II. Bayes motion estimation
Visual motion processing

Perception

Physiology
Visual motion

- Physiology: “motion pathway” heavily studied; arguably the strongest extrastriate success story

- Perception: Human motion perception heavily studied. Humans are adept at tasks which require motion processing.

- Provides a rich source of visual information for prediction, depth perception, material properties, etc [Gibson, 1950]
“Aperture Problem”

Figure: Movshon, Adelson, Gizzi, Newsome, 1985
Intersection-of-constraints (IOC)

[Adelson & Movshon, 1982]
Visual motion ambiguity
Simple plaid perception = IOC

[Adelson & Movshon, 1982]
Simple plaid perception = IOC
IOC failure

[Stone et al. 1990]
Stone, Watson, Mulligan 1990

Perceived Direction Bias (degrees) vs. Log Contrast Ratio

Total contrast:
- 5%
- 10%
- 20%
- 40%

Log Contrast Ratio:
-4 -3 -2 -1 0 1 2 3 4 5 10 20 30 40

Perceived Direction Bias (degrees):
-5 -4 -3 -2 -1 0 1 2 3 4 5 10 20 30 40
The “Thompson effect”

Contrast affects perceived speed

[Thompson ‘82]
Helmholtz (1866)
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Perception is our best guess as to what is in the world, given our current sensory input and our prior experience
Helmholtz (1866)

Perception is our best guess as to what is in the world, given our current sensory input and our prior experience

=> Bayes
Some Bayesian perceptual models

- Shading/lighting [Kersten 90; Knill, Kersten, Yuille 96; Mamassian, Landy, Maloney 01]

- Motion [Simoncelli 93; Weiss et al. 02; Stocker & Simoncelli 06]

- Surface orientation [Bülthoff & Yuille 96; Saunders & Knill 01]

- Color constancy [Brainard & Freeman 97]

- Contours [Geisler, Perry, Super 01]

- Sensory-motor tasks [Körding & Wolpert 04]
Brightness Constancy

• Assume translational motion (locally)

• Differential approximation (Taylor series)

\[ \vec{\nabla}I \cdot \vec{v} + I_t = 0, \quad \vec{\nabla}I = [Ix, Iy] \]

• Insufficient constraint, so combine over a neighborhood (space and/or time):

\[ \min \sum (\vec{\nabla}I \cdot \vec{v} + I_t)^2 \]

[Fennema & Thompson ‘79; Horn and Schunck ‘81]
With noise...

• Additive Gaussian noise in temporal derivative:

\[ \nabla I \cdot \vec{v} + I_t = n \]

• Likelihood (combined over neighborhood):

\[ P(\nabla I, I_t|\vec{v}) \propto \exp[-\sum(\nabla I \cdot \vec{v} + I_t)^2/2\sigma^2] \]

[Simoncelli, Adelson, Heeger ‘91]
With prior...

• Simplest prior choice: Gaussian (preference for slow speeds)

\[ P(\vec{v}) \propto \exp\left[-||\vec{v}||/2\sigma_p^2\right] \]

• Posterior:

\[ P(\vec{v}|\nabla I, I_t) \propto \exp\left[-||\vec{v}||/2\sigma_p^2 - \sum(\nabla I \cdot \vec{v} + I_t)^2 / 2\sigma^2\right] \]

[Simoncelli, Adelson, Heeger '91]
Bayesian posteriors
Bayesian posteriors
Bayesian perception

world

\( v \)

noise!

measurement

\( m \)

memory

observer

\( \hat{v} \)

estimate
Bayesian perception

world

observer

measurement

estimate

noise!

memory

\[ P(m|v) \]

likelihood

probability

\[ v \quad m \]
Bayesian perception

\[ P(m|v) \]

\[ P(v) \]
Bayesian perception

world → measurement → estimate

P(m|v) P(v)

noise!

prior
Bayesian perception

\[ P(m|v) \times P(v) \sim P(v|m) \]
Bayesian perception

world

measurement

observer

\[ \hat{v} \]

\[ m \]

noise!

\[ v \]

\[ \text{prior} \]

\[ \text{estimate} \]

probability
Bayesian perception

world

measurement

observer

\[ \hat{v} \]

\[ v \]

\[ \text{noise!} \]

\[ \text{prior} \]

\[ \text{estimate} \]

\[ \text{probability} \]

\[ v \]

\[ \hat{v} \]
stimulus

idealization

model

[Simoncelli & Heeger, ARVO '92]
stimulus

idealization

model
Stone et al. 1990

[Simoncelli & Heeger, ARVO ‘92]
Ferrera & Wilson, 1991

Perceived Speed (relative to IOC) vs. Plaid angle (degrees)

