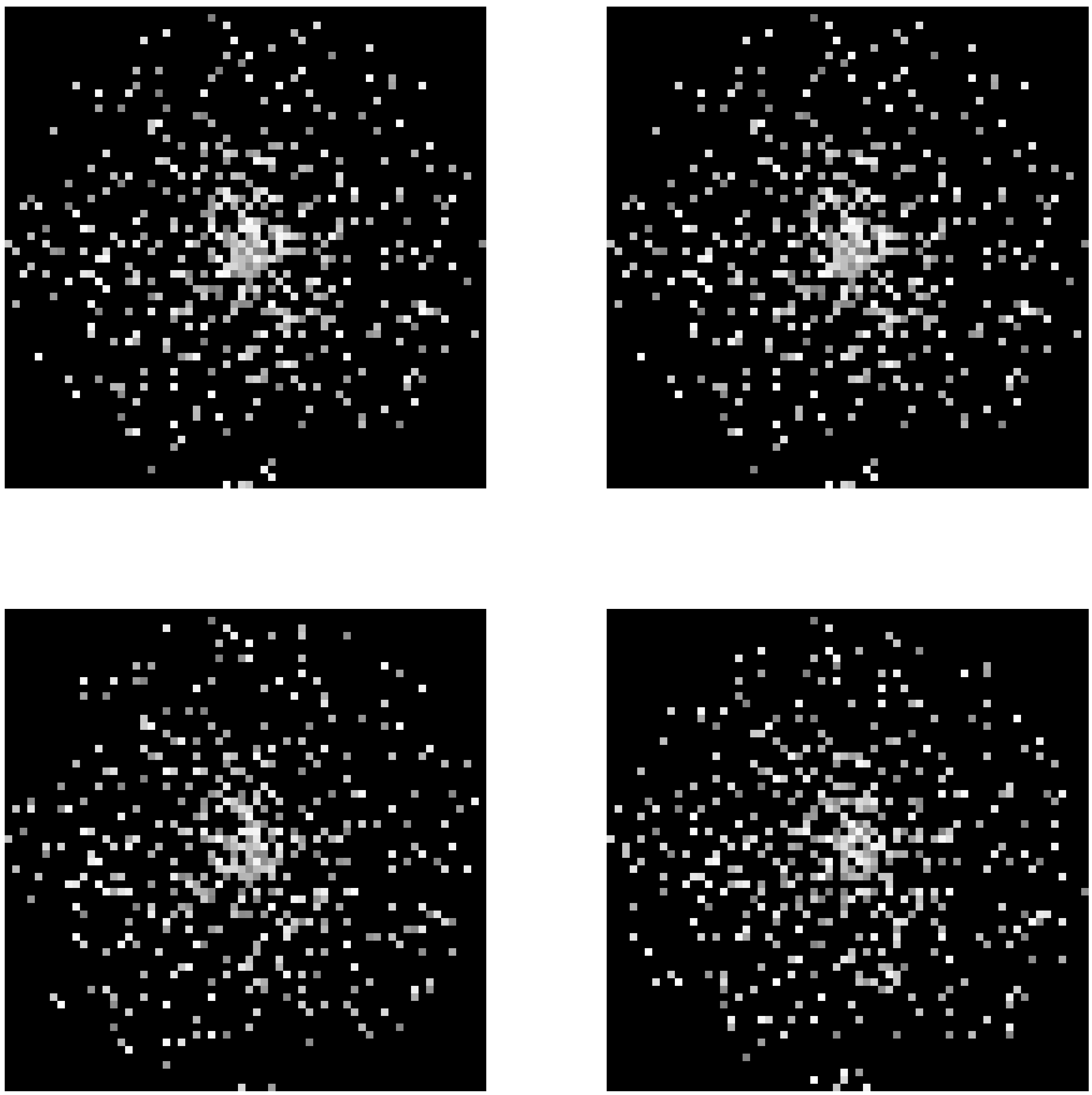
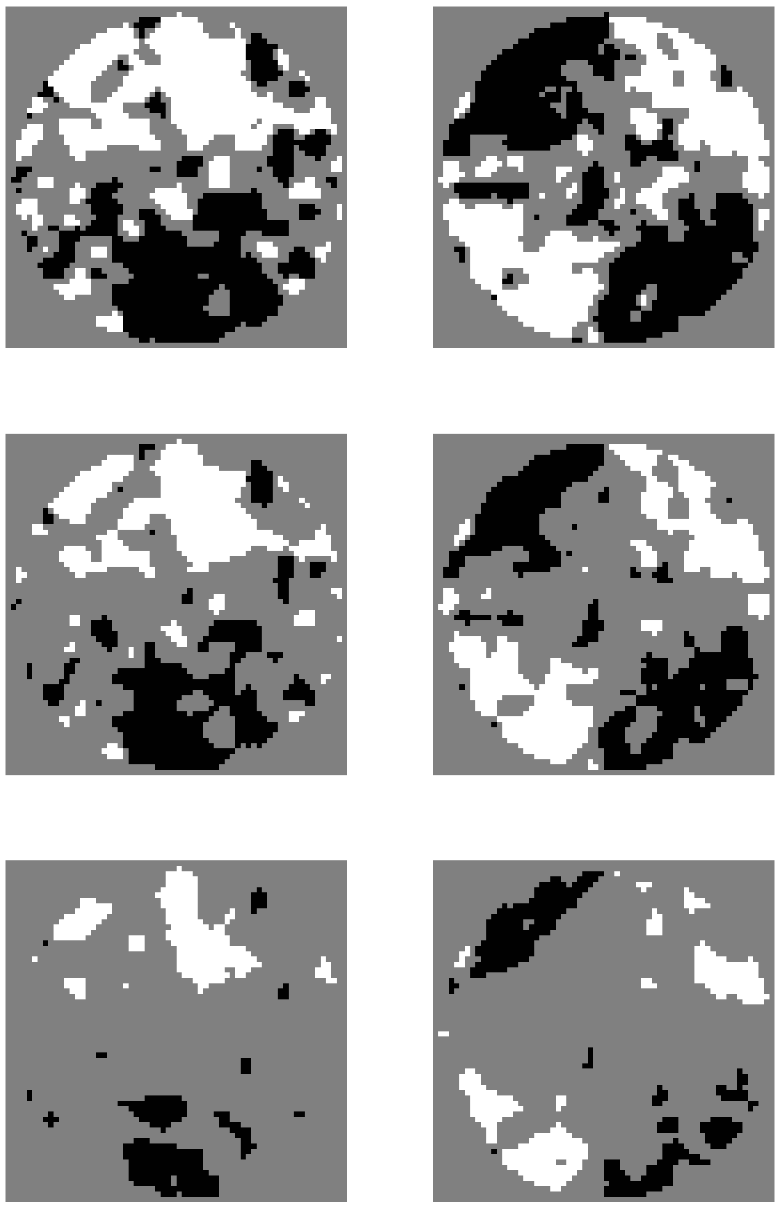


Figure 9: Simulated Experiment: Sign of the Longitudinal Flow for *hit* (left) and *miss* (right) situations. In the first row the points with less that 0.125 pixels of longitudinal flow were ignored. The comparable figure for the second row was 0.25 pixels and for the third row was 0.5 pixels.



6 Conclusion

In this paper we have considered the effect of fixation on optical flow on a hemispherical retina. The exact relationship between retinal flow, the movement of the observer and the geometry of the physical world has been captured in a systematic way, both for a 2D universe and for a 3D universe.

The theoretical analysis and a simulated experiment reveal that the information along the periphery of the retina appears to be sufficient for determining whether the observer will eventually hit a target if he continues moving in his current direction (assuming the target will not move). This is possible precisely because the observer actively fixates on the target while moving.

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