Image Statistics

space of all images

typical images
Statistical Image Models

Image Processing / Graphics:

• How compactly can we represent images?
• How easily can we detect (and remove) artifacts or distortions?
• Can we enhance images by increasing resolution or spatial extent?
• Can we synthesize realistic-looking images?

Theoretical Neurobiology:

• Do sensory neurons perform optimal decomposition of images?
• If so, how does the system learn this decomposition?
Loosely:

- Neural architecture optimized for ensemble of images
- Instantaneous pattern of responses represents relevant information about current image
Statistical Optimality

Two basic frameworks (with overlap):

- Bayes (prior)
- Efficient coding [Pam’s next lecture]

How do we test these??

- Experimental
- Theoretical
- Hybrid (!)
Building a statistical model

We want to look for statistical properties that provide constraints that are both strong and reliable.

- make measurements...
  - but measurements of what?
  - beware the curse of dimensionality
- make structural assumptions (e.g., translation-invariance, locality)
Pixels

Original image

Range: [0, 237]
Dims: [256, 256]

histogram

0 50 100 150 200 250
Pixels

Original image

Range: [0, 237]
Dims: [256, 256]

Equalized image

Range: [1.99, 238]
Dims: [256, 256]
Pixels
Pixel Correlations

**a.**

```
I(x,y)  I(x+1,y)  I(x+2,y)  I(x+4,y)
0      50        100       5
```

**b.**

```
Spatial separation  Correlation
5                   1.0
10                  0.9
15                  0.8
20                  0.7
25                  0.6
30                  0.5
35                  0.4
40                  0.3
```

CSH-02
Relationship between Covariance matrix and Fourier on board...
Principal Component Analysis (PCA)

Find linear transform (specifically, rotation and axis re-scaling) that transforms the covariance matrix to the identity.

Well-known eigenvalue/eigenvector solution

Assuming translation-invariance, Fourier transform suffices
Power spectra of natural images fall as $1/f^\alpha$, $\alpha \sim 2$.

[Ritterman ’52; DeRiugin ’56; Field ’87; Tolhurst ’92; Ruderman & Bialek ’94; etc]
Scale-invariance

If $\tilde{g}(\omega) = A\omega^p$, then $\tilde{g}(s\omega) = As^p\omega^p$.

i.e., for power-law spectrum, shape is preserved under scaling.

Conversely, if shape is preserved under scaling, spectrum must follow a power law!
Maximum Entropy

The distribution over response $r$ with maximum entropy subject to a constraint of the form:

$$\mathcal{E}(f(r)) = c$$

is

$$\mathcal{P}(r) \propto \exp(-\lambda(c)f(r))$$

Examples:

- $f(r) = r^2$
- $f(r) = |r|$
Multi-dimensional Gaussians

- Characterized by mean (vector) and covariance (matrix)
- Remains Gaussian under linear transformation of space
- Conditionals (slices) and marginals (projections) are Gaussian
- Unique property: separable products are spherically symmetric
- Central limit theorem: sums of i.i.d. random variables become Gaussian
- Heisenberg: Fourier transform is Gaussian, and minimizes variance product
Most image processing engineering is based on this “classic” model
Synthesis

\[ P(c) \rightarrow \frac{1}{f^2} \rightarrow F^{-1} \rightarrow \text{Image} \]
Whitening
Windowed PCA

[Hancock et al, '91]