- Subject 2
- Model
- Cosine

[Simoncelli & Heeger, ARVO '92]
Stone & Thompson, ‘90

max contrast 70%

max contrast 40%

Log contrast ratio

[Weiss, Simoncelli, Adelson, ‘02]
Bayesian motion model
Bayesian motion model

+ Theory: Optimal solution
  - unknown likelihood
  - unknown prior
Bayesian motion model

+ Theory: Optimal solution
  - unknown likelihood
  - unknown prior

+ Perception: Accounts for psychophysical data
  - qualitative
  - deterministic (what about response variability?)
Bayesian motion model

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+ Physiology: Seems loosely plausible...
  - but mechanism unspecified and non-unique
Bayesian motion model

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[Stocker & Simoncelli, NIPS*04 / Nature Neurosci 06]
Prior/likelihood from psychophysics

- Assume Gaussian likelihood, with contrast-dependent width
- Assume prior is smooth
- Assume MAP estimates (max posterior)
- Speed-matching and speed-discrimination data are sufficient to determine prior and
Which is faster?

Which is faster?
Effect increases with contrast ratio, decreases with speed

[Stocker & Simoncelli, ‘06]
Effect increases with contrast ratio, decreases with speed

[Stocker & Simoncelli, ‘06]
Bayesian perception

\[ P(m|v) \times P(v) \sim P(v|m) \]
Trial-to-trial variability

\[ \hat{v}(\tilde{m}) \quad \text{noise in } \tilde{m} \]

\[ \text{prior} \quad \text{posterior} \quad \text{likelihood} \]

\[ p(\hat{v}(\tilde{m}) \mid v) \]
stimulus

observer model

subject response

estimation stage
decision stage

prior

likelihood

p(v_1 | v)

m_1

m_2

v_1

v_2

P(\hat{v}_1 > \hat{v}_2)

"v1 seen faster"

psychometric function
Model accounts for perceptual data

![Graphs showing relative matching speed and relative threshold versus velocity. The graphs illustrate the relationship between velocity (\(v\) in deg s\(^{-1}\)) and relative matching speed or threshold, with parameters \(c_1\) and \(c_2\) indicated.]
Model comparison

- Weibull fit
- Coin-flipping model
- Subject

- New model (non-parametric)
- New model (semi-parametric)
- Weiss et al.
- Hürlimann et al.

(avg. probability of data)
Probability

Prior

Likelihood width

$V$ [deg s$^{-1}$]

$V$ [deg s$^{-1}$]

Contrast
Speed tuning in area MT is approximately constant in log(v)

- Maunsell & Van Essen 83
- also Nover et. al. 05
Area MT contrast-response function:

\[ r(c) = \alpha \frac{c^k}{c^k + c^k_{50}} + \beta \]

- Sclar et. al. 90
Area MT contrast-response function:

\[ r(c) = \alpha \frac{c^k}{c^k + c^{k}_{50}} + \beta \]

Likelihood width under Poisson variability:

\[ w(c) \propto \left( \frac{1}{r(c)} \right)^2 \]

- Sclar et. al. 90
Probability

Prior

<table>
<thead>
<tr>
<th>Speed [deg s⁻¹]</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>10⁻¹⁰</td>
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</table>

Likelihood width

<table>
<thead>
<tr>
<th>Speed [deg s⁻¹]</th>
<th>Contrast</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.6</td>
</tr>
<tr>
<td>10</td>
<td>0.6</td>
</tr>
</tbody>
</table>
Log-spaced tuning curves
Probability

Prior

Likelihood width

Speed [deg s$^{-1}$]

Log-spaced tuning curves

Contrast

Contrast-response + Poisson variability
Prior

Log-spaced tuning curves

Contrast-response + Poisson variability
Hypothesis 1: MT encodes likelihood

Responses are separable in speed and contrast
Prior is imposed on readout
Hypothesis 2: MT population encodes posterior

Each MT cell provides a “labelled line” for posterior at a particular velocity [Simoncelli, ‘03]

=> Speed and contrast are linked
Hypothesis 2a

Cell speed tuning depends on contrast

=> Should prefer higher speeds at lower contrast

Recent physiological evidence suggests not

[Priebe & Lisberger ‘05; Pack & Born ‘05; Krekelberg & Albright (unpublished)]
Hypothesis 2b

Responses are separable in speed and contrast
Contrast-response functions linked to speed tuning
Credits

• Reverse-engineered prior/likelihood: Alan Stocker

• Plaid motion modeling: Edward Adelson, David Heeger, Yair Weiss

• Physiological model: David Heeger